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Dynamic Modelling and Simulation of Salient Pole Synchronous Motor Using Embedded Matlab

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ABSTRACT: *This paper sets to present the dynamic model and simulation of a three-phase salient pole synchronous motor based on a model and computer program simulation in a rotor reference frame to avoid the complexity involved in the course of solving time varying differential equations obtained from the dynamic model with the aid of MATLAB/Simulink. While several methodologies are feasible with MATLAB/Simulink, an embedded MATLAB toolbox is utilized in this paper due to its uniqueness in offering the user the opportunity of programming the differential equations rather than obtaining the complete block diagram representation of those equations. This approach also enables the user the opportunity to easily crosscheck the model for ease of identification of errors. Equations in the phase variable reference frame, derivation of expression for the machine inductances, transformation of the stator variables to a frame of reference fixed in the rotor, as well as power input and torque equations are presented. The developed Simulink model is presented with results for all machine's variable characteristics investigated and discussed. This dynamic model is capable of predicting the machine's behaviour for this machine type with damper windings (amortisseur windings) taken into consideration.*

Keywords: *d-q rotating reference frame, Dynamic modeling, Embedded MATLAB, Park's transformation, Salient-pole machine*

I. INTRODUCTION

A salient pole synchronous motor is an a.c. rotating machine whose speed under steady state condition is proportional to the frequency of the current in its armature. The magnetic field produced by the armature currents rotates at the same speed as that produced by the field current on the rotor, which is rotating at the synchronous speed. Synchronous machines are used in many industrial applications due to their high power ratings and constant speed operation. The electrical and electromechanical behaviour of most synchronous machines can be predicted from the equations that describe the three phase salient pole synchronous machine [1]. These equations can be used directly to predict the performance of hydro and steam turbine synchronous generators and synchronous motors. The rotor of synchronous machine is equipped with field winding and one or more damper windings, which is magnetically unsymmetrical. Salient pole synchronous machines (SPSMs) have salient pole or projecting poles with concentrated field windings. In synchronous motors, damper windings act as rotor bars and help in self-starting of the motor. Salient pole synchronous machine is used to provide independent control of mechanical torque and deliver electric power. It provides mechanical torque to the rotating assembly and transmits electric power across an air gap to electric equipment mounted on the rotating assembly. The rotor is then driven by external means producing a rotating magnetic field, which induces a three phase voltage within the stator winding. Slip power recovery systems can provide both the mechanical and electrical power transmission, but these are coupled and not independently controllable [2].

The stator self and mutual inductances and stator-rotor mutual inductances present computational difficulty when used to solve the phase quantities directly. To obtain the phase currents from the flux linkages, the inverse of the time varying inductance matrix will have to be computed at every time step [3] - [4]. The computation of inverse at every time step is time consuming and could produce numerical stability problems. To remove the time varying quantities in voltages, currents, flux linkages and phase inductances, stator quantities are transformed to a d-q rotating reference frame using Park's transformation. This results in the machine equations having time-invariant coefficients. The idealized machines have the rotor windings along the d- and q-axes. Stator winding quantities need transformation from three phases to two phase d-q rotor rotating reference frame. Park's transformation is used to transform the stator quantities to d-q reference frame, the d-axis aligned with the magnetic axis of the rotor and q-axis is leading the d-axis by $\pi/2$ [5] - [6].

Simulation of the synchronous machine is well documented in the literature and digital computer solutions can be performed using various methods such as numeric programming [7] - [8].

This paper discusses the use of the embedded MATLAB function in the modelling and simulation of salient pole synchronous motor. This is due to its uniqueness in offering the user the opportunity of programming the differential equations rather than obtaining the complete block diagram representation of those equations. The approach also enables the user the opportunity to easily crosscheck the model for ease of identification of errors. Modeling of a three-phase salient pole synchronous motor using Park's Transformation is presented. At first, stationary variables are transformed into dq0 reference frame and in Park's Transformation. And later,dq0 reference frame transformed into rotor reference frame.

II. MODELING OF SYNCHRONOUS MOTOR

The ultimate purpose of this section is to develop a detailed mathematical model of salient pole synchronous machine because the model and its understanding (reference frame theory) are the basis for all modern motor modeling and control schemes. The d-q model eliminates time varying coefficients dependent on rotor position. The model considers direct and quadrature stator windings. Here, the synchronous machine is operated as motor; it is convenient to assume that the direction of negative stator current is into the terminals. The voltage equations of stator and rotor are written as:

$$v_{abcs} = r_s i_{abcs} + p\lambda_{abcs} \tag{1}$$

$$v_{dqfr} = r_r i_{dqfr} + p\lambda_{dqfr} \tag{2}$$

where v_{abcs} represents the phase voltages across stator windings

r_s is the resistance of each stator winding

i_{abcs} are the stator currents

$\lambda_{abcs}$are the flux linkages

r_r, i_{dqfr} , λ_{dqfr} are the resistance, currents and flux linkages of rotor winding respective. Complex space vectors of the three phase stator and rotor voltages, currents and flux linkages, all expressed in the stationary reference frame fixed to the stator and rotating reference frame to the rotor, respectively [2].

They are defined as:

$$v_{abcs} = [v_{as} \; v_{bs} \; v_{cs}]^t, \quad v_{dqfr} = [v_{dr} \; v_{qr} \; v_{fr}]^t \tag{3}$$
$$r_s = diag\ [r_{as} \; r_{bs} \; r_{cs}], \quad r_r = diag\ [r_{qr} \; r_{dr} \; r_{fr}]$$

where v_{as} ,v_{bs} v_{cs} are the values of stator instantaneous phase voltages and v_{dr} ,v_{qr} ,v_{fr} are the rotor phase voltages.

Since three stator and three rotor windings exist with mutual coupling between each possible pair of windings, the problem to be solved is formidable. In general, the stator flux linkages for any orientation of the rotor are given as in equation (4).

$$\begin{aligned}
\lambda_{as} &= L_{asas} i_{as} + L_{asbs} i_{bs} + L_{ascs} i_{cs} + L_{asdr} i_{dr} + L_{asqr} i_{qr} + L_{asfr} i_{fr} \\
\lambda_{bs} &= L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bscs} i_{cs} + L_{bsdr} i_{dr} + L_{bsqr} i_{qr} + L_{bsfr} i_{fr} \\
\lambda_{cs} &= L_{csas} i_{as} + L_{csbs} i_{bs} + L_{csc\,s} i_{cs} + L_{csdr} i_{dr} + L_{csqr} i_{qr} + L_{csfr} i_{fr}
\end{aligned} \tag{4}$$

Similarly, the rotor flux linkages are given in equation (5).

$$\begin{aligned}
\lambda_{dr} &= L_{dras} i_{as} + L_{drbs} i_{bs} + L_{drcs} i_{cs} + L_{drdr} i_{dr} + L_{drqr} i_{qr} + L_{drfr} i_{fr} \\
\lambda_{qr} &= L_{qras} i_{as} + L_{qrbs} i_{bs} + L_{qrcs} i_{cs} + L_{qrdr} i_{dr} + L_{qrqr} i_{qr} + L_{qrfr} i_{fr} \\
\lambda_{fr} &= L_{fras} i_{as} + L_{frbs} i_{bs} + L_{frcs} i_{cs} + L_{frdr} i_{dr} + L_{frqr} i_{qr} + L_{frfr} i_{fr}
\end{aligned} \tag{5}$$

where L_{asas}, L_{bsbs}, L_{cscs} are the self-inductances of the stator a, b and c-phase respectively. L_{asbs}, L_{bscs} and L_{ascs} are the mutual inductances between the ab, bc and ac phases respectively. In similar manner, L_{drdr}, L_{qrqr} and L_{frfr} are the self-inductances of the rotor windings, L_{drqr},L_{drfr}, L_{qrfr} are the mutual inductances between the windings of the rotor. The self-inductances of the stator a-phase winding which is define as the ratio of flux linkage λ_{aa} by the a-phase current I_a is given by:

$$L_{aa} = L_{ls} + L_o - L_{ms}\cos 2\theta_r \tag{6}$$

Where L_{ls} is the leakage inductance of the stator winding due to the armature leakage flux. L_o is the average inductance due to the space fundamental air-gap flux; $L_o = 1/2(L_d + L_q)$. L_{ms} is the inductance fluctuation (saliency) due to the rotor position dependent on flux $L_{ms} = 1/2(L_d - L_q)$. Similar to that of L_{aa} but with θ_r replaced by $\left(\theta_r - \frac{2\pi}{3}\right)$ and $\left(\theta_r - \frac{4\pi}{3}\right)$ for b-phase and c-phase self-inductances. Hence,

$$L_{bb} = L_{ls} + L_o - L_{ms}\cos 2\left(\theta_r - \frac{2\pi}{3}\right) \tag{7}$$

$$L_{cc} = L_{ls} + L_o - L_{ms}\cos 2\left(\theta_r - \frac{4\pi}{3}\right) \tag{8}$$

The mutual inductance between the a-phase and b-phase of the stator can be express as:

$$L_{ab} = L_{ba} = -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) \tag{9}$$

Similarly the mutual inductance between a-phase and c-phase which is displaced $\frac{4\pi}{3}$ away from a-phase can be written as:

$$L_{ac} = L_{ca} = -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) \tag{10}$$

Also the mutual inductance between b-phase and c-phase which are $\frac{2\pi}{3}$ away from each other can be written as:

$$L_{bc} = L_{cb} = -\frac{1}{2}L_o - L_{ms}\cos 2\theta_r \tag{11}$$

In matrix form, the stator self and mutual inductances are given as:

$$\begin{bmatrix} L_{ls} + L_o - L_{ms}\cos 2\theta_r & -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_o - L_{ms}\cos 2\left(\theta_r - \frac{2\pi}{3}\right) & -\frac{1}{2}L_o - L_{ms}\cos 2\theta_r \\ -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2}L_o - L_{ms}\cos 2\theta_r & L_{ls} + L_o - L_{ms}\cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \tag{12}$$

The rotor self-inductances and mutual inductances do not depend on the rotor position and therefore, may be given as:

$$\left.\begin{aligned} L_{dd} &= L_{ldr} + L_{dr} \\ L_{qq} &= L_{lqr} + L_{qr} \\ L_{ff} &= L_{lfr} + L_{fr} \end{aligned}\right\} \text{Rotor self-inductances} \tag{13}$$

Where L_{ldr}, L_{lqr}, and L_{lfr} are the leakage inductances for d-, q- and field coils but here leakages are not equal as these windings are unlike the stator windings, not identical.

$$\left.\begin{aligned} L_{dq} &= L_{qd} = 0 \\ L_{df} &= L_{fd} = L_{fd} \\ L_{qf} &= L_{fq} = 0 \end{aligned}\right\} \text{Rotor Mutual inductances} \tag{14}$$

The mutual inductances between the rotor windings along the d-axis and q-axis are zero because of the 90^o displacement.

In matrix form, the rotor self and mutual inductances are given as:

$$L_{dqfrr} = \begin{bmatrix} L_{ldr} + L_{dr} & 0 & L_{fd} \\ 0 & L_{lqr} + L_{qr} & 0 \\ L_{fd} & 0 & L_{lfr} + L_{fr} \end{bmatrix} \tag{15}$$

On the other hand, the mutual inductances between the rotor and stator circuits are function of rotor position. These inductances are maximum when the rotor axis in consideration coincides with the stator phase a-, b- or c-. Since the q-axis of the rotor is along the north-pole and its position θ_r to the stator a-phase, the inductances can be developed as follows:

$$\left.\begin{aligned} L_{aq} &= L_{aq}\cos\theta_r \\ L_{ad} &= L_{ad}\sin\theta_r \\ L_{af} &= L_{af}\sin\theta_r \end{aligned}\right\} \text{Stator a-phase and rotor mutual inductances} \tag{16}$$

Similarly,

$$\left.\begin{aligned} L_{bq} &= L_{bq}\cos\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{bd} &= L_{bd}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{bf} &= L_{bf}\sin\left(\theta_r - \frac{2\pi}{3}\right) \end{aligned}\right\} \text{Stator b-phase and rotor mutual inductances} \tag{17}$$

$$\left.\begin{aligned} L_{cq} &= L_{cq}\cos\left(\theta_r + \frac{2\pi}{3}\right) \\ L_{cd} &= L_{cd}\sin\left(\theta_r + \frac{2\pi}{3}\right) \\ L_{cf} &= L_{cf}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{aligned}\right\} \text{Stator c-phase and rotor mutual inductances} \quad (18)$$

In matrix form, stator –rotor mutual inductances are given by:

$$L_{sr}(\theta_r) = \begin{bmatrix} L_{ad}sin\theta_r & L_{aq}cos\theta_r & L_{af}sin\theta_r \\ L_{bd}sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bq}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{bf}sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{cd}sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cq}cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{cf}sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \quad (19)$$

III. D-Q MODEL OF SYNCHRONOUS MOTOR

Using Park' transformation, the d-q variables in the synchronous rotating reference frame can be obtained from the phase (a, b and c) quantities. The Park's transformation is as defined in equation (20).

$$i_{dq0} = T(\theta_r) i_{abc} \quad (20)$$

where

$$i_{dq0} = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) \\ \sin\theta_r & \sin(\theta_r - 120^\circ) & \sin(\theta_r + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \quad (21)$$

and

$$T(\theta_r) = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) \\ \sin\theta_r & \sin(\theta_r - 120^\circ) & \sin(\theta_r + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (22)$$

where i_{ds} is the direct axis stator current, i_{qs} is the quadrature axis stator current, and i_0 is the zero sequence stator current.

Equation (21) can be written with either the d-q axis linkage fluxes (λ_{ds}, λ_{qs}, λ_{os}) replacing d-q currents and phase flux linkages $(\lambda_{as}, \lambda_{bs}, \lambda_{cs})$ replacing phase currents or d-q axis voltages (v_{ds}, v_{qs}, v_{0s}) replacing d-q currents and phase voltages (v_{as}, v_{bs}, v_{cs}) replacing the phase currents. With the d-q transformation equation (26) and assuming balanced three phase supply, the d-q voltage equations referred to the stator side can be obtained as shown in equations (23) – (28).

$$V_{qs} = r_s I_{qs} + p\lambda_{qs} + \omega_r \lambda_{ds} \quad (23)$$
$$V_{ds} = r_s I_{ds} + p\lambda_{ds} - \omega_r \lambda_{qs} \quad (24)$$
$$V_{os} = r_s I_{os} + p\lambda_{os} \quad (25)$$
$$V_{qr} = r_{qr} I_{qr} + p\lambda_{qr} \quad (26)$$
$$V_{dr} = r_{dr} I_{dr} + p\lambda_{dr} \quad (27)$$
$$V_{fr} = r_{fr} I_{fr} + p\lambda_{fr} \quad (28)$$

Flux Linkage Equations:

$$\lambda_{qs} = L_{ls} I_{qs} + L_{mq}(I_{qs} + I_{qr}) \quad (29)$$
$$\lambda_{ds} = L_{ls} I_{ds} + L_{md}(I_{ds} + I_{dr} + I_{fr}) \quad (30)$$
$$\lambda_{os} = L_{ls} I_{os} \quad (31)$$
$$\lambda_{qr} = L_{lqr} I_{qr} + L_{mq}(I_{qs} + I_{qr}) \quad (32)$$
$$\lambda_{dr} = L_{ldr} I_{dr} + L_{md}(I_{ds} + I_{dr} + I_{fr}) \quad (33)$$

$$\lambda_{fr} = L_{lfr}I_{fr} + L_{md}\left(I_{ds} + I_{dr} + I_{fr}\right) \tag{34}$$

The relationship between torque and rotor speed is given as shown below

$$T_e = J\left(\frac{2}{p}\right)p\omega_r + T_L \tag{35}$$

And of course

$$\theta_r = \int \omega_r dt \tag{36}$$

IV. DYNAMIC SIMULATION IN MATLAB/SIMULINK®

Computer-based analysis of electrical machines requires that appropriate measures are made towards the proper selection of a simulation tool. The complex models of some electrical machines need computing tools capable of performing dynamic simulations with greater efficiency and accuracy. SIMULINK® has the advantages of being capable of complex dynamic simulations, graphical environment with visual real time programming and broad selection of toolboxes [8].

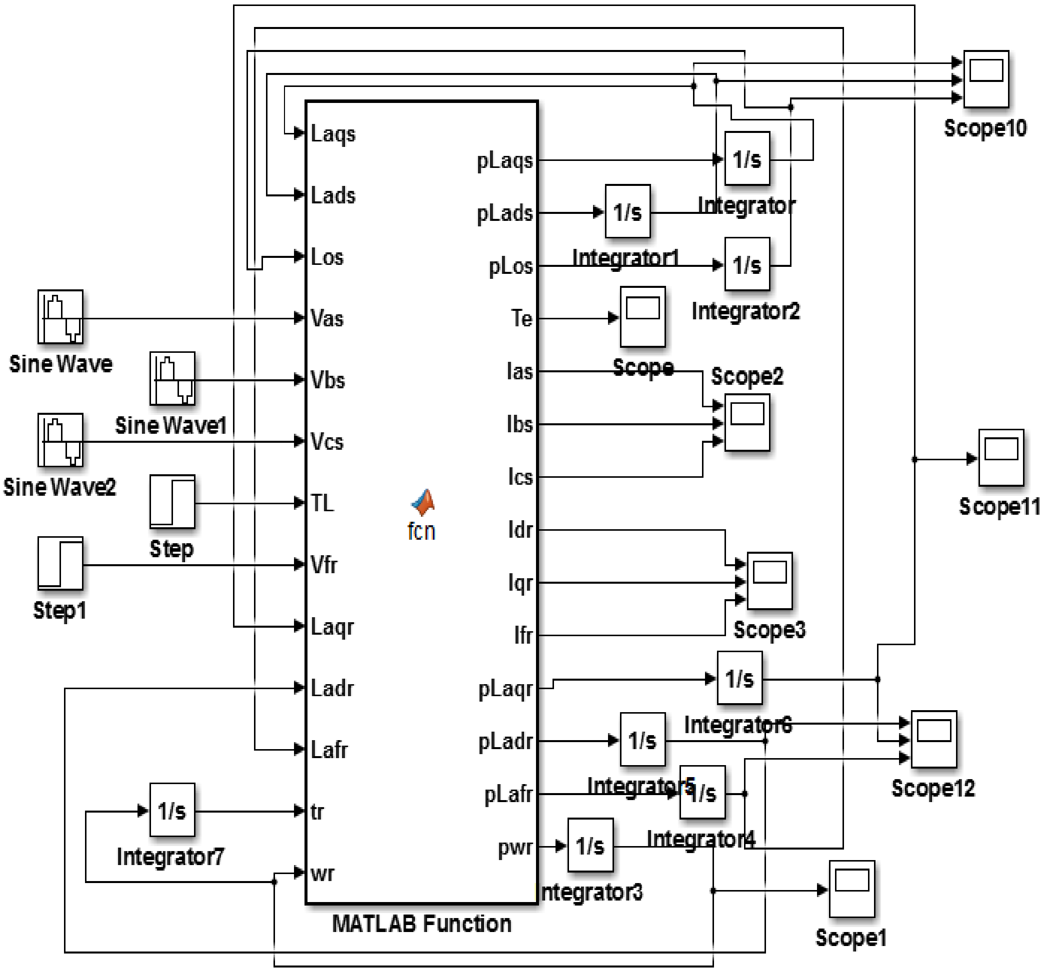

Figure 1: Simulink block diagram of a salient pole synchronous motor using embedded MATLAB function

V. RESULTS AND DISCUSSION

With the Simulink model developed in figure 1, the model is debugged and simulated. The simulation is set to run for 5 computer seconds. The rated load torque of 48Nm was set to be applied at 3 seconds when the machine has attained full speed and all the transients had died down. The parameters of 400kW salient pole synchronous motor are as shown in Table 1.

Table 1: Machine parameters

Parameter	Value	Parameter	Value
d-axis mutual inductance	$L_{md} = 0.01686$	Inertia	J = 36233
q-axis mutual inductance	$L_{mq} = 0.01447$	Poles	P = 4
Stator winding leakage inductance	$L_{ls} = 0.00076$	Rated power	400kW
Field winding leakage inductance	$L_{lfr} = 0.0075$	Frequency	50Hz
q-axis leakage inductance	$L_{lqr} = 0.0011$		
d-axis leakage inductance,	$L_{ldr} = 0.0011$		
Rotor *d*-axis winding resistance	$r_{dr} = 2.916$		
Rotor *q*-axis winding resistance	$r_{qr} = 2.916$		
Stator winding resistance	$r_s = 0.0738$		
	$r_{fr} = 0.434$		

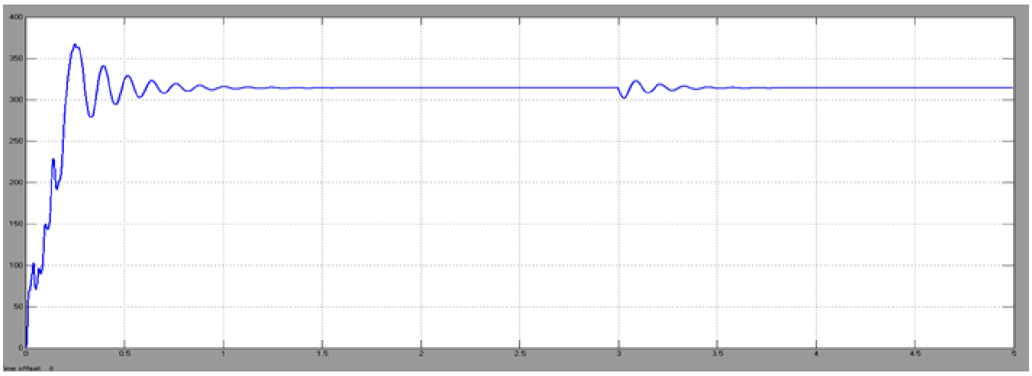

Figure 1: Rotor Speed during starting and its response to a load torque

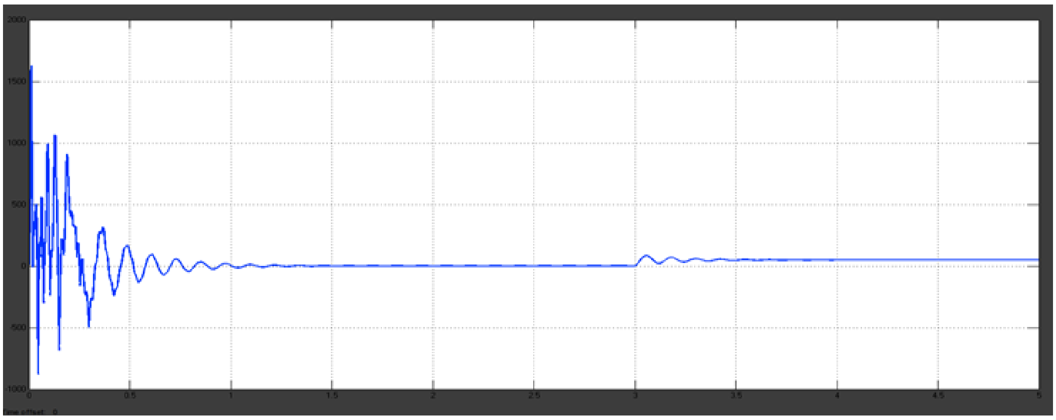

Figure 2: Electromagnetic torque.

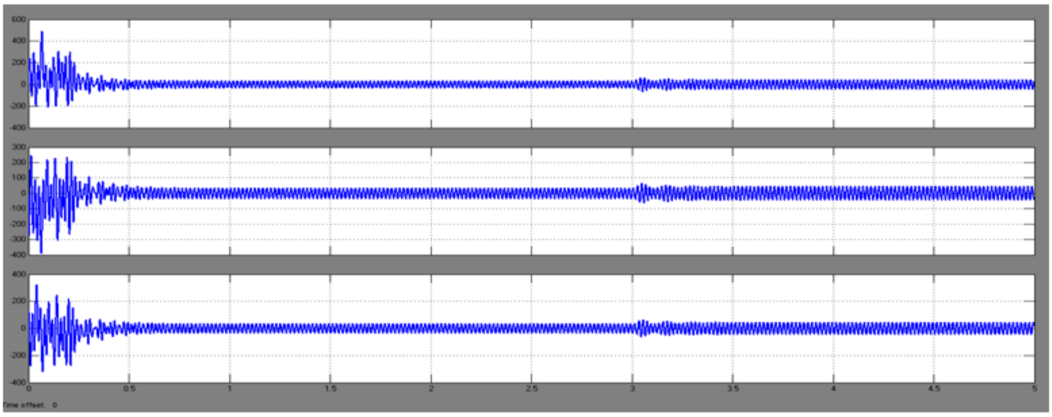

Figure 3. Stator phase currents

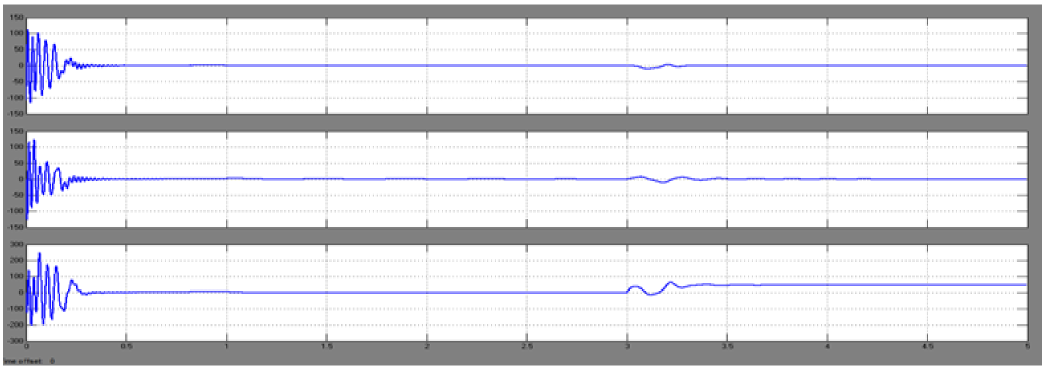

Figure 4: Rotor Currents

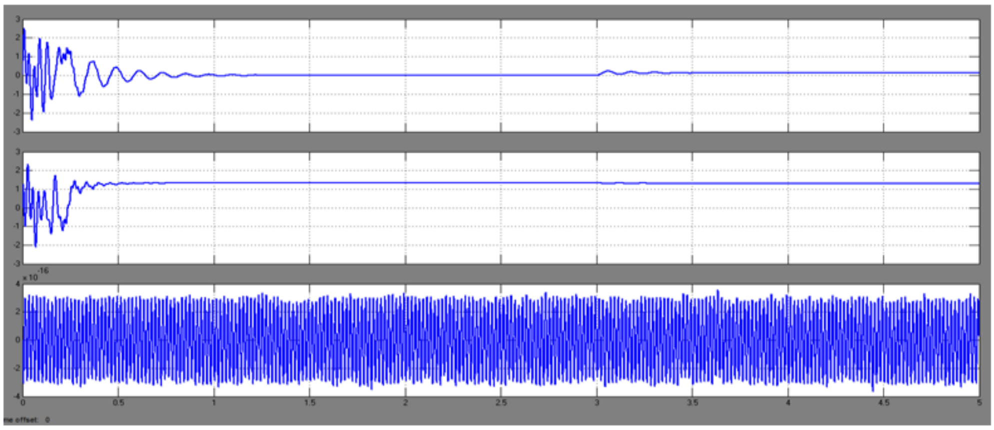

Figure 5: Stator axes flux linkages

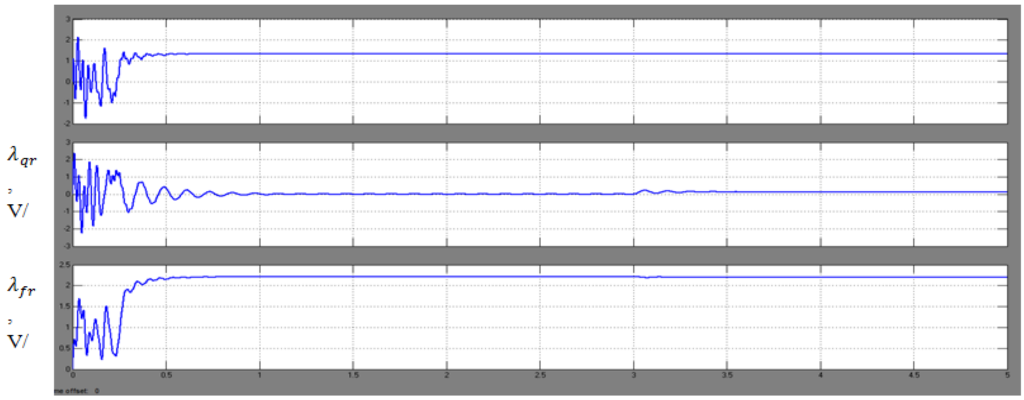

Figure 6: Rotor flux linkages

Figure 1 represents the rotor speed during starting and its response to the load torque. It indicates run up speed of the motor. Preceding the attainment of full synchronous speed, there was speed overshoot and oscillations before the speed finally settled.

Figure 2 represents the electromagnetic torque over time which shows that the torque naturally settled to zero value but reported its full rated value on application of load.

Figure 3 is a representation of stator phase currents i_{as} , i_{bs} *and* i_{cs} against time. It shows that the currents are noticeably high at starting but settles down to the rated no load current. However, on application of rated load torque, the current rose rapidly and later settles at rated full load current.

Figure 4 indicates the three rotor currents i_{dr} , i_{qr} *and* i_{fr} , which were observed to be very high but transient values were at the onset of the simulation. It also shows that as the rotor attained full speed, the rotor currents i_{dr} *and* i_{qr} became zero while the field current i_{fr} settled to 27.6A.

Figure 5 denotes stator flux linkages in dq0 frame. Transient variations were also noticed at the start of the motor. However, after the attainment of full speed by the motor, the d-axis flux linkage λ_{ds} became zero on no load synchronous speed.

Figure 6 depicts rotor flux linkages λ_{dr} , λ_{qr} *and* λ_{fr} . On application of input voltage, transients occurred but attained steady state values when the rotor speed settled to synchronous value.

VI. CONCLUSION

The objective of this paper which is to introduce a new approach into the modeling of a salient pole synchronous machine through the utilization of embedded MATLAB/Simulink Toolbox is attained. The motor is modeled using Park's Transformation and the developed model equations properly simulated in MATLAB Simulink with some programming techniques. The approach produced all the machine state variables, both in the transient and steady-state. This same approach can serve as a feasible solution to equations of any electrical machine provided the model equations are explicitly specified together with the machine parameters.

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