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A Generality of Mathematical Thought by Carl Friedrich Gauss through his contributions to science

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ABSTRACT: *Mathematical thinking is composed in the systematization and contextualization of the knowledge of mathematics present in each historical period of humanity. This study aimed to present a generality of mathematical thinking by Carl Friedrich Gauss through his numerous contributions to the development of science. Gauss presented humanity with an extensive work of studies and discoveries essential for the understanding of new knowledge and scientific and technological progress. It is hoped that this work can contribute significantly to understanding how essential mathematical thinking is for the cognitive and intellectual development of humanity.*

Keywords:
KEYWORDS: *Mathematical Thinking, Carl Friedrich Gauss, Knowledge and Knowledge.*

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I. INTRODUCTION

Since the beginning, the individual has always sought to create environments to improve his evolution, either intuitively, through common sense, or through observations, decompositions, deductions, relationships and / or experiments.

According to [1], the evolutionary process of science and the cognitive development of man were decisive for understanding the origins of the ideas that produced the shape of things and presenting mathematics as a propelling instrument.

For [2] the ability to deal with mathematics is combined with multiple mental attributes developed by the individual, among them: numerical sense, algorithmic capacity, logical reasoning, capacity for abstraction, spatial / geometric vision, ability to elaborate new things.

It is noted that the combination of these attributes inherent to humans, which contributed so much to the increase of knowledge are connected to an essential characteristic of the individual for the full development of science: Mathematical thinking.

This study aimed to present a generality of the mathematical thought of Carl Friedrich Gauss (1777 - 1855) through his countless contributions to the development of science. Gauss considered the "prince of mathematics" presented humanity with a vast work of studies and discoveries indispensable for the conception of new knowledge and scientific and technological advancement.

The theoretical framework of this work is divided into two essential parts: In the first part, we approach the composition of mathematical thinking and its importance in the development of new knowledge and knowledge. In the second part, we look for the history of Gauss and his admirable contributions to science originated by his intense and exacerbated mathematical thinking. Then, we present a discussion about the strong relationship between mathematical thinking and Gauss's scientific contributions. Finally, the final considerations will be exposed.

II. MATHEMATICAL THINKING

Thinking is a word that comes from the Latin “pensare”, etymologically means to weigh or evaluate the weight of something. Thought is the act of becoming aware, preparing for action translates everything that originates through existence through intellectual activity, finding oneself at the service of human beings to solve problems, make decisions and create models. It represents the product of the mind that derives from the rational actions of the intellect or abstractions of the imagination and harmonizes the individual to model his clairvoyance of the world.

Therefore, mathematical thinking is composed in the systematization and contextualization of the knowledge of mathematics, focusing on the study of principles, concepts, conjectures, relationships, techniques and algorithms present in each historical period of humanity. When an individual register his desires and aphorisms suggestive of mathematical ideas, at this moment, it is possible to capture his intuitive mathematical thinking.

According to [3], the nature of mathematical thinking is fundamentally connected to the cognitive procedures that give rise to mathematical knowledge. Understanding, as it happens, is a process occurring in the subject's mind, it can be accelerated, a click in the mind, but it is, repeatedly, based on a long sequence of learning activities during which a wide variety of mental procedures take place and interact.

[4] presents a systematic action of the evolution of mathematical thinking in a cognitive approach. Human activity is separated by three devices: input would be perception, internal thinking and output, action, so that mathematical thinking remains linked to the mathematical activities of perceiving objects, thinking about them and implementing actions on them.

Mathematical thinking is characterized by several properties inherent to other types of thinking. (Figure 1):

a. Inductive Thinking: observes to reach a conclusion.
b. Deductive Thinking: observes, analyzes and tests to reach a conclusion.
c. Analytical Thinking: explains things by breaking them down into simpler parts.
d. Systemic Thinking: complex view of multiple elements with their diverse interrelationships.

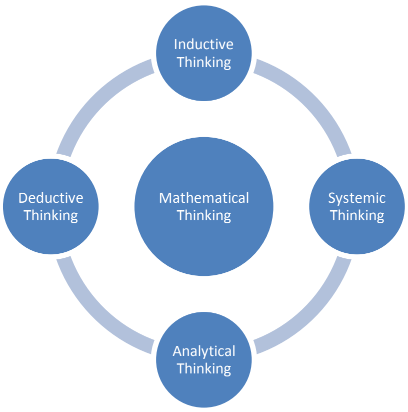

Figure 1: Mathematical Thinking.
Source: Authors' elaboration.

In contemporary school, mathematical thinking is an indispensable mental attribute, so that the individual can develop his intellect and cognitive aspects, so that he can efficiently follow scientific and technological evolution. Learning mathematics linked to a technological world leads us to demand that our learners are fully connected with the idea of periodic use of mathematical thinking and all other thoughts that connect with it.

In view of this, it is necessary to look in the past for ideas that truly contributed to our scientific and technological evolution, and one of the main responsible people was Carl Friedrich Gauss.

III. JOHANN CARL FRIEDRICH GAUSS HIS HISTORY AND CONTRIBUTIONS TO SCIENCE

Monitoring Johann Carl Friedrich Gauss was born in Braunschweig, Germany on April 30, 1777, known as the prince of mathematicians, he was also an astronomer and physicist. Considered the greatest mathematician of the time - perhaps of all time - Gauss had an estimated IQ of 240.

He contributed in several areas of knowledge, among them, statistics, differential geometry, differential equations, number theory, astronomy, physics, and cartography. Due to Gauss's extraordinary aptitude, the Duke of Brunswick financed his studies through scholarships for high school, the University in Göttingen and the completion of his doctorate at the University of Helmstădt.

Figure 2: Johann Carl Friedrich Gauss (1777 – 1855).
Source: https://www.google.com/url

[5] tells that one day, a teacher, to keep the class busy, established that students add up all the numbers from one to one hundred, with instructions for each one to put his slate on a table as soon as he completed the task, almost Carl promptly placed his slate on the table, saying, "There it is"; the professor looked at him with little regard as the others worked diligently. When the master finally looked at the results, Gauss's slate was the only one to display the correct answer, 5050, without any calculation.

In 1787, Gauss was only ten years old and intuitively reports the discovery of the sum of the terms of an arithmetic progression. The professor was so amazed by the young man's achievement that he wrote arithmetic books out of his own pocket. Before that, he had already learned to read and add by himself.

Gauss's mathematical thinking is strongly observed to find the answer to the exercise proposed by his teacher. The proposal was to add from 1 to 100, figure 3, and Gauss realized that when we add the first number in set 1 to the last 100, we get the result of 101, even though, when we add the second number 2 with the penultimate 99, we also get 101, then when we add the third number 3 with the antepenultimate 98, we get 101, and so on.

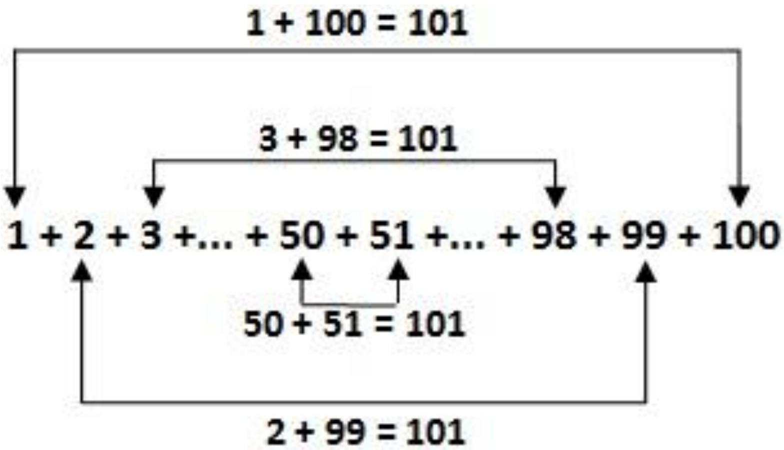

Figure 3: Sum of Gauss.
Source: https://www.google.com

Repeating this procedure, the moment will come when we add the central numbers, 50 and 51, of the set and find 101. In this way, instead of adding the 100 numbers, proposed by the teacher, we will have 101 repeated 50 times, hence, 101 · 50 = 5050.

In 1796, Johann Carl Friedrich Gauss, achieved the feat of sketching a regular 17-sided polygon, the heptadecagon, only with the aid of ruler and compass. In addition, it proved that the regular seven-sided polygon could not be built with a ruler and compass. In his studies with regular plane figures, he proved how to construct any regular polygon of n sides with ruler and compass, in the hypothesis that n is a prime number of the form $2^{2^k} + 1$, a cousin of Fermat.

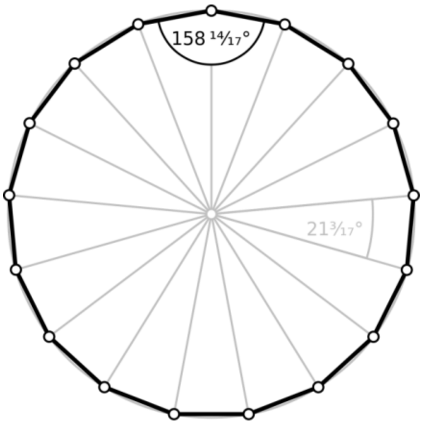

Figure 4: Regular 17 – sided polygon – heptadecagon.
Source: https://www.google.com

In 1798, Gauss defended his doctorate at the University of Helmstãdt and his thesis was to demonstrate the Fundamental Theorem of Algebra, proving that every polynomial equation $f(x) = 0$ has at least one real or imaginary root and for that purpose was based on geometric concepts. The Fundamental Theorem of Algebra states that "Every non-constant polynomial with complex coefficients has a complex root".

Gauss established the graphic representation of complex numbers by looking at the real and imaginary parts as coordinates of a plane. Each complex number $z = a + bi$ can be associated with a point P in the Cartesian plane, where ρ and θ are, the $module\ of\ z$ and the argument of z, respectively.

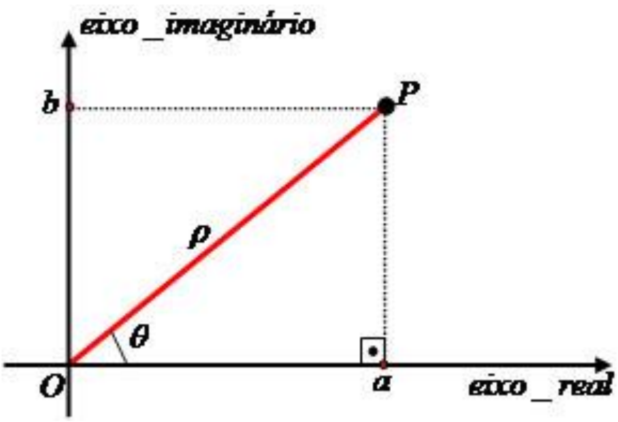

Figure 5: Argand-Gauss Traffic.
Source: https://www.google.com

In 1795, he developed the fundamental bases of the Least Squares Method (MMQ). MMQ is a mathematical optimization procedure that seeks to find the best fit for a data set (X, Y), in order to minimize the sum of the squares of the differences between the estimated value and the observed data. In 1809, Gauss published a work, in Theoria motus, about MMQ.

$$\min \sum_{i-1}^{N} (y_i^e - y_i^o)^2$$

In 1801, [6] published the book "*Disquisitiones Arithmeticaé"* where he develops studies in the area of algebraic number theory exposing a notation for the congruence relation: a is congruent to b, module m, which means, a and b are integers that leave even rest in the division by m.

$$a \equiv b \ (mod\ m)$$

Also in this same work, he exposes the law of quadratic reciprocity, Theorema Aureum, considered, by him, as the "jewel of arithmetic".

$$x^2 \equiv p \ (mod\ q)$$
$$y^2 \equiv q \ (mod\ p)$$

Where p and q are odd prime numbers.

In other words, if none of the primes p or q belongs to the arithmetic progression $4k + 1$ then one of the congruences has a solution if and only if the other has no solution. If any of the cousins belongs to the $4k + 1$ progression then either congruence has a solution, or neither has a solution.

Gauss proved the theorem in which every positive integer can be represented in one way as a product of primes. This shows that the smallest positive prime number is 2. For, if 1 were prime, we could represent, for example, the number 10 in two different ways: $10 = 1 \cdot 2 \cdot 5 \ and \ 10 = 2 \cdot 5$.

At the beginning of the century XIX, in 1801, he gave up Arithmetic to dedicate himself to Astronomy, creating a method to follow the orbit of satellites. In 1807, he was appointed director of the Gôttingen observatory, where he spent 40 years.

In 1809, in the third section of the book Theoria motus, Gauss presented the famous law of normal distribution (Gaussian distribution or Gauss-Laplace distribution or Laplace distribution) to analyze astronomical data. Gauss made a series of general propositions about observable observations and errors and supplemented them with a purely mathematical assumption. Then, in a very simple configuration, he was able to obtain the equation of the curve that corresponded to his empirical results.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \text{ onde } x \in (-\infty, +\infty).$$

The graph resembles a bell and is sometimes called a bell-shaped curve. The Gaussian curve has two fundamental parameters: Arithmetic mean and standard deviation. The greater the standard deviation, the greater the variability of the data around the arithmetic mean. It is also noticed that, according to the values of the arithmetic mean and the standard deviation, the curve may become flatter or more tapered.

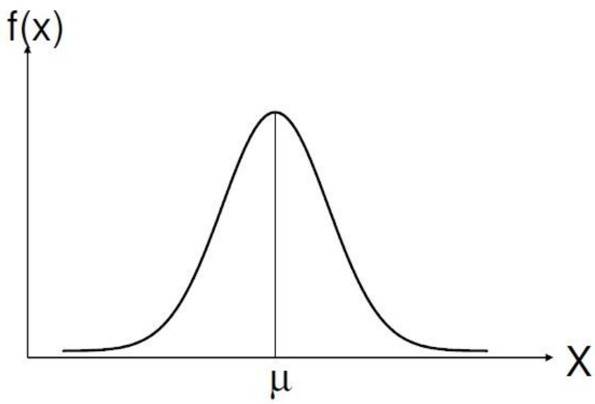

Figure 6: The Normal Curve.
Source: https://www.google.com

In 1817, Gauss began his work in relation to Geodesics, which resulted in an invention: heliotrope, an instrument that transmits signals by means of reflected light. In 1822 Gauss won the University of Copenhagen Prize with *Theoria attractionis*, with the idea of sketching one surface on top of another so that the two are similar in their smaller parts. In 1823, Gauss published *Theoria combinationis observationum erroribus minimus obnoxiae*, this is the theory. observable errors. His mathematical research continued on the theory of functions and geometry applied to Newton's theory. In Electromagnetism he invented the bifilar magnetometer and the electric telegraph. His only claim was the progress of mathematics that he endured until the moment he became aware of the end because he suffered from cardiac dilation. Gauss died on February 23, 1855, at the age of 78, in the city of Göttingen.

IV. A DISCUSSION ON GAUSSIAN MATHEMATICAL THINKING AND ITS CONTRIBUTIONS

In fact, the protagonism of mathematical thinking is notorious in the face of a world full of technological innovations. It is not enough to "know mathematics" and "do mathematics", it is necessary to "think mathematically", to increase the individual's capacity for understanding and articulation in the face of contemporary challenges. Mathematics is essential for the design of the survival structures of contemporary society.

The relentless search in the past for ideas and conjectures produced by expert mathematical thinkers has contributed immeasurably to scientific and technological advances today. Johann Carl Friedrich Gauss is one of those thinkers, his studies took the world to an early stage of technological prominence and scientific progress. Gauss' mathematical thinking produced significant and essential results for the understanding of many phenomena of nature and things, always explained by mathematical models, mostly extremely abstract and complex.

One of the mathematical episodes, probably the most prodigious of all humanity, was the resolution proposed by Gauss, at the age of 10, in the sum of the first hundred natural numbers. It is observed in this event his noticeable mathematical thinking in intuitively deducing one of the main formulas of Progressions, used tirelessly by young high school students.

In Gauss's proposal for the construction of a regular 17 – sided polygon with ruler and compass, there is an exceptional spatial perception and an accentuated mathematical thinking focused on geometry.

In his doctoral work, Gauss demonstrates one of the most important theorems of algebra, showing all his deductive mathematical thinking. [7] states that the *Algebra Fundamental Theorem* is extraordinarily accurate for mathematical manipulations with polynomial equations and for understanding algebraic structures and their properties.

In another contribution Gauss presents a way to sketch a complex number through an orthogonal plane. Again, it shows all its geometric ability based on a spatial mathematical thought. This idea by Gauss is a necessary requirement for the study of complex numbers in Brazilian high schools.

Through his mathematical thinking, Gauss, aged 24, proposed a method – Least Squares Method – to handle observations in order to estimate the parameters of a function. The mathematical procedures for calculating orbits of celestial bodies in Gauss's day required a large number of observations taken over a reasonably long period of time. In this way, with the skills inherent to the young mathematician, it was possible to provide an extremely efficient and quite usual model in the various areas of knowledge.

Another notable contribution by Gauss was in the area of number theory, particularly in the relations of congruence and equivalence. A numerical mathematical thought with countless applications from arithmetic to coded systems, widely used at different levels of education and in mathematics Olympiad programs.

In the early nineteenth century, perhaps Gauss made the greatest contribution: the normal curve. Mathematical thinking strongly directed towards a scientific model with diverse applications in practically all areas of knowledge. According to [8] and [9], this curve is the most important distribution of probabilities and has different applications in physical, biological, financial and social phenomena. Some applications linked to the curve: the duration of human pregnancy, the number of times an adult breathes per minute, the height of individuals, the number of rain drops that fall in a storm, the intelligence quotient of university students, the relationship the number of daughters and sons of Brazilian couples, the weight of men and women, the amount of hemoglobin in men, per 100 ml of blood, the level of glucose in human blood, among others.

V. CONCLUSION

There [10] ensures that mathematics is everywhere and relates to the demands of our day-to-day lives. From politics to theology, from economics to public health, from everyday life to the cosmos, mathematical thinking offers us proof that everyday problems can be faced with mathematics.

In view of the above, it is said with praise that Johann Carl Friedrich Gauss was a point outside the curve, for the progress of science and mathematics, represented by his endless relevant contributions to technological innovation and scientific and technological advancement in the transformation of society.

It is hoped that this work can instigate other researchers to expose the mathematical thinking of other great thinkers of science, adding their contributions to the scientific and technological development of the current world. It is undeniable that mathematical thinking becomes a unique reference to highlight how much science lacks these models for the evolution of humanity.

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