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Mathematical modeling of the running performances in endurance exercises: comparison of the models of Kennelly and Péronnet-Thibaut for World records and elite endurance runners.

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Abstract *In 1906 Kennelly proposed a model for the relations between time (t_{lim}) versus speed ($S = kt_{lim}^{g-1}$) or distance ($D_{lim} = kt_{lim}^{g}$). In 1989, Péronnet and Thibault proposed a logarithmic model based on the decrease in the fractional use of maximal aerobic speed (MAS) for t_{lim} beyond the exhaution time (t_{MAS}) corresponding to MAS [$S/MAS = C – EI \ln(t_{lim}/t_{MAS})$] where C was equal to 100 and EI was an endurance index. Both models can accurately describe the relationships between S and t_{lim} for the World records and the performances of elite endurance runners. Exponent g and EI are linearly correlated ($r > 0.999$) which confirms that g can be considered as an estimator of endurance ability in runners. The effect of the value of t_{MAS} (assumed to be equal to 7 min in Péronnet-Thibault model) on EI should be more important in low-level endurance runners than in elite runners. Nevertheless, the logarithmic model is interesting because the effect of t_{MAS} on EI is relatively small when compared with the range of EI. In theory, the best performances of low-level runners cannot be described by both models. Therefore, it would be interesting to study what is the best model.*

Keywords: *Aouita, Gebrselassie, Nurmi, Radcliffe, Virén, Zatopek.*

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I. Introduction

Many models of running records have been proposed since the beginning of the 20th century [1]. Empirical and descriptive models were first proposed [2] before more recent models based on biomechanics and physiology [3, 4, 5, 6, 7, 8, 9].

1.1.1 Power law model (Kennelly)

In 1906, Kennelly [2] studied the relationship between running speed (S) and the time of the world records (t_{lim}) and proposed a power law:

$$D_{lim} = k\, t_{lim}^{\ g} \qquad \text{Eq.1}$$

where k is a constant and g an exponent. This power law between distance and time corresponds to a power law between time and speed (S):

$$S = k/t_{lim}^{\ b} = k\, t_{lim}^{\ -b} = k\, t_{lim}^{\ g-1} \qquad \text{Eq.2}$$

where b is an exponent equal to 1 - g.

Recently, Kennelly's model has been applied the best performances of elite endurance runners [10].

1.2 Logarithmic model (Péronnet-Thibault)

In 1989, Péronnet and Thibault [9] poposed a model that took into account the contributions of aerobic and anaerobic metabolism to total energy output in function of the duration of the race. A runner is only capable of sustaining his maximal aerobic power (MAP) for a finite period of time (t_{MAS}). The inertia of the aerobic metabolism at the beginning of the exercise was also included in the model. When exercise duration is longer than t_{MAS}, B was equal to:

$$B = (MAP – BMR) + E \ln(t_{lim}/t_{MAS}).$$

Péronnet and Thibault proposed the slope of the relationship between the natural logarithm of running duration and the fractional utilization of MAS as an index of endurance capability.

$$100\ S/MAS = C - EI\ \ln(t_{lim}/t_{MAS}) \quad \text{Eq. 3}$$

where C was a constant close to 100 and EI an endurance index.

This endurance index was significantly related (r = 0.853) to ventilatory threshold, expressed as a percentage of MAS, in a group of marathon runners [11]. The lower the absolute value of EI, the higher the endurance capacity is assumed to be. For example, endurance indexes computed from personal best performances were equal to 8.14 for Ryun, an elite middle-distance runner and 4.07, for Derek Clayton, an elite long distance runner.

In the present study, the model of Péronnet and Thibault was modified as following:

$$S/MAS = 1 - EI'\ \ln(t_{lim} / t_{MAS})$$

where EI' = EI/100

$$S = MAS - MAS\ EI'\ \ln(t_{lim} / t_{MAS}) = MAS - E\ \ln(t_{lim} / t_{MAS}) \quad \text{Eq.4}$$

where E = MAS*EI/100

1.3 Aims of the present study

The models of Kennelly and Péronnet-Thibault have been separately applied to world records [9, 12] but not compared. In the present study, both models are applied not only to world record in men (WRm) and (WRw) but also to the individual performances of elite endurance runners. Moreover, the relationships between the indices of endurance (exponent g , E and EI) are studied.

II. Methods

2.1 World records and individual performances of elite endurance runners

The modeling of running performances assumes that the running data correspond to the maximal performance for each distance. The best performances of world elite runners generally correspond to the results of many competitions against other elite runners and the motivation is probably optimal during these races. The first studies on the modeling of running performances were based on the world records. Now, the best performances of endurance runners who ran on different distances and were the best of their times, can be found in Internet (Wikipedia…). Therefore, it is possible to study the characteristics of the different models which have been proposed for the endurance exercises not only with the world records but also with the best performances of elite endurance runners.

2.2 relationships between S and t_{lim} for the world records

The relationships between S and t_{lim} for the world records (2016) of the running performances on track are computed from data in table 1.

Table 1: World records in seconds in men and women (distances and one hour record)

Distances (m) of World records

	1000	*1500*	*1609*	*2000*	*3000*	*5000*	*10000*	*20000*	*25000*	*30000*	one hour
Men	132	206	223	285	441	757	1578	3386	4345	5207	21285 m
Women	149	230	253	324	486	851	1757	3927	5226	6350	18517 m

2.3 relationships between S and t_{lim} for elite endurance runners

The best times of elite endurance runners who participated to international competitions over 3000, 50000 and 10000 m are presented in table 2 with the computed values (3) of exponent g in Kennelly's model. The data of the female elite runner P. Radcliffe who performed the same distances are also presented.

Distances (m)

	3000	*5000*	*10000*
Nurmi	500	868	1806
Zatopek	488	837	1734
Virén	463	796	1658
Aouita	449	778	1646
Gebrselassie	445	759	1583
Radcliffe	502	869	1801

Table 2: individual performances of elite endurance runners.

2.4 Computation of exponent g

The individual power laws between S and t_{lim} are determined by computing the regressions between the natural logarithms of S and t_{lim}:

$$\ln(S) = \alpha + \beta \ln(t_{lim}) = \ln(k) - b \ln(t_{lim}) \quad \text{and} \quad k = e^{\ln(k)} \qquad \text{Eq.8}$$

The value of exponent g was equal to 1 – b

2.5 Range of t_{lim} used in the comparison of exponent g and endurance index EI

The study of the relationship between t_{lim} and S in World records must not include the distances shorter than 1000 m. Indeed, it has been showed [12] that the slopes of the regressions between velocity and the logarithm of time was different for races under and beyond 150-180 s, that is, distances under and over 1000 m. It was suggested that this difference was the expression of the switch from the anaerobic metabolism to the aerobic metabolism. Similarly, in the model of Péronnet-Thibault, the data corresponding to tlim < 7 min, which corresponds to distances shorter than 3000 m, should not be included in the computation of EI.

III. Results

3.1.1 Kennelly's model applied to World records

The application of Kennelly's to the World record is presented in Fig. 1A.

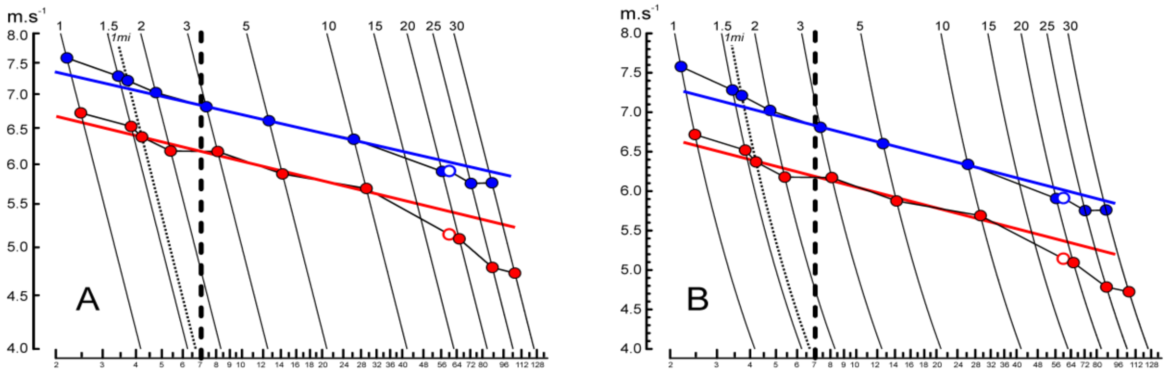

Fig. 1: in A, Kennelly'model applied to the running world record for men (blue data) and women (red data). In A, both abcissa (time in minute) and ordinate (running speed in $m.s^{-1}$) scales are logarithmic. In B, Péronnet-Thibault model applied to the running world records. The only abcissa scale is logarithmic. Oblique black lines (A) and curves (B) correspond to the time-speed relationships for the different distances. The dotted lines correspond to 1 mile (1609m). Vertical dashed lines correspond to 7 min (420 s).

For all the distances (1000-30000 m, n = 11) the relationships between speed (S) and time (t_{lim}), computed fom the regressions between the natural logarithms of S and t_{lim}, are:

$S = 10.73\ t_{lim}^{-0.0732} = 10.73\ t_{lim}^{g-1} = 10.73\ t_{lim}^{0.927-1}$ r = 0.997 in men

g = 0.927

$S = 10.58\ t_{lim}^{-0.0893} = 10.58\ t_{lim}^{g-1} = 10.58\ t_{lim}^{0.911-1}$ r = 0.985 in women

g = 0.911

For 3000-10000 m (n = 3):

$S = 9.56\ t_{lim}^{-0.0559} = 9.56\ t_{lim}^{g-1} = 9.56\ t_{lim}^{0.944-1}$ r > 0.999 in men

g = 0.944

$S = 9.03\ t_{lim}^{-0.0623} = 9.03\ t_{lim}^{g-1} = 9.03\ t_{lim}^{0.938-1}$ r = 0.981 in women

g = 0.938

3.1.2 Kennelly's model applied to the individual performances of elite endurance runners

The values of exponent g of elite endurance runners are presented in table 3.

	Exponent g
Nurmi	0.938
Zatopek	0.950
Virén	0.944
Aouita	0.927
Gebrselassie	0.948
Radcliffe	0.943

Table 3: values of exponent g computed from 3000 to 10000 m in elite endurance runners.

3.2.1. Application of the Péronnet-Thibault model to world records

The application of the model of Péronnet-Thibault to the World record is presented in Fig. 1B. For all the distances (from1000 to30000 m, n = 11), the relationships between speed (S) and time (t_{lim}), computed fom the regressions between S and the natural logarithm of t_{lim} (4) with t_{MAS} equal to 420, are:

$S = 6.917 - 0.479 \ln (t_{lim}/420)$ $r = 0.996$ in men
$S/MAS = 1 – 0.0692 \ln (t_{lim}/420)$

$S = 6.187 - 0.505 \ln (t_{lim}/420)$ $r = 0.989$ in women
$S/MAS = 1 – 0.0816 \ln (t_{lim}/420)$

For 3000-10000 m (n = 3):
$S = 6.823 - 0.367 \ln (t_{lim}/420)$ $r > 0.999$ in men
$S/MAS = 1 – 0.0538 \ln (t_{lim}/420)$
$EI = 5.38$
$S = 6.193 - 0.369 \ln (t_{lim}/420)$ $r = 0.978$ in women
$S/MAS = 1 – 0.0596 \ln (t_{lim}/420)$
$EI = 5.96$

3.2.2 Péronnet-Thibault model applied to the individual performances of elite runners

The individual endurance index can be studied in endurance runners (table 3) who were the best of their time and ran on 3,000, 5,000 and 10,000 m distances.

	G	E	MAS_{6min}	EI_{6min}	MAS_{7min}	EI_{7min}	MAS_{8min}	EI_{8min}
Nurmi	0.938	0.357	6.1	5.85	6.05	5.9	6.00	5.95
Zatopek	0.950	0.299	6.23	4.80	6.19	4.83	6.15	4.86
Virén	0.944	0.351	6.56	5.35	6.51	5.39	6.46	5.43
Aouita	0.927	0.467	6.78	6.88	6.71	6.96	6.65	7.02
Gebrselassie	0.948	0.337	6.82	4.94	6.77	4.98	6.73	5.01
Radcliffe	0.943	0.329	6.07	5.42	6.02	5.47	5.98	5.51
WRm	0.944	0.367	6.88	5.34	6.82	5.38	6.77	5.42
WRw	0.938	0.369	6.25	5.90	6.19	5.96	6.14	6.01

Table 4: values of exponent g (Kennelly model), E and the estimations of MAS ($MAS_{6min,}$ $MAS_{7min,}$ MAS_{8min}) and EI ($E_{6min,}$ $E_{7min,}$ E_{8min}) for different values of t_{MAS} (6, 7 or 8 min), computed from performances over 3000, 5000 and 10000 m. The last rows (WRm and WRw) correspond to E, MAS and EI computed from the values of the World records in men and women.

3.3 Comparison of the models of Kennelly and Péronnet-Thibault

Both Kennelly and Péronnet-Thibault models can describe the world records in running (Fig. 1). The value of E is independent of the value of t_{MAS} and is significantly correlated with exponent g (Fig.2A). The relationship between EI_7 and g is almost perfectly linear (Fig. 2B). As MAS increases when t_{MAS} decreases, the values of EI depend on t_{MAS} (Table 4). However, all the relationships between EI and g for different values of t_{MAS} (6, 7 and 8 min) are almost perfectly linear ($r > 0.999$) and depend on t_{MAS}:

$EI_{6min} = 91.5 – 91.3\, g$ $r > 0.999$
$EI_{7min} = 93.2 – 93.1\, g$ $r > 0.999$
$EI_{8min} = 94.6 – 94.5\, g$ $r > 0.999$

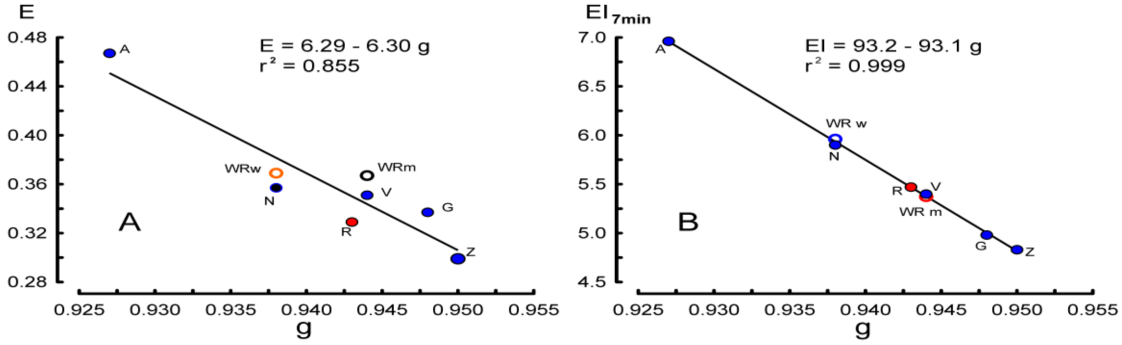

Fig. 2: relationships between exponent g in the model of Kennelly and slopes E (4A) and EI_{7min} (4B) in the model of Péronnet-Thibault.

IV. Discussion

For distances between 3000 and 10000 m, both the models of Kennelly and Péronnet-Thibault can accurately describe not only the World records as previously found [9, 12] but also the best performances of elite endurance runners [13, 14]. The races on one-hour and 25 or 30 km are much less frequent than the races on 3000, 5000 and 10000 m, which could partly explain that the running performances over 25 and 30 km could be submaximal and are below the 3000-10000 regression lines (blue and red lines in Fig.1A and 1B). In men, the best time over 20 km (blue empty circle on Fig. 1) was measured during the one-hour World record by H. Gebrselassie who accelerated at the end of race, which explains that 20-km speed was slightly lower than one-hour speed. For the 3000-100000 range, both the model of Kennelly and Péronnet-Thibault can accurately describe the World records and the best times of elite endurance runners. The differences between men and women for the slopes corresponding to exponent g (Fig. 1A) or E (Fig. 1B) are low (< 0.65 %) for the 3000-100000 range. However, the difference between men and women is higher for EI (10.7 %).

The value of exponent g is independent of scaling: the value of g is independent of the expression of t_{lim}, S and D_{lim}. Exponent g is probably an expression of the endurance capability. Indeed, it is likely that the curvatures of the t_{lim}-S and t_{lim}-D_{lim} relationships depend on the decrease in the fraction of maximal aerobic metabolism that can be sustained during long lasting exercises. The t_{lim}-D_{lim} relationship is linear when g is equal to 1. The hypothesis that exponent g is an index of endurance is confirmed by its highly significant correlation (r > 0.999) with EI_{7min} (Fig. 2B). The interest of EI as an endurance index can be questioned because it depends on MAS. Indeed, the value of t_{MAS} is debated. In Péronnet-Thibault model [9], t_{MAS} is assumed to be equal to 7 min but, in a review on the exhaustion time at VO2max [15], the value of t_{MAS} was 6 min. In contrast, the value of t_{MAS} was 14 min in a study on the energetics of the best performances in middle distance running [7].

However, the effect of t_{MAS} on the estimated value of EI is not important in elite endurance runners as demonstrated by the small differences between EI_6, EI_7 and EI_8 in table 4. The effect of t_{MAS} on EI can be estimated by computing the relationship between t_{MAS} and ratio EI_T / EI_{7min} (Fig. 3).

Let $T = t_{MAS}$ and $T/420 = m$

$S = MAS_T - E\ln(t_{lim}/T) = MAS_T - E\ln(t_{lim}/420\ m) = MAS_T + E\ln(m) - E\ln(t_{lim}/420)$

$S = MAS_{7min} - E\ln(t_{lim}/420)$

$MAS_T = MAS_{7min} - E\ln(m)$

$EI_T = 100\ E/MAS_T$ and $EI_{7min} = 100\ E/MAS_{7min}$

$EI_T / EI_{7min} = MAS_{7min} / MAS_T = MAS_{7min} / (MAS_{7min} - E\ln(m))$

$EI_T / EI_{7min} = 1 / (1 - E\ln(m)/MAS_{7min}) = 1 / (1 - EI_{7min}\ln(m)/100)$

On Fig. 3, the relationship between ratio EI_T / EI_{7min} and T is computed for 3 theoretical runners: an elite endurance runner ($EI_{7min} = 4$), an endurance runner ($EI_{7min} = 8$) and a low-level endurance runner ($EI_{7min} = 16$). The effect of t_{MAS} is much more important in the low-level endurance runner than in the elite endurance runner (fig.3).

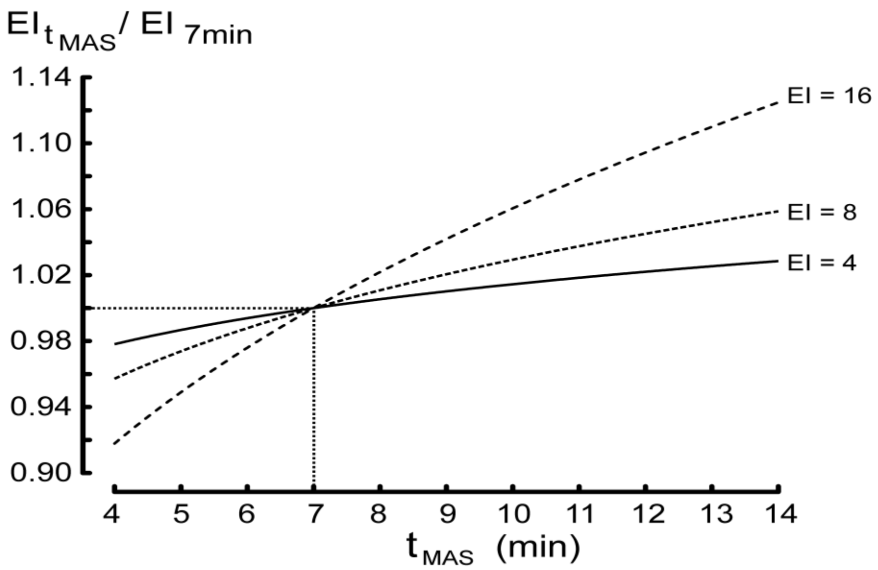

Fig. 3: effect of t_{MAS} on the endurance index (EI)

However, large variations in t_{MAS} have small effects on the classification of runners because the differences in EI between elite and medium or low-level runners are very large (from 4 to 16). For example, if the individual t_{MAS} is equal to 14 min instead of 7 min, the medium endurance runner would be still considered as a medium runner in spite of the increase of EI (8.47 instead of 8). Similarly, the elite endurance runner would be still considered as an elite runner in spite of the increase in EI (4.11 instead of 4) if his t_{MAS} is also equal to 14 min instead of 7 min. On the other hand, if the individual t_{MAS} is equal to 4 min instead of 7 min, the medium endurance runner would be still considered as a medium runner in spite of the decrease of EI (7.66 instead of 8.00). Similarly, the low-level endurance runner would be still considered as a low-level runner in spite of the decrease in EI (14.7 instead of 16) if his t_{MAS} is also equal to 4 min instead of 7 min.

In contrast, the same percentages of variations in exponent g would modify the classification of the runners because the difference in g between elite and medium or low-level runners is small (from 0.7 to 0.95). Therefore, the Péronnet-Thibault model is as useful as the Kennelly model for the evaluation of the runners even if the value of t_{MAS} is debatable.

In spite of the difference between equations 2 and 4, both models can describe the World records and the best performances of elite endurance runners. In Fig. 4A, the speed-time curves corresponding to different values of exponent g (from 0.6 to 0.95) of the Kennelly model are superimposed with Péronnet-Thibault speed-time curves with different values of EI corresponding to the same values of S at t_{lim}/t_{MAS} equal to 20. The curves are almost perfectly superimposed for g equal to 0.90 and 0.95, only. In contrast, in subjects whose values of g are low (0.6 and 0.7 [16]), the curves are not superimposed (Fig. 4A).

In Fig.4B, the values of D_{lim} corresponding to t_{lim} equal to 420 and 1500 s have been selected and used to compute the values of parameters k and exponents g of the D_{lim}-t_{lim} relationships of Kennelly's model (continuous curved lines). For EI_{7min} equal to 6, the corresponding curve of Kennelly's model (g = 0.9376, k = 5.333) is almost superposed to the curve of the Péronnet-Thibault model. Similarly, the curves corresponding to the models of Péronnet-Thibault and Kennelly are also almost superposed for EI_{7min} = 10, g = 0.8930 and k = 6.707. On the other hand, the curves corresponding to the models of Péronnet-Thibault and Kennelly diverges beyond 1500 s when EI_{7min} = 20, g = 0.7692 and k = 12.69.

Therefore, as demonstrated in Fig. 4A and B, both models of Kennelly and Péronnet-Thibault cannot describe the best performances of the low-level endurance runners with the same accuracy.

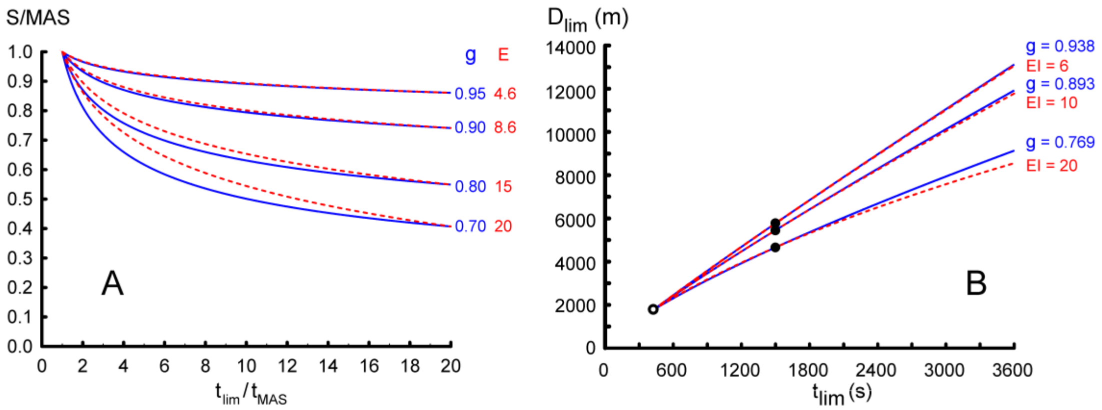

Fig.4. In A: superposition of theoretical speed-time curves computed from the models of Kennelly (blue solid curves) and Péronnet-Thibault (dashed red curves) with values of slope EI corresponding to the same values of S at t_{lim}/t_{MAS} equal to 20. In B: D_{lim}-t_{lim} relationships for MAS equal to15 $km.h^{-1}$ and 3 values of EI_{7min} (6, 10 and 20) in the model of Péronnet and Thibault (dashed red curves) and D_{lim}-t_{lim} relationships according to Kennelly's model (continuous blue curves).

V. Conclusions

Both the models of Kennelly and Péronnet-Thibault can accurately describe the World records and the best performances of elite endurance runners for distances between 3000 and 10000 m. When computed from performances between 3000 and 10000 m, the relationships between exponent g of Kennelly's model and the endurance index (EI) of the model of Péronnet-Thibault are almost perfectely linear for the different values of t_{MAS} (from 6 to 8 min) in elite endurance runners. In theory, the S-t_{lim} and D_{lim}-t_{lim} curves cannot be described by both models for the subjects whose values of g are low (< 0.8) and EI_{7min} are high (> 15), that is in low-level endurance runners. Therefore, it would be interesting to study what is the best model (Kennelly or Péronnet-Thibault) in non-elite runners.

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