

Managing Uncertainty in Artificial Intelligence and Expert Systems Using Bayesian Theory and Probabilistic Reasoning

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ABSTRACT: *Uncertainty is a common phenomenon in everyday interaction and in all areas of human lives especially when dealing with information from different sources. These information may be unreliable based on their sources or method of collection such as random sampling or other statistical methods rather than categorical means. Uncertainty may arise from incomplete data or information, ambiguous and inconsistent information. In most tasks that requires intelligent behavior, the problem of uncertainty cannot be completely ruled out. In AI and expert systems, uncertainty is measured by using relative frequencies or by combining various statistical models based on data and information collected from various sources. Some of these measures are objective in nature while others may be from domain experts. All these measures all these measures are usually combined to make inference and decisions. That is, the expert system should be able to justify its assessments of the uncertainty and its reasoning procedures. This paper discussed managing uncertainty in artificial intelligence and expert systems using Bayesian Theory and Probability reasoning.*

Keywords: *Uncertainty, artificial intelligence, expert systems, Bayesian theory, Probabilistic reasoning.*

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I. INTRODUCTION

One of the rapidly developing areas in artificial intelligence (AI) is managing uncertainty [1] [2] [3]. It is not surprising that intelligent systems are expected to be able to exploit uncertain or vague information since human beings also reason and decide without having precise and certain information [4] [5]. Uncertainty is a common phenomenon in everyday interaction and in all areas of human lives especially when dealing with information from different sources. These information may be unreliable based on their sources or method of collection such as random sampling or other statistical methods rather than categorical means [6]. Uncertainty may arise from incomplete data or information, ambiguous and inconsistent data or information. In most tasks that requires intelligent behavior, the problem of uncertainty cannot be completely ruled out. For example, tasks such as planning, reasoning, complex problem solving, decision-making and classification problems there are elements of uncertainty to some extent because all the tasks require intelligence; may it be humans or machines. Even for machine which are expert systems, the software is developed by human experts and they are also liable to errors [8] [9].

Most times, the problem of misclassification often arise even when some of the best classification tools and algorithms are used. The simple reason is because these tools and algorithms are developed by human experts which may be liable to errors [10]. These errors arising from misclassification can be false positive (i.e., a situation in which the result of a test is positive when it ought to be negative); this situation can occur for instance in pregnancy test where the test result indicate that a woman is pregnant when indeed she is not or when a person test positive to HIV when indeed the person does not have HIV [11] [12]. The other condition is false negative (i.e., a situation in which the result of a test is negative when it ought to be positive); this situation can occur for instance in pregnancy test where the test result indicate that a woman is not pregnant when indeed she is pregnant or when a person test negative to HIV when indeed the person indeed have HIV. It can also occur in other test conditions such as in cancer, tuberculosis, etc. these are misclassification problems. They could cause serious problems especially in medical conditions as they may render a supposed patient to have

wrong diagnosis and wastage of money and serious anxiety and problems that may arise from wrong diagnosis of ailments [13].

In AI and expert systems, uncertainty is measured by using relative frequencies or by combining various statistical models based on data and information collected from various sources. Some of these measures are objective in nature while others may be from domain experts [14]. All these measures are usually combined to make inference and decisions. However, for users to be convinced about an expert system especially when the user requested that the model and its conclusion be made explicit, there is need for the human expert and those who built the expert system to provide documentation and detailed explanation about how the expert system was built and how it works [15] [16]. That is, the expert system should be able to justify its assessments of the uncertainty and its reasoning procedures. Some of its assumptions and assessments should be able to amended and modified if required by the user. This way, the user can accept the level of uncertainty measures based on the concluding evidence [17].

To ensure that the level of uncertainty is reduced to the minimum, there is need to measure the level of uncertainty especially if the source of the data or information been used is inconsistent or ambiguous or the data is secondary. In order to make the right decision, there is need to measure and know the level of uncertainty in the information or data used for the computation. This is the major reason why uncertainty must be managed. This is the focus of this paper. the rest part of the paper is as follows. Section 2 discuss uncertainty in expert systems using probabilistic reasoning and Bayes' theory. section

II. UNCERTAINTY IN EXPERT SYSTEMS

There are two basic types of uncertainty: one caused by uncertain or vague information, and the second by unknown, imprecise, or stochastic relations between variables that are part of a model of a reality. They are referred to as virtual (likelihood) evidence and soft evidence respectively. The likelihood evidence is evidence with uncertainty and can be represented as a likelihood ratio. The soft evidence of uncertainties represented as a probability distribution of one or more variables. Thus uncertain evidence specifies the probability distribution of a variable. Various frameworks are proposed to solve the problem of reasoning with uncertain and vague information. These include: 1) Dempster–Shafer theory of evidence, 2) theory of imprecise probabilities, 3) possibility theory, 4) fuzzy set theory, 5) Bayesian theory, 6) probability reasoning, etc. [18] [19]

Virtual (or likelihood) evidence is the type of evidence that reflects the uncertainty one has about a reported observation. This evidence type is an evidence with uncertainty and is usually represented as a likelihood ratio [20]. The virtual evidence method is used to deal with Bayesian Network belief update when one is uncertain about a claim of an event, say $X_i = a$. Suppose we believe with probability p that this claim is actually true due to the occurrence of $X_i = a$, then the probability that it is not occurring is $1 - p$. Virtual evidence technique requires this uncertainty be given as a likelihood ratio $L(X_i) = p + (1 - p)$, not necessarily the specific probabilities. In this technique Bayesian Network was extended by creating a virtual node, U with state u representing event where $X_i = a$ is said to have occurred. The virtual node, U has X_i as its only parent and its conditional probability table (CPT) satisfies $P(U | X_i = a) = P(U | X_i \neq a) = L(X_i)$; after which the belief update can be done by instantiating U to u .

In the soft evidence, uncertainty is represented as a probability of one or more variables with a given distribution $R(Y)$, $Y \subseteq X$. This type of evidence is very common. It refers to evidence specified by local probability distributions that define constraints on the posterior probability distribution and cannot be changed by further information, i.e., these probability distributions are fixed. Each observed local probability distribution on a subset of variables is different from the encoded prior probability distributions for those variables associated with the Bayesian network [21] [22]. Thus soft evidence is an evidence of uncertainty. Soft evidence is a true representation of observation of the distributions of some events which should all be preserved in the updated "posterior" distribution. For example, suppose $R(X_i)$ is a soft evidence, even if we are uncertain about the specific state X_i is in, we are certain about its distribution. In other words, $R(X_i)$ is a true (and certain) observation which must be preserved in the updated joint distribution Q^* (i.e., $Q^*(X_i) = R(X_i)$).

Suppose there is a distribution $P(X)$ and a soft evidence $R(Y)$, $Y \subseteq X$. All possible instantiations of Y , $Y_{(1)}$, $Y_{(2)}$, \dots , $Y_{(i)} \in Y$, from a mutually exclusive and exhaustive set of events. $R(Y)$ can then be converted to a virtual evidence with the likelihood ratio:

$$L(y) = \frac{R(y_{(1)})}{P(y_{(1)})} : \frac{R(y_{(2)})}{P(y_{(2)})} : \dots : \frac{R(y_{(i)})}{P(y_{(i)})} \quad (1)$$

Therefore, propagating the likelihood finding $L(X)$ with Pearl's method provides the same results as propagating $R(Y)$. Thus the posterior probability of Y after propagating $L(Y)$ using Pearl's method is equal to $R(X)$. This paper discussed theory of imprecise probabilities and possibility theory using Bayesian theory and probability reasoning.

There are four methods of manage uncertainty in expert systems and artificial intelligence [23] [24]. They are: 1) default or non-monotonic logic, 2) probability, 3) fuzzy logic, 4) truth-value as evidential support, Bayesian theory, and 6) probability reasoning. In default or non-monotonic logic, a reasoning system is said to be monotonic if the truthfulness of a conclusion does not change when new information is added to the system. Therefore, the set of theorems can only monotonically grow when new axioms are added. Such reasoning is characteristic of commonsense reasoning, where default rules are applied when case-specific information is not available. nonmonotonic reasoning often require jumping to a conclusion and then retracting that conclusion as further information becomes available. All systems of nonmonotonic reasoning are concerned with the issue of consistency. Inconsistency is resolved by removing the relevant conclusion(s) derived previously by default rules.

III. MEASURING UNCERTAINTY

In this section, probabilistic reasoning and Bayesian probability theory are used as the tools for the measurement.

3.1 Probabilistic Reasoning

Probability theory is used to represent and process uncertainty. In probabilistic reasoning, the truth value of a proposition is extended from (0, 1) to [0, 1], with binary logic as its special case. This is because the uncertainty with highest probability is often preferred even though no conclusion is absolutely true. Under certain assumption, probability theory gives the optimum solutions. Most often, the Boolean connectives to probability functions is used as:

Negation: $P(\sim A) = 1 - P(A)$

Conjunction: $P(A \wedge B) = P(A) * P(B)$ if A and

B are independent of each other.

Disjunction: $P(A \vee B) = P(A) + P(B)$ if A and

B never happens at the same time.

Furthermore, the conditional probability of B given A is $P(B|A) = P(B \wedge A) / P(A)$, for which Bayes' Theorem is derived, and it is often used to update a system's belief according to new information:

$$P(B|A) = \frac{P(A|B)*P(B)}{P(A)} \quad (2)$$

3.2 Bayesian Probability Theory

Bayesian probability theory provides a mathematical framework for performing inference, or reasoning using probability [25]. The foundations of Bayesian probability theory were laid down some two centuries ago by people such as Bernoulli, Bayes, and Laplace, but it has been very controversial by modern statistician. However, the last few decades have witnessed what is referred to as "Bayesian revolution," and Bayesian probability theory is now commonly used in many scientific disciplines including expert systems, machine learning, and AI in general. It is mostly used to determine the relative validity of hypothesis in the face of noisy, sparse, or uncertain data, or to adjust the parameters of a specific model. Therefore, Bayes' theorem plays an increasingly prominent role in statistical applications but remain controversial among statisticians. Bayes' theorem is thus an algorithm for combining prior experience with current evidence [26] [27]. The Bayes' theorem is stated thus:

3.2.1 Definition (Bayes' Theorem):

If $P(A)$ is the probability of event A and $P(B)$ is the probability of B, then the conditional probability of A given B is $P(A/B)$ and the conditional probability of B given A is $P(B/A)$. Thus the theorem is stated mathematically as:

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right) P(A)}{P(B)} \quad (3)$$

$$P(B) = \sum_A P(B/A)$$

$$= \sum_A P(B/A)P(A) \quad (4)$$

Using probability to quantify uncertainty in Bayesian inference, we have:

$$P(\emptyset | \hat{X}, X) \propto P(\hat{X} | \emptyset) P(\emptyset | X) \quad (5)$$

where,

$P(\emptyset | \hat{X}, X)$ = Posterior,

$P(\hat{X} | \emptyset)$ = Likelihood function, and

$P(\emptyset | X)$ = Prior

$$\text{Hence, } P(A/B) = \frac{P(A, B)}{P(B)} = \frac{P(B/A)}{P(B)} P(A) = \frac{P(B/A)P(A)}{P(B/A)P(A) + P(B/A)P(A)} \quad (6)$$

The posterior probability of an uncertain proposition is the conditional probability that is assigned after the relevant evidence or background is taken into account. "Posterior" in this context means after taking into account the relevant evidence related to the particular case being examined [28] [29]. The posterior probability is the probability of the parameters given evidence X: $P(\emptyset|X)$. This is in contrast to the likelihood function, which is the probability of the evidence given the parameters: $P(X|\emptyset)$. The two are related as follows: Suppose we have a prior belief that the probability distribution function is $P(\emptyset)$ and observations with the likelihood $P(x|\emptyset)$, then the posterior probability is obtained as:

$$P(\emptyset|x) = \frac{P(x|\emptyset)P(\emptyset)}{P(x)} \quad (7)$$

Thus the posterior probability can be written in the form:

Posterior probability is directly propositional to the likelihood function multiplied by the prior probability. That is:

Posterior probability \propto likelihood function X Prior knowledge. This is the same as the formula stated earlier in (2). The "prior" information we need

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \quad (8)$$

Thus Bayes' theorem can be regarded as a rule to update an initial probability $p(A)$, also called the prior probability, into a revised probability $p(A|B)$, called the posterior probability that takes into account the updated knowledge currently available. As an example, consider a married woman who believed she that she may be pregnant after a single sexual intercourse, but she is unsure [30] [31] [32] Assuming she takes a pregnancy test which is known to be 80% accurate, i.e., the test gives positive result to positive case 90% of the time and the test produces a positive result. Thus she would like to know the probability she is pregnant, given a positive test ($p(\text{preg} | \text{test} +)$). However, what she knows is the probability of obtaining a positive test result if she is pregnant ($p(\text{test} + | \text{preg})$). In another similar example, suppose a young man conducts a test on prostate cancer and that this test is 80% accurate. Again, the individual would like to know the probability that he has prostate cancer, given the positive test, but the information available is simply the probability of testing positive if he has prostate cancer, coupled with the knowledge that he tested positive [33]. Therefore, Bayes' Theorem provides a way to reverse conditional probabilities and, hence provides a way to answer these questions.

3.2.2 Proof of Bayes' Theorem

$$\text{Theorem: } P(B|A) = \frac{P(A|B) P(B)}{P(A)}$$

Proof

As noted earlier, $p(A, B) = p(A|B) p(B)$ and $p(B, A) = p(B|A) p(A)$. From the foregoing, one can see that $p(A, B) = p(B, A)$. Thus

$$p(B|A)/p(A) = p(A|B) p(B) \quad (9)$$

Divide both sides by $p(A)$ leaves us with equation (6). The same is true for equation (2.1) if we divide both sides by $P(A)$.

The left-hand-side (L.H.S) of equation 9 is the conditional probability ($P(B|A)$) which we are interested. However, the righthand side (R.H.S) of equation 9 has three components: the conditional probability, $p(A|B)$, probability B, ($p(B)$), and probability A, ($p(A)$). $P(A|B)$ is the conditional probability we are interested in. $P(B)$ is the unconditional (marginal) probability of the event of interest. Finally, $p(A)$ is the marginal probability of event A. This quantity is computed as the sum of the conditional probability of A under all possible events B_i in the sample space: Either the woman is pregnant or she is not.

This is stated mathematically for a discrete sample space as follows:

$$P(A) = \sum_{B_i \in S_B} P(A|B_i) p(B_i) \quad (10)$$

3.3 Combining Rules in Bayesian Probability Theory

Two types of rules are often used in Bayesian probability Theory. They are: 1) sum rule and 2) product rule. The sum rule states that:

$$P(A|B) + P(A|\bar{B}) = 1 \quad (11)$$

while the product rule is as follows:

$$P(AB|C) = P(A|C) P(B|AC) \quad (12)$$

Here, $p(A|B)$ denotes the probability of A on the condition that B is true. These rules correspond to the negation and conjunction operations of Boolean algebra. The disjunction does not need a separate rule because it can be derived from negation and conjunction:

$$A + B = \sim(\sim A \sim B)$$

In fact, only one operation would suffice since other operations can be derived from either NAND or NOR operation alone. The NAND operation, for example, yields the following rule, starting from which every other rule of Bayesian probability theory can be derived:

$$P(\sim A + \sim B|C) + P(A|C) P(B|AC) = 1 \quad (13)$$

From equation 12, we know that:

$$P(AB|C) = P(A|C) P(B|AC) = P(B|C) P(A|BC) \quad (14)$$

and from equation 9,

$$P(A|B) + P(\sim A|B) = 1 \quad (15)$$

Equation 12 is called the product rule and equation 13 is called the sum rule.

Therefore, there is no loss of generality. Thus to determine the plausibility any logic function $f(A_1, \dots, A_n)$ from those of $\{A_1, \dots, A_n\}$ using equations 14 and 15, we need formulae for the plausibility of the conjunction AB and the negation $\sim A$. However, since:

$$(A|C) > (C|B) \quad (16)$$

The conjunction and negation are adequate set of operations from which all logic functions can be constructed. Therefore, for our basic rules is possible through repeated applications of the product rule and the sum rule in order to arrive at the plausibility of any proposition in the Boolean algebra generated by $\{A_1, \dots, A_n\}$. In order

to verify this, we need to find a formula for the logical sum $A + B$. This we can do by applying the product rule and sum rule repeatedly as follows:

$$\begin{aligned} P(A+B|C) &= 1 - P(\sim A \sim B|C) = 1 - P(\sim A|C) P(\sim B|\sim AC) \\ &= 1 - P(\sim A|C) [1 - P(B|\sim AC)] = P(A|C) + P(\sim AB|C) \\ &= P(A|C) + P(B|C) P(\sim A|BC) = P(A|C) + P(B|C) [1 - P(A|BC)] \end{aligned} \quad (17)$$

and finally,

$$P(A+B|C) = P(A|C) + P(B|C) - P(AB|C) \quad (18)$$

This generalized sum rule is one of the most useful rule in applications. Thus the primitive sum rule (15) is a special case of (18), with the choice $B = \sim A$.

Suppose we consider our earlier example on the pregnant woman with 80% chance of being pregnant. However, there is always the common error of misclassification such as false – positive results give 30%. That is, a woman will test positive 30% of the time when she is not even pregnant. Thus we have two possible outcomes or events B_1 : $B_1 = \text{preg}$ and $B_2 = \text{not preg}$. In addition, given the accuracy of 80% and false – positive rate of 30%, the conditional probabilities of obtaining a positive test in these two events are: $P(\text{test} + | \text{preg}) = 0.8$ and $P(\text{test} + | \text{not preg}) = 0.3$. Combining these information with some “prior” information on the probability of becoming pregnant from a single sexual intercourse, Bayes’ theorem provides a prescription for determining the probability of interest.

Since our marginal probability of being pregnant is $P(B) = P(\text{preg})$, this is our “prior” information apart from the fact that we know that the woman only had sexual intercourse once. This information is said to be prior because it is relevant information that exists prior to the test. Suppose we know from previous research that the probability of conception for any single sexual encounter is approximately 10%, Then based on these information, we can compute $p(B|A) \equiv P(\text{preg} | \text{test} +)$ as:

$$P(\text{preg}|\text{test} +) = \frac{P(\text{test} + | \text{preg})P(\text{preg})}{P(\text{test} + | \text{preg})P(\text{preg}) + P(\text{test} + | \text{notpreg})P(\text{notpreg})}$$

Substituting for these parameters, we have:

$$\begin{aligned} P(\text{preg}|\text{test} +) &= \frac{(0.8)(0.1)}{(0.8)(0.1) + (0.3)(0.9)} \\ &= \frac{0.08}{(0.08) + (0.27)} = \frac{0.08}{0.35} = 0.2286 \\ &= 0.229 \end{aligned}$$

Therefore, from our computation, the probability that the woman is pregnant, given the positive test is 0.229. This probability is usually referred to as “posterior” probability because it is the estimated probability arrived at using the “prior” information and other valuable data. This probability is very small due to some limitations before the test. However, the test can be repeated a number of times until the result becomes convincing. Using the “updated” probability of being pregnant ($p = 0.229$) as our new $p(B)$, i.e.; the prior probability for being pregnant has now been updated to reflect the result of the test. Supposing the woman repeats the test and again observes a positive result and her new “posterior probability” of being pregnant is:

$$\begin{aligned} P(\text{preg}|\text{test} +) &= \frac{(0.8)(0.229)}{(0.8)(0.229) + (0.3)(0.771)} \\ &= \frac{0.1832}{0.1832 + 0.2313} = \frac{0.1832}{0.4145} \\ &= 0.4412 \end{aligned}$$

This result is still not very convincing evidence that she is pre-pregnant. However, if she repeats the test again and finds a positive result, her probability increases to:

$$\begin{aligned} P(\text{preg}|\text{test}+) &= \frac{(0.8)(0.4412)}{(0.8)(0.4412) + (0.3)(0.5588)} \\ &= \frac{0.35296}{0.35296 + 0.16764} = \frac{0.35296}{0.5206} \\ &= 0.678 \end{aligned}$$

This result though good is not very convincing. The women need to perform further test(s). Supposing she repeat the test again as test 4. Then the Bayes' probability result or outcome will be as follows:

$$\begin{aligned} P(\text{preg}|\text{test}+) &= \frac{(0.8)(0.678)}{(0.8)(0.678) + (0.3)(0.322)} \\ &= \frac{0.5424}{0.5424 + 0.0966} = \frac{0.5424}{0.639} \\ &= 0.849 \end{aligned}$$

Although, this result is better, however, she can still perform further test(s).

However, this process of repeating the test and recomputing the probability of interest is of concern to statisticians. Basically, the Bayesian probability begin with some prior probability for some event, and we then continue to update this prior probability with new information to obtain a posterior probability is the used as a new prior probability in the next computation and the process is repeated continuously. From Bayesian and statistician point of view, this is a good strategy for conducting scientific research, i.e., by continually gathering data to evaluate a particular hypothesis because previous research gives us a priori information about an hypotheses and a due about research direction.

IV. CONCLUSION

This paper discussed managing uncertainty in artificial intelligence and expert systems using Bayesian Theory and Probability reasoning. In AI and expert systems, uncertainty is measured by using relative frequencies or by combining various statistical models based on data and information collected from various sources. Some of these measures are objective in nature while others may be from domain experts. All these measures all these measures are usually combined to make inference and decisions. However, for users to be convinced about an expert system especially when the user requested that the model and its conclusion be made explicit, there is need for the human expert and those who built the expert system to provide documentation and detailed explanation about how the expert system was built and how it works. That is, the expert system should be able to justify its assessments of the uncertainty and its reasoning procedures. Some of its assumptions and assessments should be able to amended and modified if required by the user. This way, the user can accept the level of uncertainty measures based on the concluding evidence.

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