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Prediction of Tide Height Using the Discrete Fourier Transform

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ABSTRACT: *In this study, I have investigated some aspects of astronomical tide and predicted tide time and height by different methods. This thesis deals with the prediction of height and time for both high and low waters of the ports set up in several places by discrete Fourier transform. I computed the tide height using Discrete Fourier Transform (DFT). The results are found to be in an agreement with the predicted data of others. By this work, we can predict the tide height of overall stations if the sample observed data are available for any kind of stations. I think that my work could be helpful to predict the tides over all stations where the observed data are available.*

Keywords- *Coastal areas, discrete Fourier transform, gravity, port, tide height*.

I. INTRODUCTION

The Moon's gravity on the Earth creates tide in which other celestial bodies also take part. Tides are the alternating rise and fall of sea level with respect to land, as influenced mainly by the gravitational attraction of the Moon and the Sun. Other factors influence tides such as coastline configuration, local water depth, seafloor topography, winds, and weather. They can alter the arrival times of tides, their range, and the interval between high and low water. According to the World Resources Institute (WRI), 40% of the world's population lives in the Earth's coastal regions. People who live and work near sea-coasts have understood the importance of being able to predict tides and tidal currents. In our country, many people live in coastal region. Some of them are fishermen, salt farmers. They depend highly on tides. The marine industries are always greatly relied on tides. The tide phase governs currents in the coastal regions and many harbors are only accessible when certain water levels are exceeded. In order to allow scheduling harbors utilization and docking and sailing times, it is important to predict the times and expected water levels for high and low water. Another important application of water levels forecast can be found in disaster control. Therefore, we interested in predicting tide time, expected water levels for high and low water and their nature at some locations of Bangladesh. In order to do so the following terms are essential to describe [1-4].

In mathematics, the discrete Fourier transform (DFT) is a specific kind of Fourier transformation, used in Fourier analysis. The DFT requires an input function that is discrete and whose non-zero values having an inadequate (finite) period. Such efforts are very often, created by testing an unbroken function, like a person's voice. And unlike the Discrete Time Fourier Transformation (DTFT), it merely accesses frequent modules to reconstruct the limited fragment that was scrutinized well. Its inverse transform cannot reproduce the entire time domain, unless the input happens to be periodic [5-7]. Therefore, it is often said that the DFT is a transform for Fourier analysis of finite-domain discrete-time functions. The sinusoidal basis functions of the decomposition have the identical possessions [8-10]. Since the input task is a finite series of genuine or multifarious number, the DFT is ideal for processing information stored in computers. In particular, the DFT is widely employed in signal processing and related fields to analyze the frequencies containing in a sampled signal, to solve partial differential equation, and to perform other operations such as convolution. Before getting started on the DFT, we look at the Fourier transform (FT). The Fourier transform of a continuous-time signal $x(t)$may be defined as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt,\ \omega \in (-\infty, \infty) \tag{1}$$

The DFT, on the other hand, replaces the infinite integral with a finite sum:

$$X(\omega_k) = \sum_{n=0}^{N-1} x(t_n)e^{-j\,\omega_k t_n} \tag{2}$$

Where k=0, 1, 2, 3, … … … , N-1

Furthermore, in the ground of digital signal processing, the signals and spectra are developed only in sampled form, so that the DFT is what we really need anyway. In summary, the DFT is simpler mathematically, and more relevant computationally than that of the Fourier transform. At the same time, the basic concepts are the same. Signals are typically represented as time dependent function. Real signals are continuous or analog signals. However through sampling, the signal by gathering data, the signals does not contain high frequencies and is finite in duration. The information is then detached and the corresponding frequencies are separated and surrounded. Thus, in the process of gathering data, one seriously affects the frequency content of the signal. This is accurate for a superposition of signals that fixes frequencies. The circumstance turns out to be more complicated if the data has an overall non-constant trend or even exists in the presence of noise. By restricting data to a time interval (0, T) for period T, and extending the data to$(-\infty, \infty)$, one can generate a periodic function of infinite duration at the cost of losing data outside the fundamental period. This is not unphysical, as the data is typically taken over a finite period time. Thus, any physical results in the analysis can be obtained be restricting the outcome to the given period. We intend to discuss FT, DFT and to predict tide height using DFT at Boston, Ma and Chattagong, Sadarghat, Bangladesh [11].

II. HISTORICAL DEVELOPMENT OF DISCRETE FOURIER TRANSFORM (DFT)

In a prehistoric form of harmonic successions dates, back to very old Babylonian Mathematics, they were used to compute ephemeris (tables of astronomical positions). In modern times, variants of the districted Fourier transform were made with the intellectualities of Alexis Clairaut in 1754 to figure an orbit out of the asteroids, which has been described as the first formula for the DFT, and in 1759 by Joseph Louis Lagrange, it was considered in calculating the coefficients of a trigonometric sequence for a vibrating string. Technically, Clairaut's work was a cosine-only series (a form of discrete cosine transform), while Lagrange's work was a sine-only series (a form of discrete sine transform), a true (cosine + sine) DFT was used by Gauss in 1805 for trigonometric interpolation of asteroid orbits. Euler and Lagrange both discredited the vibrating string problem using samples [12-22].An early modern development towards Fourier analysis was that of by Lagrange, in which the method of Lagrange resolvents used a complex Fourier decomposition to study the solution of a cubic equation. Lagrange transformed the rootsx_1, x_2, x_3 into the resolvents:

$$r_1 = x_1 + x_2 + x_3$$
$$r_2 = x_1 + \xi x_2 + \xi^2 x_3$$
$$r_3 = x_1 + \xi^2 x_2 + \xi x_3$$

Where $\xi$is a cubic root of unity, which is the DFT of order 3. A number of authors, notably Jean le Rond d'Alembert, and Carl Friedrich Gauss used trigonometric series to study the heat equation, but the breakthrough development was the 1807 by Joseph Fourier whose vital approaching was to model the entire functions by trigonometric chains, bringing in the Fourier series. Daniel Bernoulli and Leonhard Euler had introduced trigonometric representations of functions, and Lagrange had given the Fourier series solution to the wave equation, so Fourier's contribution was mainly the bold claim that an arbitrary function could be represented by a Fourier series. An ensuing progress of the field is well-known as harmonic analysis, plus is also an early instance of representation theory. The first fast Fourier transform (FFT) algorithm for the DFT was revealed around the year of 1805 by Carl Friedrich Gauss. At that time, an interpolating measurement of the orbit of the asteroids of Juno and Pallas, particularly from FFT, algorithm is often attributed to its modern rediscoveries of Cooley and Tukey [23-32].

III. FOURIER SERIES REPRESENTATION

Recall that in Fourier series the signal, $y(t)$ valid for$t\epsilon\ [0, T]$, as

$$y(t) = \frac{1}{2}A_0 + \sum_{P=1}^{\infty}[A_p \cos(\overline{\omega_p} t) + B_p \sin(\overline{\omega_p} t)] \quad (3)$$

Where the angular frequency is given by

$$\omega_p = 2\pi f_p = \frac{2\pi}{T} p.$$

Most real analog signals have an infinite period, so we have made a first approximation. Thus, this has restricted the physical data to $0 \leq t \leq T$. The frequency content of a signal for a particular $f_p$is contained in both the cosine and sine terms when$A_p,\ B_p \neq 0$. So, often one may desire to combine these terms. This is easily done by using trigonometry identities namely, we will show that

$$A_p \cos(\omega_p t) + B_p \sin(\omega_p t) = C_p \cos(\omega_p t + \phi) \quad (4)$$

Where $\phi$is called the phase shift. Recall that

$$\cos(\omega_p t + \phi) = \cos(\omega_p t) \cos\phi - \sin(\omega_p t) \sin\phi \quad (5)$$

Inserting this expression above, we have

$$C_p \cos(\omega_p t + \phi) = C_p - C_p \sin(\omega_p t) \sin\phi \quad (6)$$

Equating the coefficients of the sines and the cosines in this expression and equation (4), we obtain

$$A_p = C_p \cos\phi \text{ and } B_p = -C_p \sin\phi$$

Therefore, $C_p = \sqrt{A_p + B_p}$and $\tan\phi = \frac{B_p}{A_p}$

Recall that we extract the Fourier coefficients (A_p, B_p) using the orthogonality of the trigonometric basis. For the trigonometric functions, this was given earlier by the relations

$$A_p = \frac{2}{\pi}\int_0^T y(t)\cos(\omega_p t)\,dt, \text{ p=0, 1, 2, } \ldots \ldots \ldots$$

$$B_p = \frac{2}{\pi}\int_0^T y(t)\sin(\omega_p t)\,dt, \text{ p=0, 1, 2, } \ldots \ldots \ldots \quad (7)$$

IV. DISCRETE SERIES

We first note that the data is sampled at N equally spaced times$t_n = n\Delta t$, n=0, 1, … … …, N-1, where Δt is the time increment. For a record length of T, we have$\Delta t = \frac{T}{N}$. We will denote the data at these times as$y_n = y(t_n)$, so the DFT representation is

$$y_n = \frac{1}{2}A_0 + \sum_{p=1}^{M}\left[A_p \cos(\omega_p t_n) + B_p \sin(\omega_p t_n), n = 0, 1, 2, \ldots \ldots \ldots, N-1\right. \quad (8)$$

The trigonometric arguments are given by$\omega_p t_n = \frac{2\pi\rho n}{N}$. Note that $p = 1, 2, \ldots \ldots \ldots, M$ thus allowing only for frequencies$f_p = \frac{\omega_p}{2\pi} = \frac{p}{T}$, or we can write $f_p = p\Delta f$ for$\Delta f = \frac{1}{T}$. We need to determine M and the unknown coefficients. As for the Fourier series, we will need some orthogonally relations, but this time the orthogonality statement will consist of a sum and not an integral. Let N equations for the unknown coefficients. Therefore, we should have N unknowns. For N samples, we want to determine N unknown coefficient $A_0, A_1, \ldots \ldots \ldots A_{\frac{N}{2}}$ and$B_1, B_2, \ldots \ldots \ldots B_{\frac{N}{2}-1}$. Thus, we need to fix$M = \frac{N}{2}$. Often the coefficients B_0 and $B_{\frac{N}{2}}$ are included for symmetry. So we claim that the DFT coefficients are

$$A_p = \frac{2}{N}\sum_{n-1}^{N} y(t_n)\cos\left(\frac{2\pi pn}{N}\right), p = 1, 2, \ldots \ldots \ldots \frac{N}{2} - 1 \quad (9)$$

$$B_p = \frac{2}{N}\sum_{n-1}^{N} y(t_n)\sin\left(\frac{2\pi pn}{N}\right), p = 1, 2, \ldots \ldots \ldots \frac{N}{2} - 1 \quad (10)$$

$$A_0 = \frac{1}{N}\sum_{n=1}^{N} y(t_n) \quad (11)$$

V. DISCRETE ORTHOGONALITY

The beginning of the discrete Fourier coefficients could be finished by means of the discrete orthogonality of the discrete trigonometric basis similar to the derivation of the above Fourier coefficients for the Fourier series.

The vectors $e^{\frac{2\pi i}{N}kn}$ form an orthogonal basis over the set of N-dimensional complex vectors:

$$\sum_{n=0}^{N-1}\left(e^{\frac{2\pi i}{N}kn}\right)\left(e^{-\frac{2\pi i}{N}kn}\right) = N\delta_{kk} \quad (12)$$

Where $\delta_{kk}$is the Kronecker delta. This orthogonality condition can be used to derive the formula for the IDFT from the definition of the DFT. The derivation of the discrete Fourier coefficient can be done using the discrete orthogonality of the discrete trigonometric basis similar to the derivation of the above Fourier coefficients for the Fourier series.

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi nk}{N}\right) = \begin{cases} 0, & k = 1, 2, \dots, N-1 \\ N, & k = 0, N \end{cases}$$
$$\sum_{n=0}^{N-1} \sin\left(\frac{2\pi nk}{N}\right) = 0, \quad k = 0, 1, \dots \dots \dots, N \tag{13}$$

This can be done more easily using exponential form

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi nk}{N}\right) + i\sum_{n=0}^{N-1} \sin\left(\frac{2\pi nk}{N}\right) = \sum_{n=0}^{N-1} e^{\frac{2\pi ink}{N}} \tag{14}$$

The exponential sum is the sum of a geometric progression. Namely we note that

$$\sum_{n=0}^{N-1} e^{\frac{2\pi ink}{N}} = \sum_{n=0}^{N-1} \left(e^{\frac{2\pi ik}{N}}\right)^n$$

The geometric progression is a sum of the form $S_N = \sum_{k=0}^{N-1} ar^k$. This is the sum of N terms in which consecutive terms have a constant ratio, r. We multiply the sum S_N by r and subtract the resulting sum from the original sum to obtain

$$S_N - rS_N = (a + ar + \cdots \dots \dots, +ar^{N-1}) - (ar + \cdots \dots \dots + ar^{N-1} + ar^N) = a - ar^N$$

Factoring on both sides, we have the desired sum $S_N = \frac{a(1-r^N)}{1-r}$.

Thus we have

$$\begin{aligned} \sum_{n=0}^{N-1} e^{\frac{2\pi ink}{N}} &= \sum_{n=0}^{N-1} \left(e^{\frac{2\pi ik}{N}}\right)^n \\ &= 1 + e^{\frac{2\pi ik}{N}} + \left(e^{\frac{2\pi ik}{N}}\right)^2 + \cdots \dots \dots + \left(e^{\frac{2\pi ik}{N}}\right)^{N-1} \\ &= \frac{1-\left(e^{\frac{2\pi ik}{N}}\right)^N}{1-e^{\frac{2\pi ik}{N}}} \\ &= \frac{1-e^{2\pi ik}}{1-e^{\frac{2\pi ik}{N}}} \end{aligned}$$

When $k \neq 0, N$ the numerator is 0. In the special case, $k = 0, N$, we have

$$\sum_{n=0}^{N-1} e^{\frac{2\pi ink}{N}} = \sum_{n=0}^{N-1} 1 = N$$

Therefore,

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi nk}{N}\right) + i\sum_{n=0}^{N-1} \sin\left(\frac{2\pi nk}{N}\right) = \begin{cases} 0, & k = 1, 2, \dots, N-1 \\ N, & k = 0, N \end{cases} \tag{15}$$

We can use this to establish orthogonality relation of the following type for $p, q = 1, 2, \dots \dots \dots \frac{N}{2}$.

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi pn}{N}\right) \cos\left(\frac{2\pi qn}{N}\right) = \frac{1}{2}\left[\sum_{n=0}^{N-1} \cos\left(\frac{2\pi(p-q)n}{N}\right) + \cos\left(\frac{2\pi(p+q)n}{N}\right)\right]$$

Now splitting the above sum into two sums, then

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi(p-q)n}{N}\right) = \begin{cases} 0, & p \neq q \\ N & p = q \end{cases}$$
$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi(p+q)n}{N}\right) = \begin{cases} 0 & p = q \neq \frac{N}{2} \\ N & p + q = N \end{cases}$$

Therefore, we have

$$\sum_{n=0}^{N-1} \cos\left(\frac{2\pi pn}{N}\right) \cos\left(\frac{2\pi qn}{N}\right) = \begin{cases} \frac{N}{2} & p = q \neq \frac{N}{2} \\ N & p = q = \frac{N}{2} \\ 0 & Otherwise \end{cases} \tag{16}$$

Similarly we find

$$\sum_{n=0}^{N-1} \sin\left(\frac{2\pi pn}{N}\right) \cos\left(\frac{2\pi qn}{N}\right) = \frac{1}{2}\left[\sum_{n=0}^{N-1} \sin\left(\frac{2\pi(p-q)n}{N}\right) + \sin\left(\frac{2\pi(p+q)n}{N}\right)\right] \tag{17}$$

and

$$\begin{aligned} \sum_{n=0}^{N-1} \sin\left(\frac{2\pi pn}{N}\right) \sin\left(\frac{2\pi qn}{N}\right) &= \frac{1}{2}\left[\sum_{n=0}^{N-1} \cos\left(\frac{2\pi(p-q)n}{N}\right) - \cos\left(\frac{2\pi(p+q)n}{N}\right)\right] \\ &= \begin{cases} \frac{N}{2}, & p = q \neq \frac{N}{2} \\ 0 & Otherwise \end{cases} \end{aligned} \tag{18}$$

VI. DISCRETE FOURIER TRANSFORM

The derivation of the coefficient for the DFT is now easily done using the discrete orthogonality used, we have

$$y_n = \frac{1}{2}A_0 + \sum_{p=1}^{\frac{N}{2}}\left[A_p \cos\left(\frac{2\pi pn}{N}\right) + B_p \sin\left(\frac{2\pi pn}{N}\right)\right],\ n = 0, 1, \ldots, N-1$$

We first sum over n , yields

$$\sum_{n=0}^{N-1} y_n = \sum_{n=0}^{N-1}\left\{\frac{1}{2}A_0 + \sum_{p=1}^{\frac{N}{2}}\left[A_p \cos\left(\frac{2\pi pn}{N}\right) + B_p \sin\left(\frac{2\pi pn}{N}\right)\right]\right\}$$

$$= \frac{1}{2}A_0 \sum_{n=0}^{N-1} 1 + \sum_{p=1}^{\frac{N}{2}}\left[A_p \sum \cos\left(\frac{2\pi pn}{N}\right) + B_p \sum_{n=0}^{N-1} \sin\left(\frac{2\pi pn}{N}\right)\right]$$

$$= \frac{1}{2}A_0 N + \sum_{p=1}^{\frac{N}{2}}(A_p.0 + B_p.0)$$

$$= \frac{1}{2}A_0 N \tag{19}$$

Therefore, we have

$$A_0 = \frac{2}{N}\sum_{n=0}^{N-1} y(t_n) \tag{20}$$

Now, we multiply both sides by $\cos\left(\frac{2\pi qn}{N}\right)$ and sum over n. Then

$$\sum_{n=0}^{N-1} y_n \cos\left(\frac{2\pi qn}{N}\right) = \sum_{n=0}^{N-1}\left\{\frac{1}{2}A_0 + \sum_{p=1}^{\frac{N}{2}}\left[A_p \cos\left(\frac{2\pi pn}{N}\right) + B_p \sin\left(\frac{2\pi pn}{N}\right)\right]\right\}\cos\left(\frac{2\pi qn}{N}\right)$$

$$= \frac{1}{2}A_0 \sum_{n=0}^{N-1}\cos\left(\frac{2\pi qn}{N}\right) + \sum_{p=1}^{\frac{N}{2}}\left[A_p \sum_{n=0}^{N-1}\cos\left(\frac{2\pi pn}{N}\right)\cos\left(\frac{2\pi qn}{N}\right) + B_p \sum_{n=0}^{N-1}\sin\left(\frac{2\pi pn}{N}\right)\cos\left(\frac{2\pi qn}{N}\right)\right]$$

$$= \begin{cases} \sum_{p=1}^{\frac{N}{2}}\left\{A_p \frac{N}{2}\delta_{p,q} + B_p.0\right\} & q \neq \frac{N}{2} \\ \sum_{p=1}^{\frac{N}{2}}\left\{A_p N\delta_{p,\frac{N}{2}} + B_p.0\right\} & q = \frac{N}{2} \end{cases}$$

$$= \begin{cases} \frac{1}{2}A_q N & q \neq \frac{N}{2} \\ A_{\frac{N}{2}} N & q = \frac{N}{2} \end{cases}$$

Thus we found that

$$A_q = \frac{2}{N}\sum_{n=0}^{N-1} y(t_n)\cos\left(\frac{2\pi qn}{N}\right),\ q \neq \frac{N}{2} \tag{21}$$

$$A_{\frac{N}{2}} = \frac{1}{N}\sum_{n=0}^{N-1} y(t_n)\cos\left(\frac{2\pi n\left(\frac{N}{2}\right)}{N}\right) = \frac{1}{N}\sum_{n=0}^{N-1} y(t_n)\cos(\pi n) \tag{22}$$

Similarly,

$$\sum_{n=0}^{N-1} y_n \sin\left(\frac{2\pi qn}{N}\right) = \sum_{n=0}^{N-1}\left\{\frac{1}{2}A_0 + \sum_{p=1}^{\frac{N}{2}}\left[A_p \cos\left(\frac{2\pi pn}{N}\right) + B_p \sin\left(\frac{2\pi pn}{N}\right)\right]\right\}\sin\left(\frac{2\pi qn}{N}\right)$$

$$= \frac{1}{2}A_0 \sum_{n=0}^{N-1}\sin\left(\frac{2\pi qn}{N}\right) + \sum_{p=1}^{\frac{N}{2}}\left[A_p \sum_{n=0}^{N-1}\cos\left(\frac{2\pi pn}{N}\right)\sin\left(\frac{2\pi qn}{N}\right) + B_p \sum_{n=0}^{N-1}\sin\left(\frac{2\pi pn}{N}\right)\sin\left(\frac{2\pi qn}{N}\right)\right]$$

$$= \sum_{p=1}^{\frac{N}{2}}\left\{A_p.0 + B_p \frac{N}{2}\delta_{p,q}\right\}$$

$$= \frac{1}{2}B_q N \tag{23}$$

Finally, we get

$$B_q = \frac{2}{N}\sum_{n=0}^{N-1} y(t_n)\sin\left(\frac{2\pi qn}{N}\right),\quad q = 1, 2, \dots \dots \dots, \frac{N}{2} - 1 \tag{24}$$

VII. APPLICATION OF DFT ON TIDE

We have

$$y_n = \frac{1}{2}A_0 + \sum_{p=1}^{\frac{N}{2}}\left[A_p\cos\left(\frac{2\pi pn}{N}\right) + B_p\sin\left(\frac{2\pi pn}{N}\right)\right],\ n = 0, 1, \dots, N-1$$

$$\text{or, } y_n = \frac{1}{2}A_0 + \sum_{p=1}^{M}\left[A_p\cos(\omega_p t_n) + B_p\sin(\omega_p t_n)\right],\ n = 0, 1, \dots, N-1$$

Where $\omega_p t_n = \frac{2\pi pn}{N}$

Suppose, to take a simple example that the tidal curve from a port was exactly

$$y(t) = \frac{1}{2}A_0 + A_1\cos(\omega_1 t) + B_1\sin(\omega_1 t) + A_2\cos(\omega_2 t) + B_2\sin(\omega_2 t) \tag{25}$$

Where the speeds ω_1 and ω_2 are known.

To predict tide height, we need to calculate the coefficients A_0, A_1, A_2, B_0, B_1, B_2. These can be done easily by using equations (9), (10), (11) and (25). We have computed all the coefficients needed and predicted tide height by developing the Matlab routine (Harman, 2005).

We choose the port Boston, Ma with identity No. 8443970 and Chittagong, Sadarghat to predict tide height.

We choose these ports because observed and predicted data are available here. As input, we have supplied observed data and predicted data was obtained. To show all the results in tabular form is space consuming, therefore, we show them in graphical form by Figs 1, 2 and 3. Fig.1 shows our predicted data (red curve) along with observed (blue curve) and predicted (dotted blue curve) data by NOAA at Boston, Ma. Fig.2 shows the same curves with the reduction of time axis. Fig.3 indicates our predicted data (red curve) along with observed (green curve) and predicted (blue) data by Chittagong port authority at Chittagong, Sadarghat, Bangladesh.

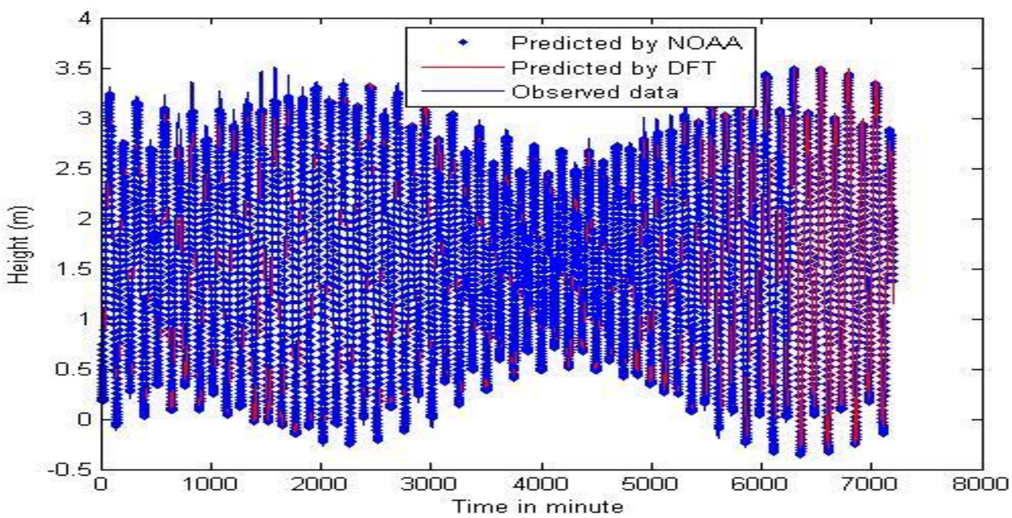

Fig.1. Tide height measuring 30 day long time range

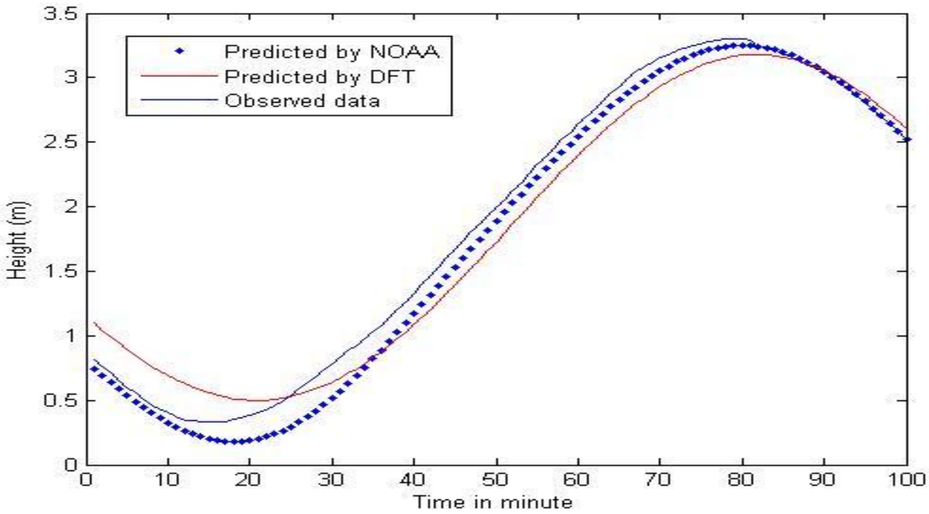

Fig.2. Tide height measuring 30 day short time range

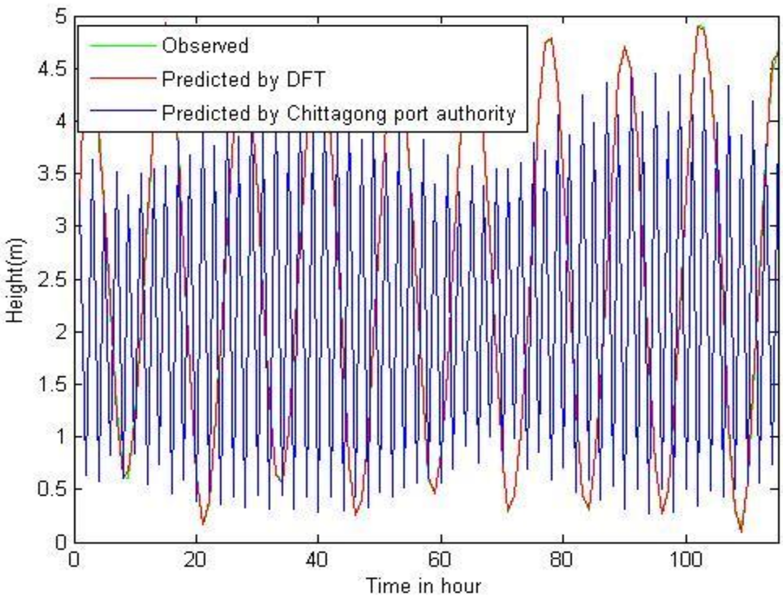

Fig.3. Tide height at, Sadarghat (Karnafully river), Chittagong

VIII. CONCLUSION

I have computed tide height using DFT. The results are found to be in good agreement with the predicted data of others. By this work, we can predict tide height over all stations if the sample observed data are available for those stations. I think, my work could be helpful to predict the tide-height overall way especially, in the coastal areas.

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