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Plane Waves in Transversely Isotropic Viscothermoelastic Medium with Two Temperatures and Rotation

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ABSTRACT: *The present investigation is to study the plane wave propagation and reflection ofplane waves in a homogeneous transversely isotropic viscothermoelastic medium with two temperature and rotation in the context of GN type-II and type-III (1993) theory of thermoelasticity. It is found that, for two dimensional assumed model, there exist three types of coupled longitudinal waves, namely quasi-longitudinal wave (QL), quasi-transverse wave (QTS) and quasi -thermal waves (QT). The different characteristics of waves like phase velocity, attenuation coefficients, specific loss and penetration depth are computed numerically and depicted graphically. The phenomenon of reflection coefficients due to quasi-waves at a plane stress free with thermally insulated boundary is investigated. Theratios of the amplitudes of the reflected waves to that of incident waves are calculated as a non-singular system of linear algebraic equations. These amplitude ratios are used further to calculate the shares of different scattered waves in the energy of incident wave. The conservation of energy at the free surface is verified. The effect of viscosity on the energy ratios are depicted graphically and discussed. Some special cases of interest are also discussed.*

Keywords: *Phase velocity, Attenuation coefficients, Energy ratios, Penetration depth, viscothermoelasticity.*

I. INTRODUCTION

The problem of elastic wave propagation in different media is an important phenomenon in the field of seismology, earthquake engineering and geophysics. The elastic wave propagating through the earth (seismic waves) have to travel through different layers and interfaces. These waves have different velocities and are influenced by the properties of the layer through which they travel. The signals of these waves are not only helpful in providing information about the internal structures of the earth but also helpful in exploration of valuable materials such as minerals, crystals and metals etc. This technique is one of the most suitable in terms of time saving and economy.

As the importance of anisotropic devices has increased in many fields of optics and microwaves, wave propagation in anisotropic media has been widely studied over in the last decades. The anisotropic nature basically stems from the polarization or magnetization that can occur in materials when external fields pass by. Mathematical modeling of plane wave propagation along with the free boundary of an elastic half-space has been subject of continued interest for many years. Keith and Crampin (1977) derived a formulation for calculating the energy division among waves generated by plane waves incident on a boundary of anisotropic media. Wave propagation in a microstretch thermoelastic diffusion solid has been investigated by Kumar (2015). Reflection of plane waves at the free surface of a transversely isotropic thermoelastic diffusive solid half-space has been discussed by Kumar and Kansal (2011).Wave propagation has remained thestudy of concern of many researchers (Marin Marin (2013),Kumar and Mukhopadhyay(2010), Lee and Lee (2010),Kumar and Gupta (2013), Othman (2010),Kaushal, Kumar and Miglani(2011), Kumar, Sharma and Ram (2008),Kaushal, Sharma and Kumar(2010)).

The theoretical study and applications in viscoelastic materialshavebecomeanimportanttaskforsolidmechanics with the rapid development of polymer science and plastic industry as well as with the wide use of materials under high temperature in modern technology and application of biology and geology in engineering.

Freudenthal (1954) pointed out that most solids when subjected to dynamic loading exhibit viscous effects. The Kelvin -Voigt model is one of the macroscopic mechanical models often used to describe the viscoelastic behaviour of a material. This model represents the delayed elastic response subjected to stress where the deformation is time dependent. Iesan and Scalia (1989) studied some theorems in the theory of thermoviscoelasticity. Borrelli and Patria (1991) investigated the discontinuity of waves through a linear thermoviscoelastic solid of integral type. Corr et al. (2001) investigated the nonlinear generalized Maxwell fluid model for viscoelastic materials. Effect of viscosity on wave propagation in anisotropic thermoelastic medium with three-phase-lag model was discussed by Kumar, Chawlaand Abbas (2012). Abd-Alla and Mahmoudstudied Magneto-Thermo-Viscoelastic Interactions in an Unbounded Non-homogeneous Body with a Spherical Cavity Subjected to a Periodic Loading .Effect of rotation, magnetic field and a periodic loading on radial vibrations thermo-viscoelastic non-homogeneous media was investigated by Basyouni, Mahmoud and Alzahrani (2014).Harry (2014) analysed coupled longitudinal 1–d thermal and viscoelastic waves in media with temperature dependent material properties.

Kumar R and Sharma N (2008), Kumar R, Sharma K.D and Garg S.K (2012), Ahmed S, El –Karamany, Magdy A and Ezzat (2015), Kumar R, Sharma K.D and Garg S.K, (2015), investigated different types of problems in viscothermoelastic media by considering various mathematical models.

Yadav, Kalkal and Deswal (2015) investigated a state problem of Two-Temperature generalized thermoviscoelasticity with fractional order strain subjected to moving heat source. Surface waves problem in a thermoviscoelastic porous half-space were analysed by Chiriţă (2015).

Chen and Gurtin (1968), Chen et al. (1968) and Chen et al. (1969) have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body (Boley and Tolins (1962)).The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen (1973).

Green and Naghdi (1991) postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearized version of model-I corresponds to classical thermoelastic model (based on Fourier's law). The linearized version of model-II and III permit propagation of thermal waves at finite speed. Green-Naghdi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy (1993) .In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green-Naghdi's third model (GN-III) admits dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi (1992) included the derivation of a complete set of governing equations of a linearized version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem.

A comprehensive work has been done in thermoelasticity theory with and without energy dissipation and thermoelasticity with two temperature. Youssef (2011), constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Quintanilla (2002) investigated thermoelasticity without energy dissipation of materials with microstructure. Several researchers studied various problems involving two temperature e.g. (Youssef and AI-Lehaibi (2007); Youssef (2006); Youssef (2011); Ezzat and Awad (2010); Sharma and Marin (2013); Sharma and Bhargav (2014); Sharma, Sharma and Bhargav (2013); Sharma and Kumar (2013); Abbas et al. (2014)).

In view of the fact that most of the large bodies like the earth, the moon and other planets have an angular velocity, as well as earth itself behaves like a huge magnet, it is important to study the propagation of thermoelastic waves in a rotating medium under the influence of magnetic field. So, the attempts are being made to study the propagation of finite thermoelastic waves in an infinite elastic medium rotating with angular velocity. Several authors (Das and Kanoria (2014); Pal, Das and Kanoria (2015); Atwa and Jahangir (2014)) have studied various problems in generalized thermoelasticity to study the effect of rotation.

Here in this paper, we analyse the reflection of plane waves incident at the stress free, thermally insulated surface of a homogeneous, transversely isotropic magnetothermoelastic solid with two temperature along with rotation in the context of GN type-II and type-III theory of thermoelasticity. The graphical representation is given for amplitude and energy ratios of various reflected waves to that of incident waves for different values of incident angle. Also phase velocity and attenuation coefficients of plane waves are computed and presented graphically for different values of wave frequencyω.

II. BASIC EQUATIONS

The simplified Maxwell's linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$\text{curl}\,\vec{h} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (1)$$

$$\text{curl}\vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t} \quad (2)$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} \times \overrightarrow{H_0}\right) \quad (3)$$

$$\text{div}\vec{h} = 0 \quad (4)$$

Maxwell stress components are given by

$$T_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}) \quad (5)$$

where$\overrightarrow{H_0}$is the external applied magnetic field intensity vector, $\vec{h}$is the induced magnetic field vector, $\vec{E}$ is the induced electric field vector, $\vec{J}$ is the current density vector, $\vec{u}$ is the displacement vector, $\mu_0 and \varepsilon_0$are the magnetic and electric permeability respectively, T_{ij} are the components of Maxwell stress tensor and δ_{ij} is the Kronecker delta.

The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T \quad (6)$$

Equation of motion for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity$\boldsymbol{\Omega} = \Omega n$, where n is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho\{\ddot{u}_i + \left(\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times u)\right)_i + (2\boldsymbol{\Omega} \times \dot{u})_i \quad (7)$$

The heat conduction equation following Chandrasekharaiah (1998) and Youssef (2006) is

$$K_{ij} \varphi_{,ij} + K_{IJ}^* \dot{\varphi}_{ij} = \beta_{ij} T_0 \ddot{e_{ij}} + \rho C_E \ddot{T} \quad (8)$$

The strain displacement relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) i,j = 1,2,3$$

where$F_i = \mu_0 (\vec{J} \times \overrightarrow{H_0})_i$ are the components of Lorentz force.

$\beta_{ij} = C_{ijkl} \alpha_{ij}$ And$T = \varphi - a_{ij} \varphi_{,ij}$

$\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i \delta_{ij}$, $K_{ij}{}^* = K_i{}^* \delta_{ij}$,i is not summed

C_{ijkl} ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$) are elastic parameters,β_{ij} is the thermal tensor,T is the temperature, $T_0$is the reference temperature,t_{ij} are the components of stress tensor,e_{kl} are the components of strain tensor,u_i are the displacement components, ρ is the density, C_E is the specific heat,K_{ij} is the materialistic constant, $K_{IJ}^*$is the thermal conductivity, a_{ij} are the two temperature parameters,α_{ij} is the coefficient of linear thermal expansion,Ω is the angular velocity of the solid .

III. FORMULATION AND SOLUTION OF THE PROBLEM

Considering a homogeneous perfectly conducting transversely isotropic magnetothermoelastic medium with two temperature and rotation in the context of GN type-II and type-III theory of thermoelasticity initially at a uniform temperatureT_0 . The origin of rectangular Cartesian co-ordinate system(x_1, x_2, x_3) is taken at any point on the plane horizontal surface. We take x_3 −axis along the axis material symmetry and pointing vertically downwards into the medium, which is thus represented by$x_3 \geq 0$. The surface (x_3=0) is subjected to stress free, thermally insulated boundary conditions. We choose x_1 −axis in the direction of wave propagation so that all particles on a line parallel to x_2 −axis are equally displaced. Therefore, all the field quantities will be independent of x_2 −co-ordinate. Following Slaughter (2002), using appropriate transformations, on the set of equations (6)-(7), we derive the basic equations for transversely isotropic thermoelastic solid. The components of displacement vector $\vec{u}$ and conductive temperature φ for the two dimensional problem have the form

$$\vec{u}(x_1, x_3, t) = (u_1, 0, u_3) , \text{ and } \varphi = \varphi(x_1, x_3, t) \quad (9)$$

We also assume that

$$\boldsymbol{\Omega} = (0, \Omega, 0) \quad (10)$$

From the generalized Ohm's law

$$J_2 = 0 \quad (11)$$

The current density components J_1 and J_3 are given as

$J_1 = -\varepsilon_0\mu_0 H_0 \frac{\partial^2 u_3}{\partial t^2}$ (12)
$J_3 = \varepsilon_0\mu_0 H_0 \frac{\partial^2 u_1}{\partial t^2}$ (13)
Equations (7) and (8) with the aid of (9)-(13), yield

$$c_{11}\frac{\partial^2 u_1}{\partial x_1{}^2} + c_{13}\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + c_{44}\left(\frac{\partial^2 u_1}{\partial x_3{}^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3}\right) - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1{}^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3{}^2}\right)\right\} - \mu_0 J_3 H_0 = \rho\left(\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t}\right) \quad (14)$$

$$(c_{13} + c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1{}^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3{}^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1{}^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3{}^2}\right)\right\} + \mu_0 J_1 H_0 = \rho\left(\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t}\right) \quad (15)$$

$$\left(k_1 + k_1{}^* \frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial x_1{}^2} + \left(k_3 + k_3{}^* \frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial x_3{}^2} = T_0 \frac{\partial^2}{\partial t^2}\left\{\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right\} + \rho C_E \ddot{T} \quad (16)$$

and $t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T$ (17)
$t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T$ (18)
$t_{13} = 2c_{44}e_{13}$ (19)
where $T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)$
$\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3$, $\beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$
In the above equations we use the contracting subscript notations $(11 \to 1, 22 \to 2, 33 \to 3, 23 \to 4, 31 \to 5, 12 \to 6)$ to relate c_{ijkl} to c_{mn}
In order to account for the material damping behavior the material coefficients c_{ij} are assumed to be a function of time operator D $=\frac{\partial}{\partial t}$, i.e.
$c_{ij} = c_{ij}{}^*$ where $c_{ij}{}^* = c_{ij}(D)$ (*)
Assuming that the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity (Kaliski (1963)), we write
$c_{ij}(D) = c_{ij}\left(1 + \tau_0 \frac{\partial}{\partial t}\right)$, (**)
where τ_0 is the relaxation time assumed to be identical for each c_{ij}.
To facilitate the solution , following dimensionless quantities are introduced:

$$x_1{}' = \frac{x_1}{L},\ x_3{}' = \frac{x_3}{L},\ u_1{}' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1,\ u_3{}' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3,\ T' = \frac{T}{T_0},\ t' = \frac{c_1}{L}t,\ t_{11}' = \frac{t_{11}}{\beta_1 T_0},$$

$$t_{33}' = \frac{t_{33}}{\beta_1 T_0},\ t_{31}' = \frac{t_{31}}{\beta_1 T_0},\ \varphi' = \frac{\varphi}{T_0},\ a_1' = \frac{a_1}{L},\ a_3' = \frac{a_3}{L},\ h' = \frac{h}{H_0},\ \Omega' = \frac{L}{C_1}\Omega \quad (20)$$

Making use of (20), (*), (**) in equations (14)-(16), after suppressing the primes, yield

$$\frac{\partial^2 u_1}{\partial x_1{}^2} + \delta_4 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \delta_2\left(\frac{\partial^2 u_1}{\partial x_3{}^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3}\right) - \frac{\partial}{\partial x_1}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial x_3{}^2}\right)\right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1\right)\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t} \quad (21)$$

$$\delta_1 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \delta_2 \frac{\partial^2 u_3}{\partial x_1{}^2} + \delta_3 \frac{\partial^2 u_3}{\partial x_3{}^2} - \frac{\beta_3}{\beta_1}\frac{\partial}{\partial x_3}\left\{\varphi - \left(\frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial x_3{}^2}\right)\right\} = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1\right)\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t} \quad (22)$$

$$\varepsilon_1\left(1 + \frac{\varepsilon_3}{\varepsilon_1}\frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial x_1{}^2} + \varepsilon_2\left(1 + \frac{\varepsilon_4}{\varepsilon_2}\frac{\partial}{\partial t}\right)\frac{\partial^2 \varphi}{\partial x_3{}^2} = \varepsilon_5{}' \beta_1{}^2 \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x_1} + \frac{\beta_3}{\beta_1}\frac{\partial u_3}{\partial x_3}\right) + \frac{\partial^2}{\partial t^2}\left(\left\{\varphi - \frac{a_1}{L}\frac{\partial^2 \varphi}{\partial x_1{}^2} + \frac{a_3}{L}\frac{\partial^2 \varphi}{\partial x_3{}^2}\right\}\right) \quad (23)$$

$\delta_1 = \frac{(c_{13}{}^* + c_{44}{}^*)}{c_{11}{}^*}$, $\delta_2 = \frac{c_{44}{}^*}{c_{11}{}^*}$, $\delta_3 = \frac{c_{33}{}^*}{c_{11}{}^*}$, $\delta_4 = \frac{c_{13}{}^*}{c_{11}{}^*}$, $\varepsilon_1 = \frac{k_1}{\rho C_E c_1^2}$, $\varepsilon_2 = \frac{k_3}{\rho C_E c_1^2}$, $\varepsilon_3 = \frac{k_1{}^*}{L\rho C_E c_1}$

$\varepsilon_4 = \frac{k_3{}^*}{L\rho C_E c_1}, \varepsilon_5{}' = \frac{T_0}{\rho^2 C_E c_1^2}$

IV. PLANE WAVE PROPAGATION

we seek plane wave solution of the equations of the form

$$\begin{pmatrix} u_1 \\ u_3 \\ \varphi \end{pmatrix} = \begin{pmatrix} U_1 \\ U_3 \\ \varphi^* \end{pmatrix} \exp\{i\left(\omega t - \xi(x_1 sin\theta - x_3 cos\theta)\right)\} \tag{24}$$

where $(sin\theta, cos\theta)$ denotes the projection of the wave normal onto the x_1, x_3 plane, ξ and ω are respectively the wave number and angular frequency of plane waves propagating in $x_1 - x_3$ plane.
Upon using (24) in (21)-(23) and then eliminating $U_1, U_3 and \varphi^*$ from the resulting equations yields the following characteristic equation

$$A\xi^6 + B\xi^4 + C\xi^2 + D = 0 \tag{25}$$

where $A = \zeta_4\zeta_5\zeta_6 - cos^2\theta\zeta_2\zeta_7 p_1 - \delta_1\zeta_8^2\zeta_1\zeta_4 + \zeta_2\zeta_7\zeta_8 + \zeta_2\zeta_7\zeta_8^2\zeta_1$

$$B = -\zeta_1\zeta_4\zeta_6 - \zeta_1 cos^2\theta\zeta_2\zeta_7 p_1 + \omega^2\zeta_6\zeta_5 - \zeta_4\zeta_5\zeta_1 + \zeta_5 cos^2\theta\zeta_7 p_1 - \delta_1\zeta_4\zeta_3\zeta_8 + \zeta_2\zeta_7\zeta_8 - \delta_1\omega^2\zeta_1\zeta_8^2 + \zeta_3\zeta_8\zeta_1\zeta_4 - \zeta 1_8^2\zeta_1\zeta_7 - \zeta_2\zeta_8\zeta_3\zeta_7 + \delta_1\zeta_8\zeta_7 + p_1\zeta_7\zeta_6 sin^2\theta - sin^2\theta\zeta_2\zeta_7 p_1$$

$$C = -\zeta_5\zeta_1\omega^2 - \zeta_1\zeta_6\omega^2 + \zeta_1^2\zeta_4 - \zeta_1 cos^2\theta\zeta_7 p_1 - \zeta_3\delta_1\omega^2\zeta_8 + \zeta_3^2\zeta_4 + \zeta_8\zeta_3\zeta_1\omega^2 - \zeta_1\varepsilon_5{}'\beta_1^2\omega^2 sin^2\theta$$

$$D = \omega^2(\zeta_1^2 - \zeta_3^2)$$

$\zeta_1 = \left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)\omega^2 + \Omega^2$, $\zeta_2 = \frac{a_1}{L} sin^2\theta + \frac{a_3}{L} cos^2\theta$, $\zeta_3 = 2i\omega\Omega$, $\zeta_4 = \zeta_2\omega^2 - sin^2\theta(\varepsilon_1 + i\varepsilon_3) - cos^2\theta\,(\varepsilon_2 + i\varepsilon_4)$, $\zeta_5 = sin^2\theta + \delta_2 cos^2\theta$, $\zeta_6 = \delta_2 sin^2\theta + \delta_3 cos^2\theta$, $\zeta_7 = \varepsilon_5{}'\omega^2\beta_1\beta_3$, $\zeta_8 = sin\theta cos\theta$, $p_5 = \frac{\beta_3}{\beta_1}$

The roots of equation (25) gives six values of, in which we are interested to those roots whose imaginary parts are positive. Corresponding to these roots, there exists three waves corresponding to decreasing orders of their velocities, namely quasi-longitudinal, quasi-transverse and quasi-thermal waves. The phase velocities, attenuation coefficients, specific loss and penetration depth of these waves are obtained by the following expressions

(4.1) Phase velocity

The phase velocity is given by

$V_j = \frac{\omega}{|Re(\xi_j)|}$, j=1, 2, 3

where V_j, j=1,2,3 are the phase velocities of QL, QTSand QTwaves respectively.

(4.2) Attenuation coefficient

The attenuation coefficient is defined by

$Q_j = Im(\xi_j)$, j=1, 2, 3

whereQ_j , j=1, 2,3 are the attenuation coefficients of QL, QTS and QT waves respectively.

(4.3) Specific loss

The specific loss is the ratio of energy (Δw) dissipated in taking a specimen through a stress cycle, to the elastic energy (w) stored in the specimen when the strain is maximum. The specific loss is the most direct method of defining internal fraction of a material. For a sinusoidal plane wave of small amplitude, the specific loss $\left(\frac{\Delta w}{w}\right)$ equals 4π times the absolute value of the imaginary part of ξ to the real part of ξ ,i.e.

$R_i = \left(\frac{\Delta w}{w}\right)_j = 4\pi\left|\frac{Im(\xi_j)}{Re(\xi_j)}\right|$, j=1, 2, 3

whereR_1, R_2, R_3 are the specific losses of QL, QTS and QT waves respectively.

(4.4) Penetration depth

The penetration depth is defined by$S_j = \frac{1}{|Im(\xi_j)|}$, j=1, 2, 3

whereS_1, S_2, S_3 are the penetration depths of QL, QTS and QT waves respectively.

V. REFLECTION AND TRANSMISSION AT THE BOUNDARY SURFACES

we consider a homogeneous transversely isotropic viscoelastic half-space occupying the region $x_3 \geq 0$. Incident quasi-longitudinal or quasi-transverse or quasithermal waves at the stress free , thermally insulated surface $(x_3 = 0)$ will generate reflected QL,reflected QTS and reflected QTwaves in the half-space $x_3 > 0$. The total displacements , conductive temperature are given by

$u_1 = \sum_{j=1}^{6} A_j e^{iM_j}$, $\quad u_3 = \sum_{j=1}^{6} d_j A_j e^{iM_j}$, $\quad \varphi = \sum_{j=1}^{6} l_j A_j e^{iM_j}$, j=1,2.........,6 (26)

where

$M_j = \omega t - \xi_j (x_1 sin\theta_j - x_3 cos\theta_j)$, j=1, 2, 3

$M_j = \omega t - \xi_j (x_1 sin\theta_j + x_3 cos\theta_j)$, j=4, 5, 6

Here subscripts j=1, 2, 3 respectively denote the quantities corresponding to incident QL, QTS and QT-mode, whereas the subscripts j=4, 5, 6 denote the corresponding reflected waves.

$$d_j = \frac{\xi_j^4(\zeta_{8j}\zeta_{2j}\zeta_{7j}-\delta_1\zeta_{8j}\zeta_{4j})+\xi_j^2\zeta_{8j}(\zeta_{7j}-\delta_1\omega^2)+\zeta_{3j}\omega^2(1+\zeta_{4j})}{-\xi_j^4(\zeta_{6j}\zeta_{4j}+\zeta_{7j}\zeta_{2j}cos\theta_j{}^2p_1)+\xi_j^2(\zeta_{4j}\zeta_{1j}-\zeta_{7j}cos\theta_j{}^2p_1-\omega^2\zeta_{6j})}, \text{ j=1, 2, 3}$$

$$l_j = \frac{-i\xi_j^3\zeta_{7j}(\zeta_{8j}cos\theta_j\delta_1-p_1\zeta_{6j}sin\theta_j)+i\xi_j\zeta_{7j}(\zeta_{3j}cos\theta_j+p_1 sin\theta_j\zeta_{1j})}{-\xi_j^4(\zeta_{6j}\zeta_{4j}+\zeta_{7j}\zeta_{2j}cos\theta_j{}^2p_1)+\xi_j^2(\zeta_{4j}\zeta_{1j}-\zeta_{7j}cos\theta_j{}^2p_1-\omega^2\zeta_{6j})} \text{ , j=1, 2, 3}$$

$$d_j = \frac{\xi_j^4(-\zeta_{8j}\zeta_{2j}\zeta_{7j}+\delta_1\zeta_{8j}\zeta_{4j})-\xi_j^2\zeta_{8j}(\zeta_{7j}-\delta_1\omega^2)+\zeta_{3j}\omega^2(1+\zeta_{4j})}{-\xi_j^4(\zeta_{6j}\zeta_{4j}+\zeta_{7j}\zeta_{2j}cos\theta_j{}^2p_1)+\xi_j^2(\zeta_{4j}\zeta_{1j}-\zeta_{7j}cos\theta_j{}^2p_1-\omega^2\zeta_{6j})}, \text{ j= 4, 5, 6}$$

$$l_j = \frac{-i\xi_j^3\zeta_{7j}(-\zeta_{8j}cos\theta_j\delta_1-p_1\zeta_{6j}sin\theta_j)+i\xi_j\zeta_{7j}(-\zeta_{3j}cos\theta_j+p_1 sin\theta_j\zeta_{1j})}{-\xi_j^4(\zeta_{6j}\zeta_{4j}+\zeta_{7j}\zeta_{2j}cos\theta_j{}^2p_1)+\xi_j^2(\zeta_{4j}\zeta_{1j}-\zeta_{7j}cos\theta_j{}^2p_1-\omega^2\zeta_{6j})} \text{ , j=4, 5, 6}$$

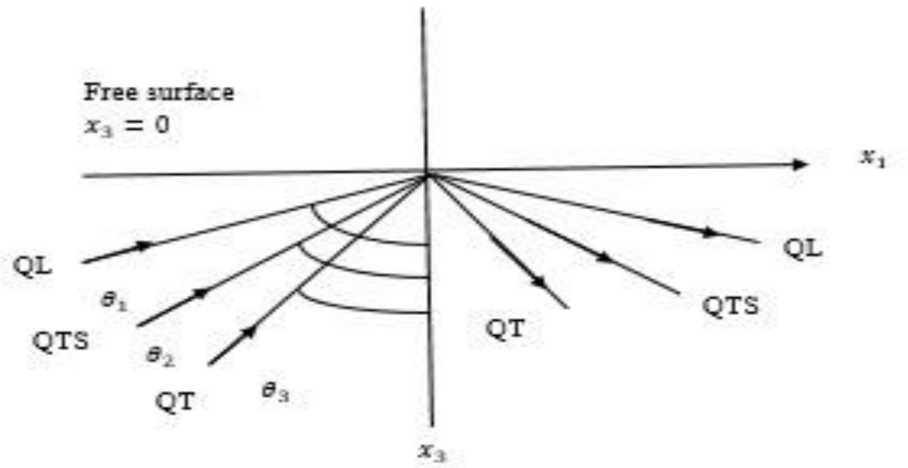

Fig.1: Geometry of the problem

VI. BOUNDARY CONDITIONS

Thedimensionless boundary conditions at the free surface$x_3 = 0$, are given by

(i) $t_{33} = 0$ (27)

(ii) $t_{31} = 0$ (28)

(iii) $\frac{\partial\varphi}{\partial x_3}$ =0 (29)

Making use of (26) into the boundary conditions (27)-(29), we obtain

$$\sum_{j=1}^{3}(-i\xi_j sin\theta_j \frac{(c_{13}{}^*}{\rho c_1^2} + id_j\xi_j \frac{c_{33}{}^*}{\rho c_1^2} - p_1 l_j (1 + a_1\xi_j^2 sin\theta_j{}^2 + a_3 cos\theta_j{}^2))\, A_j e^{iM_j(x_1,0)} + \sum_{j=4}^{6}(-i\xi_j sin\theta_j \frac{(c_{13}{}^*}{\rho c_1^2} - id_j\xi_j \frac{c_{33}{}^*}{\rho c_1^2} - p_1 l_j (1 + a_1\xi_j^2 sin\theta_j{}^2 + a_3 cos\theta_j{}^2))\, A_j e^{iM_j(x_1,0)} = 0 \quad (30)$$

$$\sum_{j=1}^{3}(-i\xi_j sin\theta_j + id_j\xi_j cos\theta_j)\, A_j e^{iM_j(x_1,0)} + \sum_{j=4}^{6}(-i\xi_j sin\theta_j - id_j\xi_j cos\theta_j)\, A_j e^{iM_j(x_1,0)}=0 \quad (31)$$

$$\sum_{j=1}^{3}(i\xi_j cos\theta_j\, l_j + h_1 l_j)\, A_j e^{iM_j(x_1,0)} + \sum_{j=4}^{6}(-i\xi_j cos\theta_j\, l_j + h_1 l_j)\, A_j e^{iM_j(x_1,0)} = 0 \quad (32)$$

The equations (30)-(32) are satisfied for all values of x_1, therefore we have,

$$M_1(x_1,0) = M_2(x_1,0) = M_3(x_1,0) = M_4(x_1,0) = M_5(x_1,0) = M_6(x_1,0) \tag{33}$$

From equations (26) and (33), we obtain

$$\xi_1 sin\theta_1 = \xi_2 sin\theta_2 = \xi_3 sin\theta_3 = \xi_4 sin\theta_4 = \xi_5 sin\theta_5 = \xi_6 sin\theta_6 \tag{34}$$

which is the form of Snell's law for stress free, thermally insulated surface of transversely isotropic viscoelastic medium with rotation. Equations (30)-(32) and (34) yield

$$\sum_{q=1}^{3} X_{iq} A_q + \sum_{j=4}^{6} X_{ij} A_j = 0, \ (i=1, 2, 3) \tag{35}$$

where

$$X_{1q} = -i\xi_q sin\theta_q \frac{(c_{13}{}^*}{\rho c_1^2} + i d_q \xi_q \frac{c_{33}{}^*}{\rho c_1^2} - p_1 l_q (1 + a_1 \xi_q^2 sin\theta_q{}^2 + a_3 cos\theta_q{}^2), q = 1,2,3$$

$$X_{2q} = -i\xi_q sin\theta_q + i d_q \xi_q cos\theta_q \quad , q = 1,2,3$$

$$X_{3q} = i\xi_q cos\theta_q l_q + h_1 l_q, \qquad q = 1,2,3$$

$$X_{1j} = -i\xi_j sin\theta_j \frac{(c_{13}{}^*}{\rho c_1^2} - i d_j \xi_j \frac{c_{33}{}^*}{\rho c_1^2} - p_1 l_j (1 + a_1 \xi_j^2 sin\theta_j{}^2 + a_3 cos\theta_j{}^2, \quad j=4, 5, 6$$

$$X_{2j} = -i\xi_j sin\theta_j - i d_j \xi_j cos\theta_j , \quad j=4, 5, 6$$

$$X_{3j} = -i\xi_j cos\theta_j l_j + h_1 l_j \quad , j=4, 5, 6 \tag{36}$$

(6.1)Incident QL-wave

In case of quasi-longitudinal wave, the subscript q takes only one value, that is q=1, which means$A_2 = A_3 = 0$. Dividing the set of equations (35) throughout byA_1, we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$A_{1i} = \frac{A_{i+3}}{A_1} = \frac{\Delta_i^1}{\Delta}, \ i=1, 2, 3 \tag{37}$$

(6.2)Incident QTS-wave

In case of quasi-transverse wave, the subscript q takes only one value, that is q=2, which means$A_1 = A_3 = 0$. Dividing the set of equations (35) throughout byA_2, we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$\mathrm{A}_{2\mathrm{i}} = \frac{\mathrm{A}_{\mathrm{i}+3}}{\mathrm{A}_2} = \frac{\Delta_{\mathrm{i}}^1}{\Delta}, \ \mathrm{i}=1, 2, 3 \tag{38}$$

(6.3)Incident QT-wave

In case of quasi-thermal wave, the subscript q takes only one value, that is q=3, which means$A_1 = A_2 = 0$. Dividing the set of equations (35) throughout byA_3, we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$A_{3i} = \frac{A_{i+3}}{A_3} = \frac{\Delta_i^1}{\Delta} \ , i=1, 2, 3 \tag{39}$$

where Z_i(i=1,2,3) are the amplitude ratios of the reflected QL, reflected QTS, reflected QT -waves to that of the incident QL-(QTS or QT) waves respectively.

Here $\Delta = \ |A_{ii+3}|_{3\times 3}$ and Δ_i^p (i=1,2,3) can be obtained by replacing , respectively ,the 1st, 2nd and 3rd columns of Δ by $\left[-X_{1p}, -X_{2p}, -X_{3p}\right]^t$.

Following Achenbach (1973), the energy flux across the surface element that is the rate at which the energy is communicated per unit area of the surface is represented as

$$P^* = t_{lm} n_m \dot{u}_l \tag{40}$$

Wheret_{lm} is the stress tensor, n_m are the direction cosines of the unit normal and $\dot{u}_l$ are the components of the particle velocity.

The time average of P^* over a period, denoted by $< P^* >$, represents the average energy transmission per unit surface area per unit time and is given at the interface $x_3 = 0$ as

$$< P^* > = < Re(t_{13}).Re(\dot{u}_1) + Re(t_{33})Re(\dot{u}_3) > \tag{41}$$

Following Achenbach (1973), for any two complex functions f and g, we have

$$< Re(f) >< Re(g) > = \frac{1}{2} Re(f\bar{g}). \tag{42}$$

The expressions for energy ratios E_i, $(i = 1,2,3)$for reflected QL, QT, and QTH-wave are given as

(i) In case of incident QL- wave,

$$E_{1i} = \frac{<P_{i+3}^*>}{<P_1^*>}, \ i=1, 2, 3 \tag{43}$$

(ii) In case of incident QTS- wave,

$$E_{2i} = \frac{<P_{i+3}^*>}{<P_2^*>}, \ i=1, 2, 3 \tag{44}$$

(iii) In case of incident QT- wave,

$E_{3i} = \frac{<P_{i+3}^*>}{<P_3^*>}$, i=1, 2, 3 (45)

where $< P_i^* >$, i=1,2,3 are the average energies transmission per unit surface area per unit time corresponding to incident QL, QTS, QT waves respectively and $< P_{i+3}^* >$, i=1,2,3 are the average energies transmission per unit surface area per unit time corresponding to reflectedQL, QTS, QT waves respectively.

VII. PARTICULAR CASES

(7.1) If $k_1{}^* = k_3{}^* = 0$, then we obtain the resulting expressions for transversely isotropic viscothermoelastic solid with rotation and without energy dissipation and with two temperature.

(7.2) If$\Omega = 0$, then we obtain the resulting expressions for transversely isotropic viscothermoelastic solid with and without energy dissipation and with two temperature without rotation.

(7.3) If $a_1 = a_3 = 0$, then we obtain the corresponding expressions for displacements, and stresses and conductive temperature for transversely isotropic viscothermoelastic solid with rotation and with and without energy dissipation.

VIII. NUMERICAL RESULTS AND DISCUSSION

For the purpose of numerical calculation, we consider the cases of incident QL, QTS, QT waves respectively and take the stress free thermally insulated boundary conditions. Copper material is chosen for the purpose of numerical calculation with numerical values as

$c_{11} = 18.78 \times 10^{10} Kgm^{-1}s^{-2}$, $c_{12} = 8.76 \times 10^{10} Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times 10^{10} Kgm^{-1}s^{-2} c_{33} = 17.2 \times 10^{10} Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10} Kgm^{-1}s^{-2}$,$C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1} \alpha_1 = 2.98 \times 10^{-5} K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$, $K_1{}^* = 0.433 \times 10^3 Wm^{-1}K^{-1}$, $K_3{}^* = 0.450 \times 10^3 Wm^{-1}K^{-1}$.$K_1 = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$, $K_3 = 0.04 \times 10^2 Nsec^{-2}deg^{-1}$.

The values of two temperatures, frequency, rotationΩ, magnetic effectH_0, are taken as
0.03, 0.06, 10S^{-1}, 4.0, 1.2 respectively.

The software Mat lab 8.4.0 has been used to determine the values of phase velocity, attenuation coefficient, specific loss, penetration depth and energy ratios of reflected QL, QTS and QT waves with respect to incident QL, QTS, and QT waves respectively. The variations of phase velocity, attenuation coefficients, specific loss and penetration depth with respect to frequency are shown in figures 2-13.The variation of magnitude of energy ratios of reflected waves subject to incident waves have been plotted in the figures 14-22 with respect to angle of incidence.

A comparison has been made to show the effect of viscosity on the various quantities.

(i) Solid line corresponds to the case τ_0=0

(ii) Small dashed line corresponds to case τ_0=1

(8.1)Phase Velocity

Figs.2-4 indicate the variations of phase velocities V_1, V_2, V_3 with respect to frequency ω respectively. From Fig2.we notice that corresponding to the case τ_0=0, the variations of phase velocity V_1 decrease for the range $1 \le \omega \le 4$ and increase monotonically in the rest. Corresponding to τ_0=1, the values of V_1 decrease for the range $1 \le \omega \le 4$ followed by an increase for the range$4 \le \omega \le 10$, then decrease monotonically in the rest.Fig3 exhibits the variations of phase velocity V_2 with respect to frequencyω. Here, we notice that corresponding to τ_0=0, variations steadily decrease and approach boundary surface with increase in wave number whereas corresponding to τ_0=1, variations are similar with a change in slope of tangent.Fig4.shows variations of phase velocity V_3 with respect to frequencyω. Here we notice that the values of phase velocity are increasing monotonically corresponding to τ_0=0, whereas decrease steadily corresponding to τ_0=1.

(8.2)Attenuation Coefficients

Fig.5 shows that the values of attenuation coefficient Q_1 increase monotonically with respect to frequency ω corresponding to τ_0=1whereas corresponding to τ_0=0, a sharp decrease is noticed for the range $2 \le \omega \le 4$ followed by a smooth decrease approaching the boundary surface.Fig6. exhibits the trends of attenuation coefficient Q_2 with respect to frequency ω.Corresponding to both the cases, the values of Q_2 increase monotonically with a change in the slope. Fig7.represents the variations of attenuation coefficient $Q_3$with respect to frequency ω.Here, corresponding to both the cases,variations increase monotonically with a change of slope with maximum variations corresponding to τ_0=0.

(8.3)Specific Loss

Fig8.exhibits the variations of Specific loss R_1 with respect to frequency. Here, we notice that the variations increase monotonically corresponding toτ_0=0 whereas corresponding to τ_0=1, the variations increase sharply for the range $2 \leq \omega \leq 8$ followed by a decrease for the range $8 \leq \omega \leq 10$ with a fall at $\omega = 11$ and decrease slowly and smoothly for the rest.Fig9.shows Variations of Specific loss R_2 with respect to frequencyω . Here, we notice that corresponding to τ_0=0, there is a sharp decrease for the range $1 \leq \omega \leq 3$,and then a small increase is noticed for the range $3 \leq \omega \leq 4$ followed by slow and smooth decrease for the rest. Corresponding to τ_0=1, we notice a small increase for the range $1 \leq \omega \leq 6$ followed by a small decrease for the range $6 \leq \omega \leq 11$with a high rise at $\omega = 11$ and then decrease for the rest.Fig10.shows Variations of Specific loss R_3 with respect to frequencyω. Here, we notice that, variations increase monotonically corresponding to the case τ_0=0 for the whole range whereas corresponding to τ_0=1, the variations increase for the range $1 \leq \omega \leq 4$ and remain stationary for the rest.

(8.4)Penetration depth

Fig11.shows the variations of penetration depth S_1 with respect to frequency ω.Here,we notice that initially , there is a sharp decrease in the values of S_1 corresponding to the case τ_0=0, followed by a smooth decrease approaching the boundary surface for the rest. Corresponding to τ_0=1, the variation increase monotonically with vibrations for the whole range. Fig12.shows the variations of penetration depth S_2 with respect to frequencyω. We notice that, there is a sharp decrease for the range $1 \leq \omega \leq$2 and the values lie upon the boundary surface for the rest whereas corresponding to τ_0=1, a sharp decrease in the values for the range $1 \leq \omega \leq 4$ is followed by a smooth decrease approaching the boundary surface.

(8.5)Energy Ratios

(8.5.1)Incident QL Wave

Fig.14.depicts the Variations of Energy ratio$E_{11}$with respect to angle of incidenceθ. It shows that the values of E_{11} decrease slowly and smoothly along with vibrations corresponding to both the cases for the whole range with a change of slope.Fig15.shows the variations of energy ratio $E_{12}$with respect to angle of incidenceθ . Here the variations increase sharply with vibrations corresponding to both the cases with change of slope.Fig.16.depicts the Variations of Energy ratioE_{13} with respect to angle of incidenceθ. It is noticed that the values of E_{13}follow opposite trends as discussed in the Fig.14.

(8.5.2)Incident QTS Wave

Fig17.depicts the Variations of Energy ratio$E_{21}$with respect to angle of incidenceθ. Here corresponding to both the cases, we notice similar slowly decreasing trends with difference in magnitudes for the whole range. Fig18.depicts the variations in Energy ratio$E_{22}$with respect to angle of incidenceθ. Here corresponding to both the cases, the variations increase monotonically and smoothly for the whole range with a change in magnitude. Variations of Energy ratioE_{23} with respect to angle of incidence $\theta$are shown in Fig19.Here, we notice that the trends are decreasing monotonically with vibrations in the whole range corresponding to both the cases.

(8.5.3)Incident QT Wave

Figs.20-22.depict the Variations of Energy ratiosE_{31}, E_{32},E_{33} with respect to angle of incidence θrespectively. Here in the Fig.20the variations are decreasing monotonically and smoothly corresponding to both the cases whereas opposite trends are noticed in the Fig21. Fig22.depicting the variations of E_{33} exhibits that the variations are decreasing slowly and smoothly for the whole range.

IX. CONCLUSION

From the graphs, we observe that while investigating the surface of earth having different layers (anisotropy) and interfaces, elastic waves propagating through the earth have different velocities and are influenced by the properties of the layer through which they travel .Frequency of waves produced in the material have significant impact on the phase velocity, attenuation coefficients, specific loss and penetration depth of various kinds of waves. Also the magnitude of energy ratios are in the impact of angle of incidence. Variations are vibrating when QL wave is incident whereas increase or decrease smoothly in the rest of the cases. The signals of these waves are not only helpful in providing information about the internal structures of the earth but also helpful in exploration of valuable materials such as minerals, crystals and metals etc.

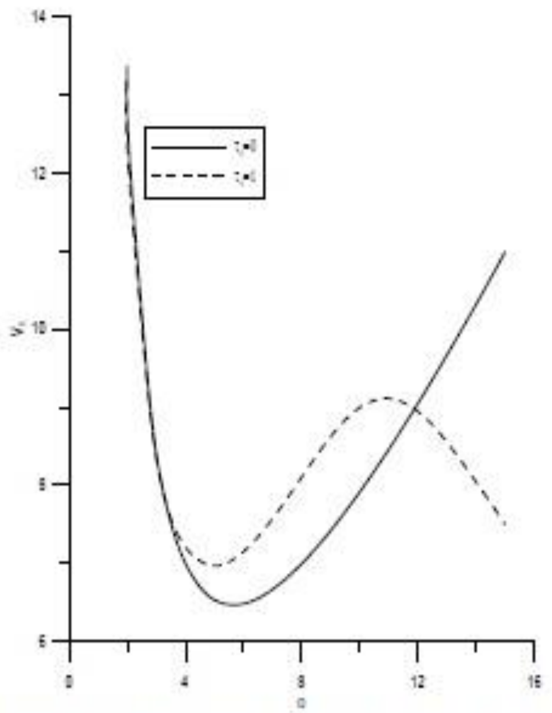

Fig.2. Variations of Phase Velocity V_1 with respect to frequency ω

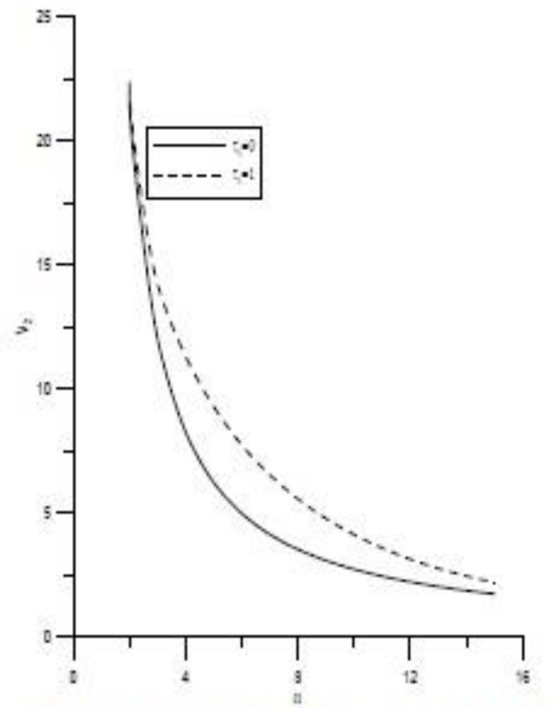

Fig.3. Variations of Phase Velocity V_2 with respect to frequency ω.

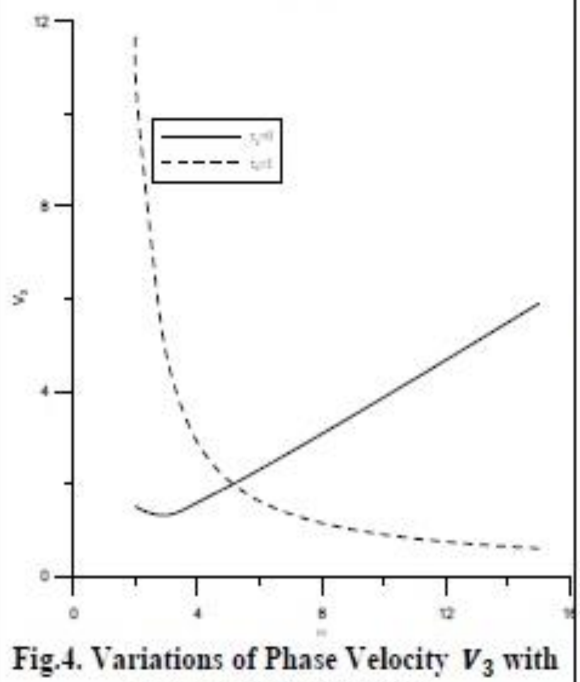

Fig.4. Variations of Phase Velocity V_3 with respect to frequency ω.

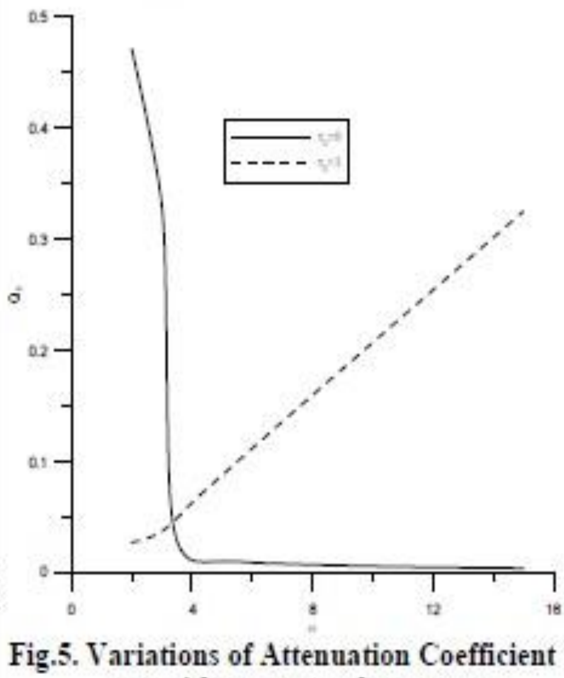

Fig.5. Variations of Attenuation Coefficient Q_1 with respect to frequency ω .

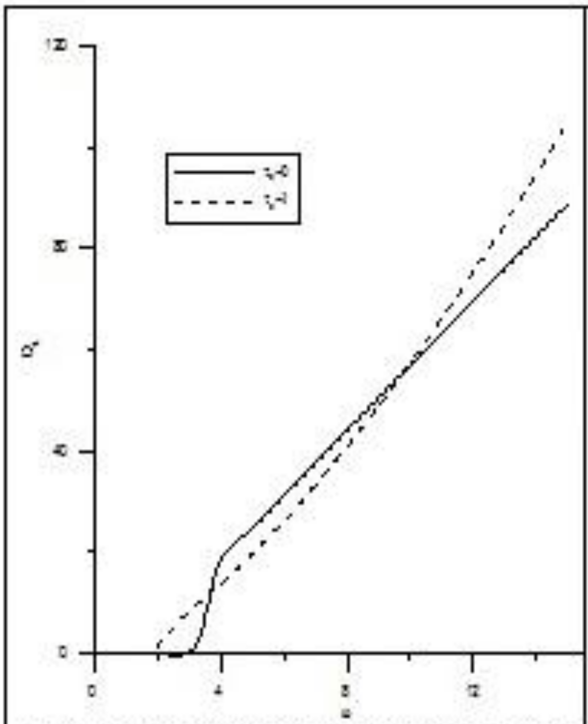

Fig.6. Variations of Attenuation Coefficient Q_2 with respect to frequency ω.

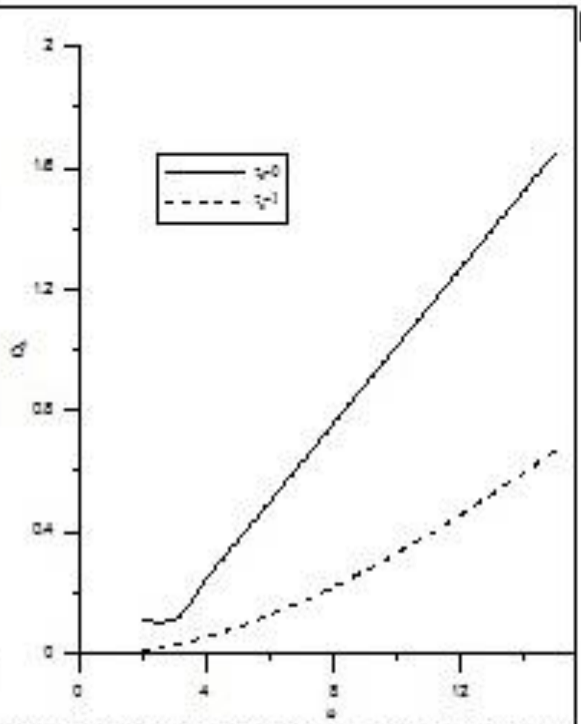

Fig.7. Variations of Attenuation Coefficient Q_3 with respect to frequency ω.

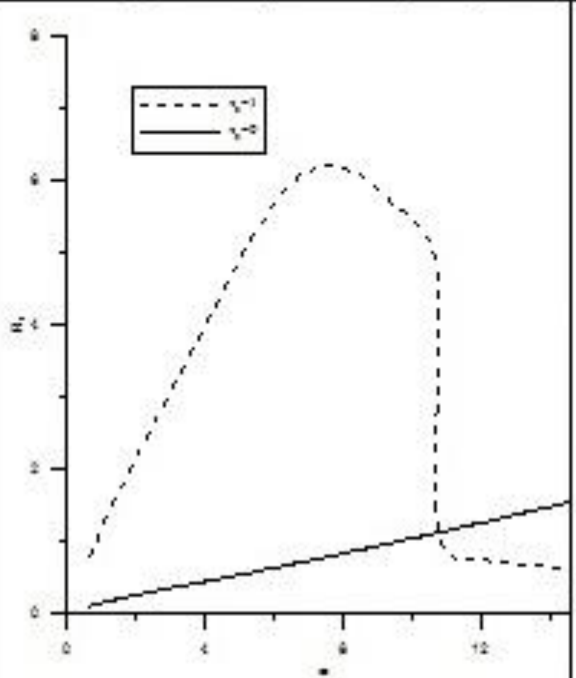

Fig.8. Variations of Specific loss R_1 with respect to frequency ω.

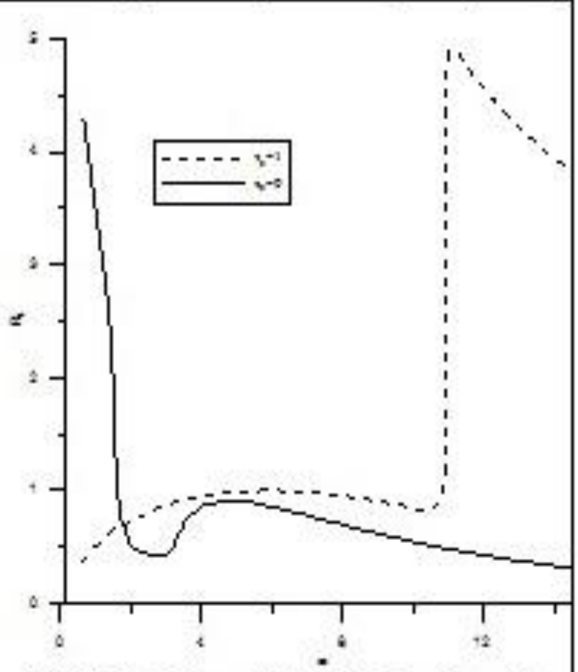

Fig.9. Variations of Specific loss R_2 with respect to frequency ω.

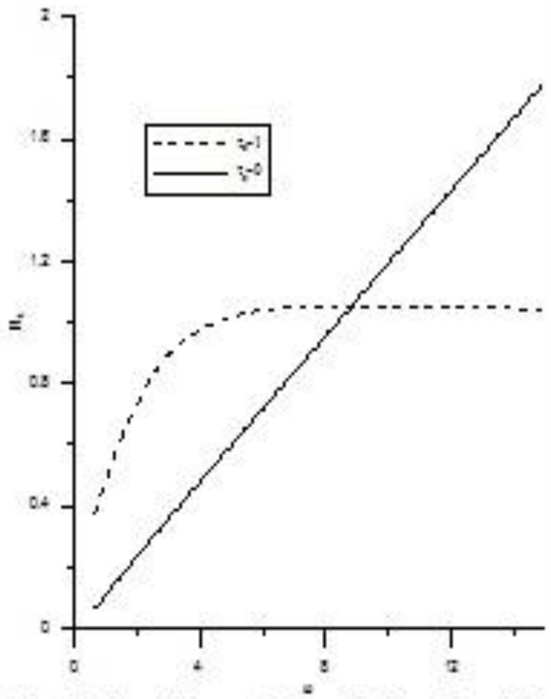

Fig.10. Variations of Specific loss R_3 with respect to frequency ω .

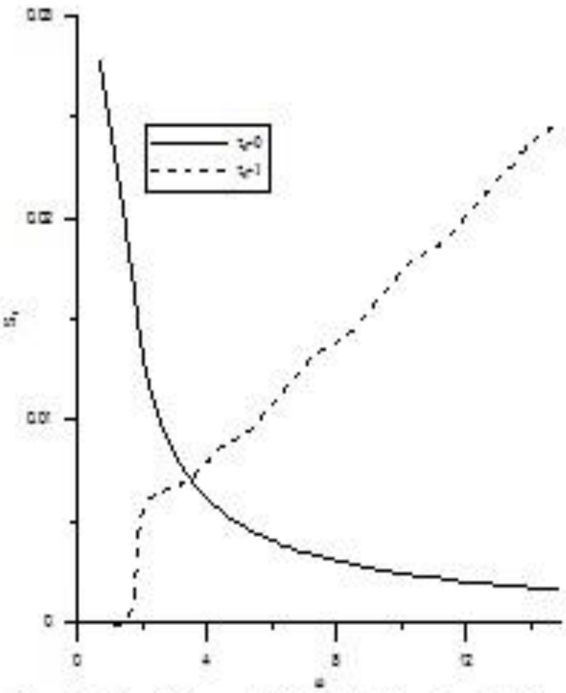

Fig.11. Variations of Penetration depth S_1 with respect to frequency ω.

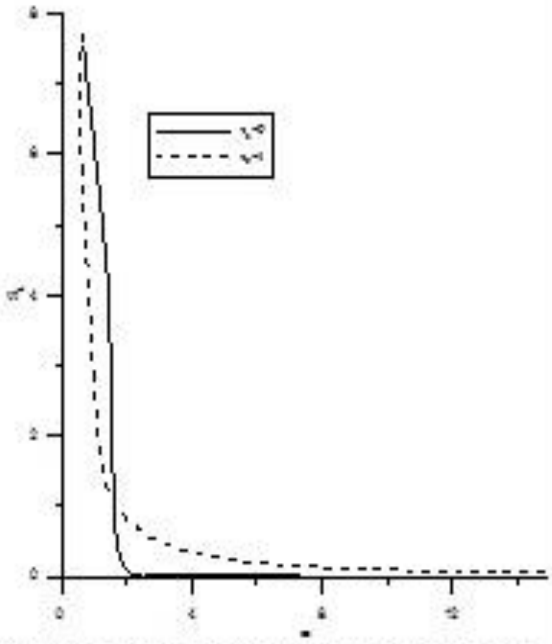

Fig.12. Variations of Penetration depth S_2 with respect to frequency ω.

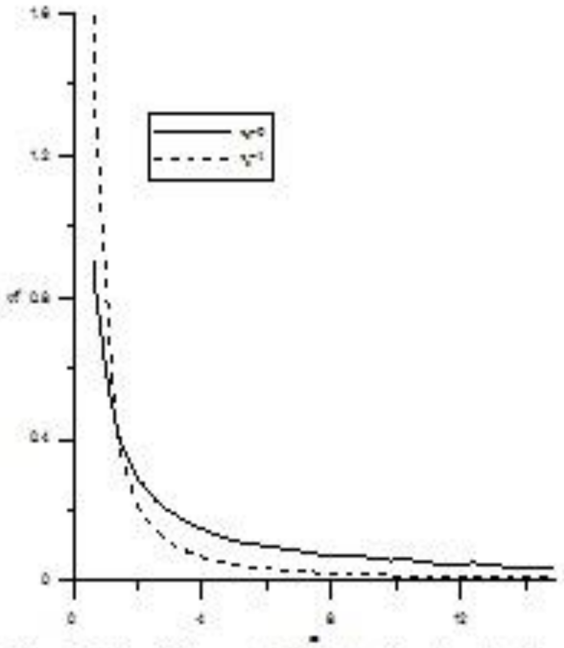

Fig.13. Variations of Penetration depth S_3 with respect to frequency ω.

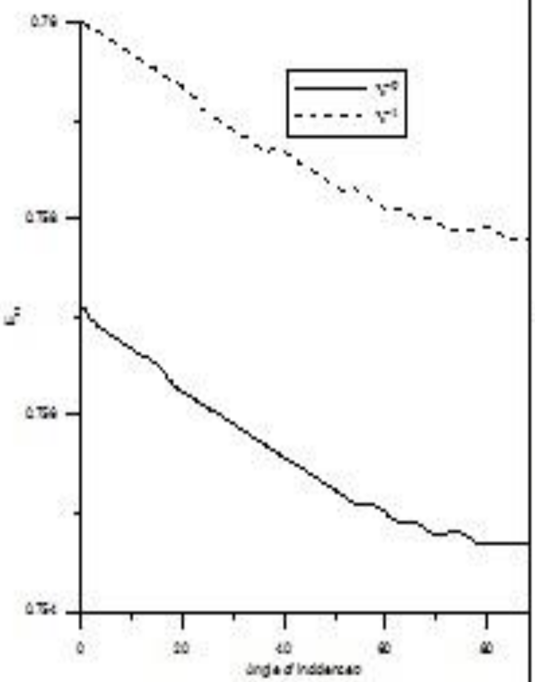

Fig.14.Variations of Energy ratio E_{11} with respect to angle of incidence θ.

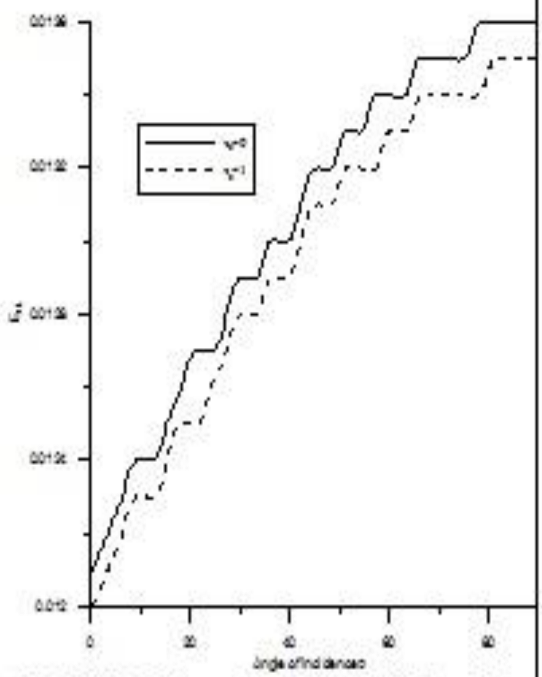

Fig.15.Variations of Energy ratio E_{12} with respect to angle of incidence θ.

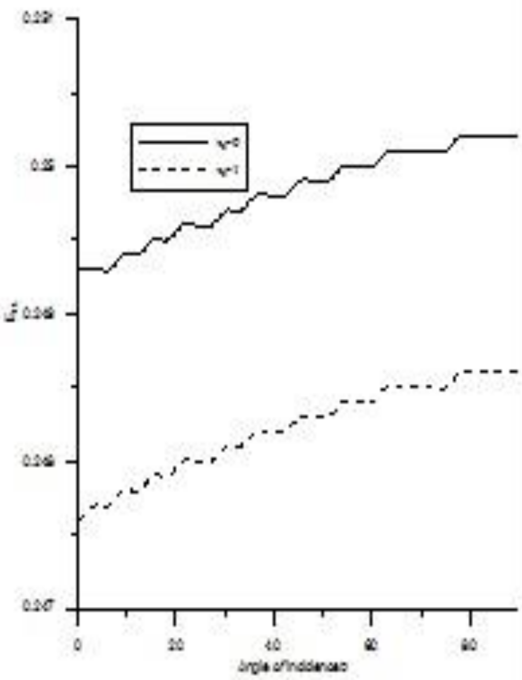

Fig.16. Variations of Energy ratio E_{13} with respect to angle of incidence θ.

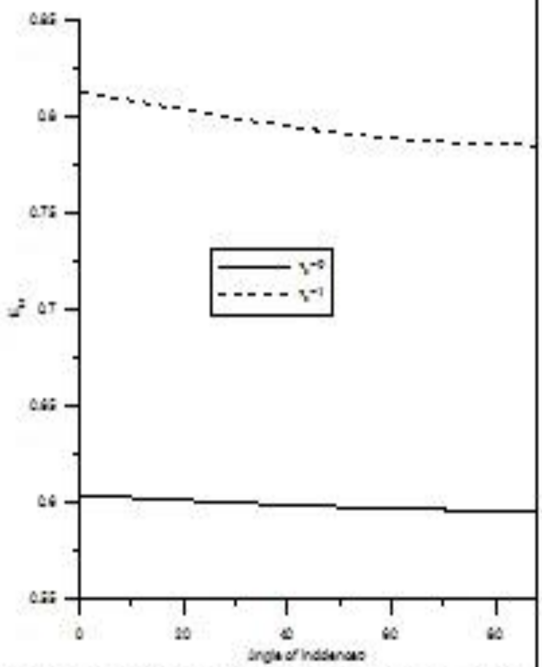

Fig.17. Variations of Energy ratio E_{21} with respect to angle of incidence θ.

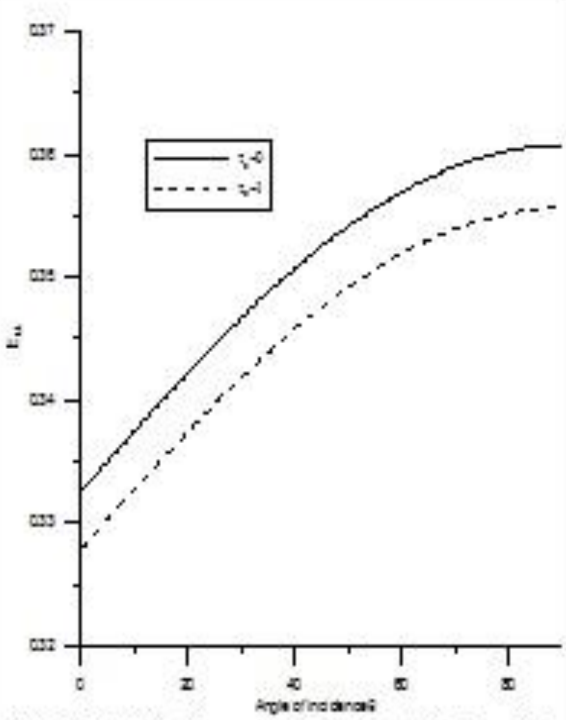

Fig.18. Variations of Energy ratio E_{22} with respect to angle of incidence θ.

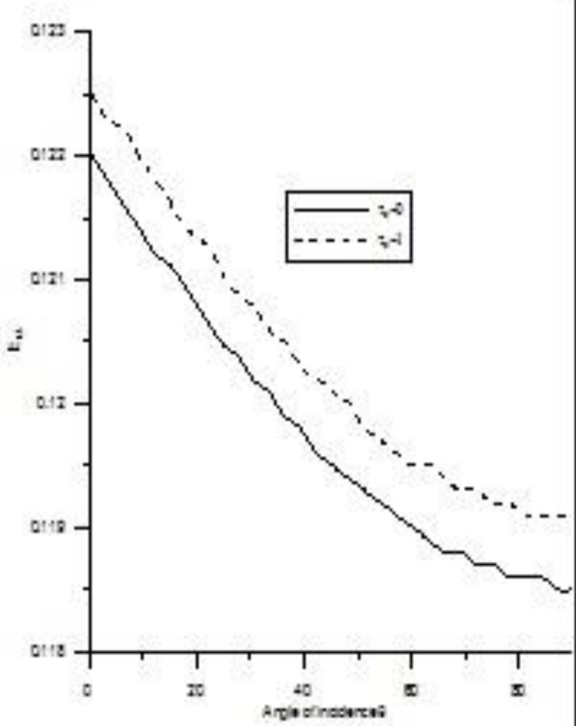

Fig.19. Variations of Energy ratio E_{23} with respect to angle of incidence θ.

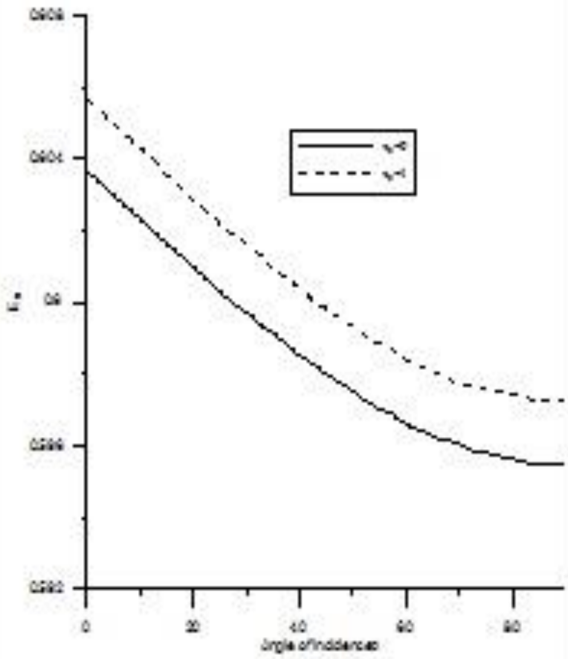

Fig.20. Variations of Energy ratio E_{31} with respect to angle of incidence θ.

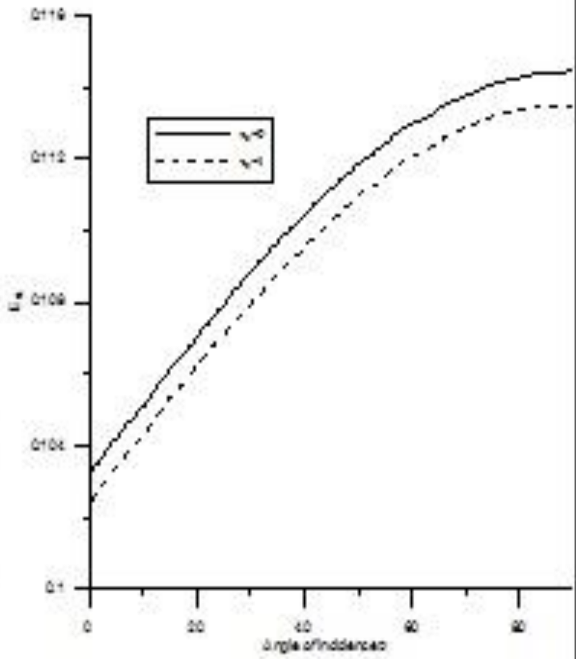

Fig.21. Variations of Energy ratio E_{32} with respect to angle of incidence θ.

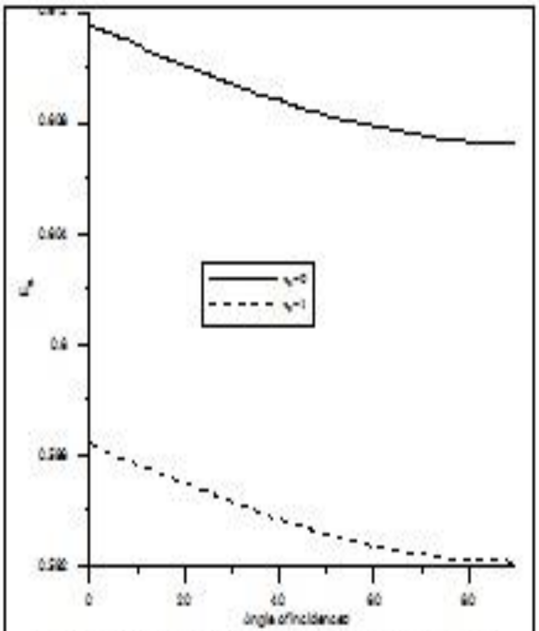

Fig.22. Variations of Energy ratio E_{33} with respect to angle of incidence θ.

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