American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-4, pp-132-140
www.ajer.org
Research Paper Open Access

Dynamics of a Holling-Tanner Model

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ABSTRACT: *In this paper, we have made an attempt to understand the dynamics of a Holling-Tanner model with Holling type III functional responses. We have carried out stability analysis of both the non-spatial model without diffusive spreading and of the spatial model. Turing and Hopf stability boundaries are found. With the help of numerical simulations, we have observed the spatial and spatiotemporal patterns for the model system. Non-Turing patterns are examined for some fixed parametric values for the predator populations.*

Keywords: *Reaction diffusion system, Globally asymptotically stable, Diffusion, Holling Tanner model.*

I. INTRODUCTION

The study of population dynamics is nearly as old as population ecology, starting with the pioneering work of Lotka and Volterra, a simple model of interacting species that still bears their joint names. This was a nearly linear model, but the predator-prey version displayed neutrally stable cycles[1,2]. From then on, the dynamic relationship between predators and their prey has long been and will continue to be one of dominant themes in both ecology and mathematical ecology due to its universal existence and importance[3].

In recent years, pattern formation in non-linear complex systems have been one of the central problems of the natural, social, technological sciences and ecological systems[4-10]. Particularly, many researchers have studied the predator-prey system with reaction-diffusion. The Holling Tanner model describes the dynamics of a generalist predator which feeds on its favourite food item as long as it is in abundant supply and grows logistically with an intrinsic growth rate and a carrying capacity proportional to the size of the prey. Starting with the pioneering work of Segel and Jackson[11], spatial patterns and aggregated population distribution are common in nature and in a variety of spatio-temporal models with local ecological interactions[12,13]. Wolpert[14] gave a clear and non-technical description of mechanisms of pattern formation in animals. Chen and Shi[15] studied a spatial Holling-Tanner model and proved the global stability of a unique constant equilibrium under a simple parameter condition.

In this paper, we have considered two interacting species, prey and predator for modelling the dynamics in spatially distributed population with local diffusion. We have considered a spatial Holling-Tanner model with Holling type III functional responses. The characteristic feature of Holling type III functional responses is that at low densities of the prey, the predator consumes it less proportionally than is available in the environment, relative to the predators' other prey[16] . We have obtained the conditions for local and global stability of the model system in the absence and the presence of diffusion. We also obtained the criteria for Turing and Hopf instability. The main objective of this paper is to see the spatial interaction and the selection of spatiotemporal patterns. This paper is organized as follows. In section 2, the model system and parameters are discussed. The model system is analyzed in the absence as well as in the presence of diffusion in section 3 and 4. We have discussed the conditions for Turing and Hopf instability in section 5. In section 6, we have discussed the numerical simulation results in both one and two dimensional spatial domain. At the end, results are discussed in section 7.

II. THE MODEL SYSTEM

We consider a reaction-diffusion model for prey-predator where at any point (x, y) and time t, the prey $N(x, y, t)$ and predator $P(x, y, t)$ populations. The prey population $N(x, y, t)$ is predated by the predator population $P(x, y, t)$. The per capita predation rate is described by Holling type III functional response. The model system satisfy the following:

$$\frac{\partial N}{\partial T} = rN\left(1-\frac{N}{K}\right) - \frac{aN^2P}{D^2+N^2} + D_N\nabla^2 N,$$
$$\frac{\partial P}{\partial T} = eP\left(1-\frac{hP}{N}\right) - sP^2 + D_P\nabla^2 P. \tag{1}$$

The parameters $r, a, D, e, h, s, K, D_N, D_P$ in model (1) are positive constants. Here, r is the prey's intrinsic growth rate in the absence of predation, a is the rate at which prey is feeded by predator and it follows Holling type III functional response, D is the half-saturation constants for prey density, e is the predator's intrinsic growth rate, h is the number of prey required to support one predator at equilibrium when P equals to N/h, s is the intra-specific competition rate for the predators, K is the carrying capacity of the phytoplankton parameters , D_N and D_P represent the diffusion coefficients of prey and predator, respectively. The units of the parameters are as follows. Time T and length $X, Y \in [0, R]$ are measured in days (d) and metres (m), respectively. Further, r, N, P, e, D are usually measured in mg of dry weight per litre $[mg.dw/l]$, the dimension of a and h is d^{-1}. The dimension of s is d^{-1}. The diffusion coefficients D_N and D_P are measured in $[m^2d^{-1}]$.

We introduce the following notation in order to bring the system of equations to a non-dimensional form:

$$u=\frac{N}{K}, v=\frac{aP}{rK}, t=rT, \gamma=\frac{D}{K}, x=\frac{X}{L}, y=\frac{Y}{L}, \beta=\frac{rh}{a}, \delta=\frac{e}{r}, \mu=\frac{sK}{a}, d_1=\frac{D_1}{rL^2}, d_2=\frac{D_2}{rL^2}.$$

We obtain the system

$$\frac{\partial u}{\partial t} = u(1-u) - \frac{u^2v}{\gamma^2+u^2} + d_1\nabla^2 u,$$
$$\frac{\partial v}{\partial t} = \delta v\left(1-\frac{\beta v}{u}\right) - \mu v^2 + d_2\nabla^2 v, \tag{2}$$

with the initial condition

$$u(x,y,0) > 0, v(x,y,0), (x,y) \in \Omega, \tag{3}$$

and with the no-flux boundary conditions

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, (x,y) \in \partial\Omega. \tag{4}$$

III. STABILITY ANALYSIS OF THE NON-SPATIAL MODEL SYSTEM

In this section, we restrict ourselves to the stability analysis of the model system in the absence of diffusion in which only the interaction part of the model system are taken into account. We find the non-negative equilibrium states of the model system and discuss their stability properties with respect to variation of several parameters[17,18].

3.1 Local Stability Analysis

We analyze model system (2) without diffusion. In such case, the model system reduces to

$$\frac{du}{dt} = u(1-u) - \frac{u^2v}{\gamma^2+u^2},$$
$$\frac{dv}{dt} = \delta v\left(1-\frac{\beta v}{u}\right) - \mu v^2, \tag{5}$$

with $u(x,0) > 0, v(x,0) > 0.$

The stationary dynamics of system (5) can be analyzed from $du/dt = dv/dt = 0$. Then, we have,

$$u(1-u)-\frac{u^2 v}{\gamma^2+u^2}=0,$$
$$\delta v\left(1-\frac{\beta v}{u}\right)-\mu v^2=0. \tag{6}$$

It can be seen that model system has two non-negative equilibria, namely, $E_1(1,0)$ and $E^*(u^*,v^*)$. It may be noted that u^* and v^* satisfies the following equations:

$$\frac{(1-u^*)(\gamma^2+u^{*2})}{u^*}-v^*=0,\ \delta-\delta\beta\frac{v^*}{u^*}-\mu v^*=0.$$

From Eqs.(6), it is easy to show that the equilibrium point E_1 is a saddle point with stable manifold in u-direction and unstable manifold in v-direction[19]. In the following theorem we are able to find necessary and sufficient conditions for the positive equilibrium $E^*(u^*,v^*)$ to be locally asymptotically stable. The Jacobian matrix of model (5) at $E^*(u^*,v^*)$ is

$$J=\begin{pmatrix} -u^*+\dfrac{u^*v^*(u^{*2}-\gamma^2)}{(\gamma^2+u^{*2})^2} & -\dfrac{u^{*2}}{\gamma^2+u^{*2}} \\ \dfrac{\delta\beta v^{*2}}{u^{*2}} & -\dfrac{\delta\beta v^*}{u^*}-\mu v^* \end{pmatrix}$$

Following the Routh-Hurwitz criteria, we get,

$$A=u^*-\frac{u^*v^*(u^{*2}-\gamma^2)}{(\gamma^2+u^{*2})^2}+\frac{\delta\beta v^*}{u^*}+\mu v^*,$$
$$B=\left(u^*-\frac{u^*v^*(u^{*2}-\gamma^2)}{(\gamma^2+u^{*2})^2}\right)\left(\frac{\delta\beta v^*}{u^*}+\mu v^*\right)+\frac{\delta\beta v^{*2}}{(\gamma^2+u^{*2})}. \tag{7}$$

Theorem 1. The positive equilibrium $E^*(u^*,v^*)$ is locally asymptotically stable in the uv− plane if and only if the following inequalities hold: $A>0$ and $B>0$.
Remark 1. Let the following inequalities hold

$$u^{*2}<\gamma^2. \tag{8}$$

Then $A>0$ and $B>0$. It shows that if inequalities in (8) hold, then $E^*(u^*,v^*)$ is locally asymptotically stable in the u − v plane.

3.2 Global Stability Analysis

In order to study the global behavior of the positive equilibrium $E^*(u^*,v^*)$, we need the following lemma which establishes a region for attraction for model system (5).

Lemma 2. The set $R=\left\{(u,v):0\le u\le 1, 0\le v\le \frac{1}{\beta}\right\}$ is a region of attraction for all solutions initiating in the interior of the positive quadrant R.
Theorem 3. If

$$\left(\frac{u^*}{v^*u}+w\mu\right)>0, \tag{9}$$
$$4\left(\frac{-uu^*(1-u-u^*)+\gamma^2}{u^2u^*}\right)\left(\frac{u^*}{v^*u}+w\mu\right)>0, \tag{10}$$

hold, then E^* is globally asymptotically stable with respect to all solutions in the interior of the positive quadrant R.

IV. STABILITY ANALYSIS OF THE SPATIAL MODEL SYSTEM

In this section, we study the effect of diffusion on the model system about the interior equilibrium point. Instability will occur due to diffusion when a parameter varies slowly in such a way that a stability condition is suddenly violated and it can bring about a situation wherein perturbation of a non-zero (finite) wavelength starts growing (perturbations of zero wave number are stable when diffusive instability sets in[20,21]). Turing instability can occur for the model system because the equation for predator is nonlinear with respect to predator population[22].

To study the effect of diffusion on the model system (5), we derive conditions for stability analysis in one- and two- dimensions cases.

4.1 One-Dimension Case

After non-dimensionalization of the model system (2) in one dimensional space takes the following form with non-zero initial condition and no-flux boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial t} &= u(1-u) - \frac{u^2 v}{\gamma^2+u^2} + d_1 \frac{\partial^2 u}{\partial x^2}, \\ \frac{\partial v}{\partial t} &= \delta v\left(1-\frac{\beta v}{u}\right) - \mu v^2 + d_2 \frac{\partial^2 v}{\partial x^2}, \end{aligned} \tag{11}$$

$$u(x,0) > 0, v(x,0) > 0, \text{ for } x \in [0,R].$$

To study the effect of diffusion on the model system, we perturb the steady state (u^*, v^*) by setting $u = u^* + U, v = v^* + V$. The linearized form of the Eqs. (11) obtained as:

$$\begin{aligned} \frac{\partial U}{\partial t} &= b_{11}U + b_{12}V + d_1 \frac{\partial^2 U}{\partial x^2}, \\ \frac{\partial V}{\partial t} &= b_{21}U + b_{22}V + d_2 \frac{\partial^2 V}{\partial x^2}, \end{aligned} \tag{12}$$

Where

$$\begin{aligned} b_{11} &= -u^* + \frac{u^* v^* (u^{*2} - \gamma^2)}{(\gamma^2 + u^{*2})^2}, b_{12} = -\frac{u^{*2}}{\gamma^2 + u^{*2}}, \\ b_{21} &= \frac{\delta \beta v^{*2}}{u^{*2}}, b_{22} = -\frac{\delta \beta v^*}{u^*} - \mu v^*. \end{aligned} \tag{13}$$

Thus, the solution of the system (12) of the form

$$\begin{pmatrix} U \\ V \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \exp(\lambda t + ikx),$$

where λ and k are the frequency and wave number respectively. The characteristic equation of the linearized system is given by

$$\lambda^2 + \rho_1 \lambda + \rho_2 = 0, \tag{14}$$

where

$$\rho_1 = A + (d_1 + d_2)k^2, \tag{15}$$

$$\rho_2 = B + d_1 d_2 k^4 + \left[\left(u^* - \frac{u^* v^* (u^{*2} - \gamma^2)}{(\gamma^2 + u^{*2})^2}\right) d_2 + \left(\frac{\delta \beta v^*}{u^*} + \mu v^*\right) d_1\right] k^2, \tag{16}$$

where A and B are defined in (7).

From Eqs.(14-16) and using the Routh-Hurwitz criteria, the following theorem follows immediately.

Theorem 4. (i)The positive equilibrium $E^*(u^*, v^*)$ is locally asymptotically stable in the presence of diffusion if and only if $\rho_1 > 0$ and $\rho_2 > 0$.

(ii)If the inequalities in Eq.(8) are satisfied, then the positive equilibrium point E^* is locally asymptotically stable in the presence as well as absence of diffusion.

(iii)Suppose that inequalities in Eq.(8) are not satisfied, i.e., either A or B is negative or both A and B are negative. Then for strictly positive wave-number $k > 0$, i.e. spatially inhomogeneous perturbations, by increasing d_1 and d_2 to sufficiently large values, ρ_1 and ρ_2 can be made positive and hence E^* can be made locally asymptotically stable.
In the following theorem, we are able to show the global stability behavior of the positive equilibrium in the presence of diffusion.

Theorem 5. (i) If the equilibrium E^* of model system (5) is globally asymptotically stable, the corresponding uniform steady state of model system (12) is also globally asymptotically stable.
(ii) If the equilibrium E^* of system (5) is unstable even then the corresponding uniform steady state of system (12) can be made globally asymptotically stable by increasing the diffusion coefficients d_1 and d_2 to a sufficiently large value for strictly positive wave-number $k > 0$.
4.2 Two-Dimension Case
In the two-dimensional case the model system can be written as

$$\begin{aligned} \frac{\partial u}{\partial t} &= u(1-u) - \frac{u^2 v}{\gamma^2 + u^2} + d_1\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \\ \frac{\partial v}{\partial t} &= \delta v\left(1 - \frac{\beta v}{u}\right) - \mu v^2 + d_2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right). \end{aligned} \tag{17}$$

We analyze the above model under the following initial and boundary conditions:

$$u(x,y,0) > 0, v(x,y,0) > 0, (x,y) \in \Omega, \tag{18}$$

and

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, (x,y) \in \partial\Omega, t > 0, \tag{19}$$

where n is the outward normal to $\partial\Omega$. We state the main result of this section in the following theorem.

Theorem 6. (i) If the equilibrium E^* of model system (5) is globally asymptotically stable, the corresponding uniform steady state of model system (17) is also globally asymptotically stable.
(ii) If the equilibrium E^* of system (5) is unstable even then the corresponding uniform steady state of system (17) can be made globally asymptotically stable by increasing the diffusion coefficients d_1 and d_2 to a sufficiently large value for strictly positive wave-number $k > 0$.

V. TURING AND HOPF INSTABILITY

The Turing instability occurs if at least one of the roots of the above (14) has a positive root or positive real part or in other words, $\mathrm{Re}(\lambda) > 0$ for some $k > 0$. Irrespective of the sign of ρ_1 and ρ_2, the diffusion-driven instability occurs when $\rho_2(k^2) < 0$. Hence the condition for diffusive instability is given by

$$H(k^2) = d_1 d_2 k^4 + \left[\left(u^* - \frac{u^* v^* (u^{*2} - \gamma^2)}{(\gamma^2 + u^{*2})^2}\right) d_2 + \left(\frac{\delta\beta v^*}{u^*} + \mu v^*\right) d_1\right] k^2 + B < 0. \tag{20}$$

H is a quadratic in k^2 and the graph of $y = H(k^2)$ is a parabola. The minimum of $y = H(k^2)$ occurs at $k^2 = k_{cr}^2$, where

$$k_{cr}^2 = \frac{1}{2 d_1 d_2}\left[\left(\frac{v^*(u^{*2} - \gamma^2)}{\left(\gamma^2 + u^{*2}\right)^2} - 1\right) d_2 u^* + \left(-\frac{\delta\beta}{u^*} - \mu\right) v^* d_1\right] > 0. \tag{21}$$

Consequently, the condition for diffusive instability is $H(k_{cr}^2)$ .Therefore

$$\frac{1}{4d_1d_2}\left[\left(\frac{v^*(u^{*^2}-\gamma^2)}{\left(\gamma^2+u^{*^2}\right)^2}-1\right)d_2u^*+\left(-\frac{\delta\beta}{u^*}-\mu\right)v^*d_1\right]^2 > B. \qquad (22)$$

Now, consider the set of parameter values $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03$. We obtain $u^* = 0.1827$ and $v^* = 0.2137$. For the set of parameter values $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03, d_1 = 0.01, d_2 = 3$, the corresponding critical value is $(k_{cr}^2, H(k_{cr}^2)) = (7.068, -1.276)$. The graph of $H(k^2)$ vs k^2 has been plotted for different values of d_2 in Fig.1. For all values of k^2 lying in the range $(0.028, 14.170)$,the system (2) is unstable. The region under the curve for which $H(k^2) < 0$ is known as Turing instability region.

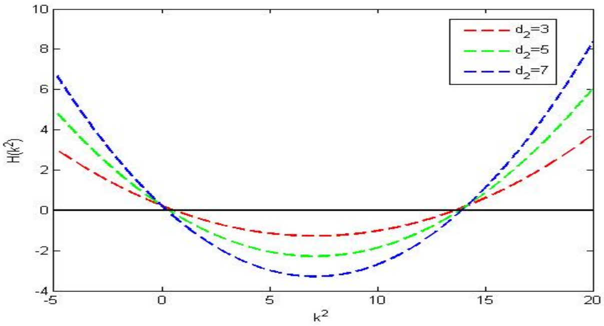

Figure 1. The graph of the function $H(k^2)$ vs k^2 for $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03$ and $d_1 = 0.01$ for different values of $d_2 = 3, 5, 7$.

The Turing bifurcation occurs when $\mathrm{Im}(\lambda(k)) = 0$ and $\mathrm{Re}(\lambda(k)) = 0$ at $k = k_T \neq 0, k_T$ is the critical wave number. If δ is considered as a bifurcation parameter, then its critical value equals to

$$\delta_T = \frac{-B_1 \pm \sqrt{B_1^2 - 4R_1C_1}}{2R_1}, \qquad (23)$$

where $R_1 = -\left(\frac{\beta v^* d_1}{u^*(d_1+d_2)}\right)^2$,

$$B_1 = \beta v^* + \frac{2\beta\gamma^2 v^{*^2}}{(\gamma^2+u^{*^2})^2} + \frac{\beta\frac{v^*}{u^*}b_{11}d_1^2 - 2\beta\mu\frac{v^{*^2}}{u^*}d_1^2 + \beta\frac{v^*}{u^*}b_{11}d_2^2}{(d_1+d_2)^2},$$

$$C_1 = \mu u^* v^* + \frac{\mu u^* v^{*^2}(u^{*^2}-\gamma^2)}{(\gamma^2+u^{*^2})^2} + \frac{b_{11}\mu v^* d_1^2 - b_{11}^2 d_2^2 - \mu^2 v^{*^2} d_1^2 + b_{11}\mu v^* d_2^2}{(d_1+d_2)^2}.$$

The Hopf bifurcation occurs when $\mathrm{Im}(\lambda(k)) \neq 0$ and $\mathrm{Re}(\lambda(k)) = 0$ at $k = 0$. The critical value of the bifurcation parameter δ is

$$\delta_H = \frac{u^*\left(\frac{u^*v^*(u^{*^2}-\gamma^2)}{(\gamma^2+u^{*^2})^2} - u^* - \mu v^*\right)}{\beta v^*}. \qquad (24)$$

At the Hopf bifurcation threshold, the temporal symmetry of the system is broken and gives rise to uniform oscillations in space and periodic oscillations in time with the frequency w_H and wavelength λ_H :

$$w_H = \sqrt{\delta\beta v^* + \mu u^* v^* + \frac{2\delta\beta\gamma^2 v^{*^2}}{(\gamma^2+u^{*^2})^2} + \frac{\mu u^* v^{*^2}(u^{*^2}-\gamma^2)}{(\gamma^2+u^{*^2})^2}},$$

$$\lambda_H = \frac{2\pi}{w_H} = \frac{2\pi}{\sqrt{\delta\beta v^* + \mu u^* v^* + \dfrac{2\delta\beta\gamma^2 v^{*^2}}{(\gamma^2 + u^{*^2})^2} + \dfrac{\mu u^* v^{*^2}(u^{*^2} - \gamma^2)}{(\gamma^2 + u^{*^2})^2}}}.$$

VI. NUMERICAL SIMULATIONS

In this section we perform numerical simulations to illustrate the results obtained in previous sections. For this purpose we have plotted the time series, spatiotemporal patterns and snapshots for one dimensional cases and two dimensional cases. The dynamics of the model system (2) is studied with the help of numerical simulation. The step lengths Δx and Δt of the numerical grid are chosen sufficiently small so that the results are numerically stable. The temporal dynamics is studied by observing the effect of time on space vs. density plot of prey and predator populations. Spatiotemporal chaos is generated as a result of breaking the homogeneity and the formation of a non-stationary irregular spatial pattern when the local kinetics of the system is oscillatory for a wide class of initial conditions. In the absence of any spatial gradient, period-doubling bifurcations serve as the generating mechanism for chaos in these model systems. For the numerical integration of the model system, we have used the Runge-Kutta fourth-order procedure on the MATLAB 7.0 platform. To investigate the spatiotemporal dynamics of the model system (2), we solved it numerically using semi implicit (in time) finite difference method. Application of the finite difference method gives rise to a sparse, banded linear system of algebraic equations. The resulting linear system is solved by using the GMRES algorithm for the two-dimensional case.

Consider the set of parameter values

$$\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03 \tag{25}$$

for the model system (5). With the above set of parameters, we note that the positive equilibrium E^* exists, and it is given by $(u^*, v^*) = (0.1827,\ 0.2137)$. We choose the same set of parameters as in (25) for the model system (2) with initial condition

$$\begin{aligned} u(x,0) &= u^* + \varepsilon(x - x_1)(x - x_2), \\ v(x,0) &= v^*, \end{aligned} \tag{26}$$

where (u^*, v^*) is the nontrivial state for the coexistence of prey and predator population and $\varepsilon = 10^{-8}, x_1 = 1200, x_2 = 2800$ is the parameter affecting the system dynamics. In order to avoid numerical artifacts we checked the sensitivity of the results to the choice of the time and space steps and their values were chosen sufficiently small.

In Figure 2, we have presented the time series of the model system (2) for the fixed set of parameters values (23) with initial condition (24). By increasing the time $t = 200, 500, 700$, the system shows from stable to limit cycle behavior. Figure 3 shows the spatiotemporal evolution of predator density for the system. Finally Figure 4 shows the snapshots of predator density for increasing values of diffusion coefficient d_2 to observe the irregular patchy non-Turing spatial distribution of predator population.

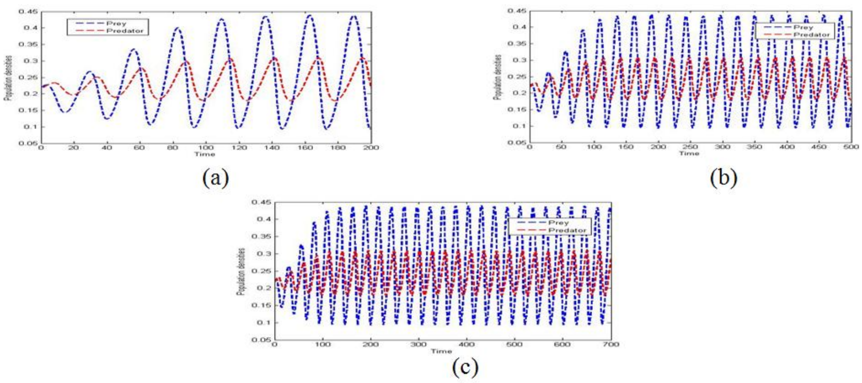

Figure 2. Time series for the model system (2) at the fixed set of parameters values $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03, d_1 = 0.0001, d_2 = 0.001$ with $t =$ (a) 200 (b) 500 (c) 700.

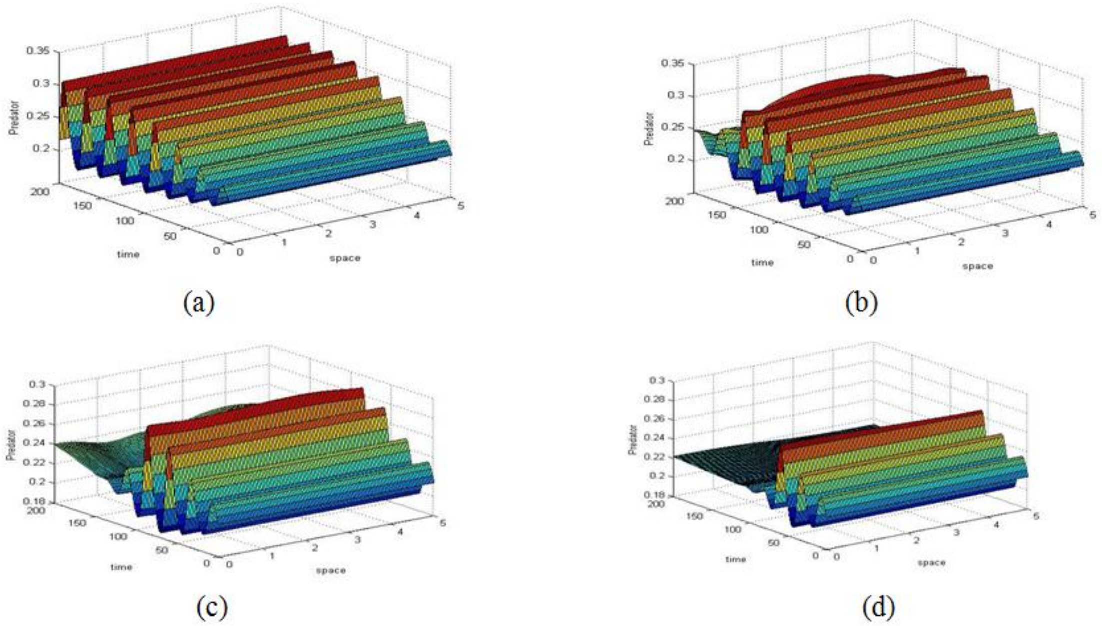

Figure 3. Spatiotemporal patterns of predator density for the model system (2) at fixed set of parameters values $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03, d_1 = 0.0001$, with d_2 = (a) 0.01 (b) 0.1 (c) 1 (d) 10.

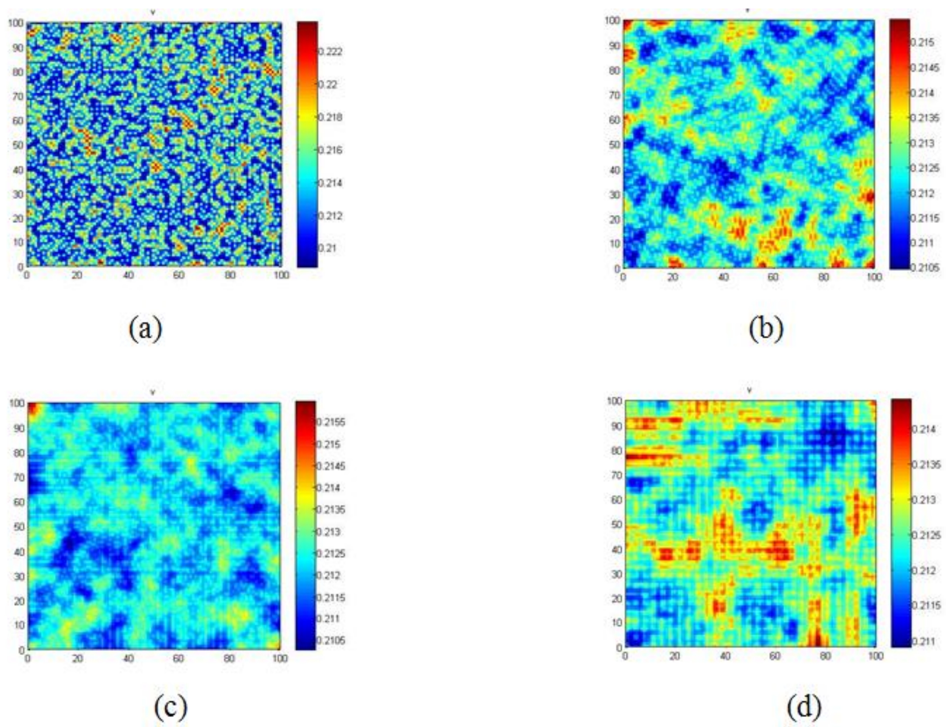

Figure 4. Snapshots of predator populations with parameters values $\gamma = 0.12, \beta = 0.8, \delta = 0.1, \mu = 0.03, d_1 = 0.001$ and d_2 = (a) 1 (b) 10 (c) 15 (d) 20 .

VII. DISCUSSION AND CONCLUSION

In this paper, we have considered a minimal model of predator-prey interaction with holling type-III functional responses. The dynamics of the Holling-Tanner predator-prey model is quite interesting for its mathematical properties and for its efficacy in describing real ecological systems such as lynx and hare and sparrow and sparrow hawk. The characteristic feature of Holling type III functional responses is that at low densities of the prey, the predator consumes it less proportionally than is available in the environment, relative to the predators' other prey . Collings[23] have used the Holling-Tanner model to study the population interaction between the predacious mite *Metaseiulus occidentalis* (Nesbitt) and its spider mite prey *Tetranychus mcdanieli* (McGregor). Recently, a modification of the Holling-Tanner model by invoking the ratio-dependent functional response is suggested by Haque and Li[24].

We have investigated the model both analytically and numerically. We have studied the reaction-diffusion model in both one and two dimensions and investigated its stability. The nontrivial equilibrium state E^* of predator-prey coexistence is locally as well as globally asymptotically stable under a fixed region of attraction when certain conditions are satisfied. We also obtained the conditions for Turing instability in terms of parameters. For a fixed set of parameter values (25), we obtained the region of Turing instability. We have plotted the time series, spatiotemporal patterns and snapshots of the model system (2). We found that for increasing value of time t = 200,500,700, the system shows from stable to limit cycle behavior. For increasing the value of d_2, we have plotted the snapshots which shows the irregular patchy non-Turing spatial distribution of predator evolved for two dimensional cases.

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