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Some Applications of Tensors in Engineering

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ABSTRACT: *Explaining what a tensor represents is not an easy task. However, in a very simple way, a tensor can be defined as a set of entities that satisfy some basic rules, something similar to the vectors. The vectors satisfy the rules of the vector space, while the tensors obey the rules of a tensor space. The vector space is contained in the tensor space. A tensor is a generalization of the concept of a vector. By the way numbers, vectors and matrices are examples of tensors, and the difference between them is that each one has a certain order. Number is an "order 0 tensor", vector is an "order 1 tensor", and arrays are "order 2 tensors" and so on. The modern formulation of physics is based on tensor calculus, for these mathematical entities better describe the physical quantities in question, just as a vector better describes a displacement than a scalar number would describe. In fact, a tensor serves, mathematically, to simplify physical information.*

KEYWORDS: *tensor, physics, applications*

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The word tensor [(1)] was introduced in 1846 by William Rowan Hamilton. It was used in its present meaning by WOLDEMAR [(2)] in 1899. The tensor calculation was developed around 1890 by Gregory Ricci-Curbastro under the title of Absolute Differential Calculus. In the 20th century the subject became known as tensor analysis and gained wider acceptance with the introduction of Einstein's theory of general relativity over 1915. Tensors are also used in other fields as, for example, the mechanics of continuity. In order to better clarify the mathematical definition of tensor, the definition of NEARING [(2)] can be used, where the tensors are related to functional. In general, a function is something that associates a scalar with a scalar. On many occasions a functional is associated with the mapping of a set of functions in a set of scalars. Still using the concepts of NEARING, a functional is a function that maps vectors in scalars. Even in many applications, these functions can be seen as vectors.

An analogy can also be made between the functions of a vector and a tensor. Tensors can also be defined as structures used to generalize the notion of scalars, vectors and matrices. Like such entities, a tensor is a form of representation associated with a set of operations such as sum and product. The order of a tensor is the dimensionality of the matrix necessary to represent it. The tensor can be represented by an array of nine elements, sixteen elements, and so on. A tensor of order n in a space with three dimensions has 3^n components.

SECOND ORDER TENSOR

In analogy to a vector that requires three components to be specified, a tensor of order 2 requires nine components:

$$\tau_{ij} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix} \quad (1)$$

SOARES [(4)] reports that the tensors used in the General Theory of Relativity (RGT) are tensors of order 0, 1 and 2, and space is 4-dimensional space-time, three spatial coordinates, and temporal coordinate. Thus, the second-order tensors of the RGT have, in principle, 16 components, and real physical problems impose constraints of symmetry that reduce to 10 the really necessary components. The first order tensors are the vectors of the RGT which have four components. In order to illustrate this [(3)], let us consider a cubic element of a given material, which may be a fluid, for example. Each face of this hypothetical cube can be specified by the direction that is perpendicular to it and pointing out of the cube. Suppose that on each face of this cube there is an acting force imposed by the remainder of the material, with components in the x, y, and z directions on each face of the cube. The quotient of this force by the area of the face on which it is applied is defined as tension.

For an infinitesimal cubic element there is an associated tension, which will be equal for each parallel face. This tension in question has two directions, that of the face and that of the force component on that face. Thus τ_{xy} would mean the value of the stress on the face perpendicular to the x-axis, in the direction parallel to the y-axis. The complete tensor would still have the components τ_{xx}, τ_{xz}, τ_{yy}, τ_{yx}, τ_{yz}, τ_{zx}, τ_{zy}, τ_{zz}. In a more compact way it is said that the tensor of the tensions is a matrix of dimension (3x3) where all the tensions are associated to an infinitesimal element in the volume element below:

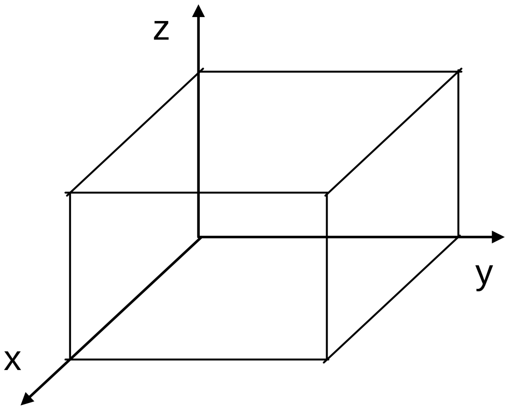

Figure 1 - Infinitesimal element in the volume element in the material

Each one of the six faces has a direction. By considering, for example, a face normal to the y direction moreover force acting on any face can act in the x, y and z directions. The force per unit face area acting in the x direction on that face is the stress τ_{yx} (first face, second stress). The forces per unit face area acting in the y and z directions on that face are the stresses τ_{yy} and τ_{yz}. Here τ_{yy} is a normal stress (acts normal, or perpendicular to the face) and τ_{yx} and τ_{yz} are shear stresses (act parallel to the face). Shear stresses similarly reverse on the opposite face are the stresses τ_{yy} and τ_{yz}. Then the tensor τ will be represented by 9 components.

$$\tau = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \tag{2a}$$

$$\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{pmatrix} \tag{2b}$$

Following an analogous reasoning it can be constructed tensors of order greater than two. Second order tensors can be represented by square matrices, third order matrices and so on. In relativity, in addition to the spatial components, the temporal component is also considered. An electric charge generates a field of electromagnetic tensors that results in interaction at each point of the electric field and the magnetic field. In general relativity theory [(5)], for example, we need to measure the curvature of space-time, whose components are (t, x, y, z), so we need to use a representation that must have four indices related to time, length, width and height. This would be done by a tensor of order 4, i.e. a number with four indices.

SOARES [(4)] reports that space-time curvature is given, mathematically, by Einstein's tensor $G_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \,, \tag{3}$$

where the indexes μ and ν assuming the values 0, 1, 2 and 3.

The tensor $R_{\mu\nu}$ is called the Ricci tensor, formed from the Riemann curvature tensor, which is a tensor of order 4, being the most general to describe the curvature of any space of n dimensions. The tensor $g_{\mu\nu}$ is the tensor of the metric of space-time and plays the role of the field in Einstein's equations. According to SOARES, the matter and energy of the Einstein equations are represented by the energy-momentum tensor $T_{\mu\nu}$. Then the Einstein field complete equations assume the following compact form

$$G_{\mu\nu} = -k\, T_{\mu\nu} \,, \tag{4}$$

where $k = 8\pi G/c^4$ is Einstein's gravitational constant, G is the universal gravitational constant, and c is the velocity of light in the vacuum.

In Newtonian language gravity is a force and in Einstein language it is itself the curvature of space-time. So the distribution of energy and matter generates the curvature. In relation to matter and energy of Einstein's equations, they are associated to the energy-momentum tensor that describes the energetic activity in space. The energy-momentum tensor quantitatively supplies the densities and flows of energy and momentum generated by the sources present in space and which will determine the geometry of space-time. The components of the energy-momentum tensor are as follows: matter and energy density, energy fluxes, moment component densities, and component fluxes which are shear stresses.

The energy-momentum tensor $T_{\mu\nu}$, in relativity, is a symmetric tensor that describes the flow of the μ component of the momentum p_μ across a hyper-surface. This tensor is useful because it can be written for any physical object containing energy, whether it is described by a system of particles or fields. According to BEZERRA DE MELLO [6] the tensor energy-momentum is given by

$$\langle T_{\mu\nu}(x)\rangle = \lim_{x'\to x} \partial_{\mu'}\partial_\nu G(x,x') + \left[\left(\xi - \frac{1}{4}\right) g_{\mu\nu}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) - \xi\nabla_\mu\nabla_\nu - \xi R_{\mu\nu}\right]\langle\phi^2(x)\rangle \quad (5)$$

The energy-momentum tensor describes the distribution and flow of energy and momentum due to the presence and movement of matter and radiation in a space-time region. In a simplified way, the energy-momentum tensor for electromagnetic fields is given by

$$T^{\mu\nu} = \frac{1}{\mu_0}\left(F^{\mu}{}_{\sigma}F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right) \quad (6)$$

In the relativistic dynamics [7] of the continuous means, for a uniform system the expected value of the energy-momentum tensor assumes the following form

$$\langle T_{\mu\nu}\rangle = (\varepsilon + p)u_\mu u_\nu + p\, g_{\mu\nu}\ , \quad (7)$$

where u_μ represents the vector velocity that describes the movement, $g_{\mu\nu}$ symbolizes the fundamental metric tensor, ε denotes the energy density and p represents the pressure.

The energy-momentum tensor can be written by a 4x4 matrix given by

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}, \quad (8)$$

where T_{00} represents the volumetric energy density; T_{10}, T_{20}, T_{30} symbolize the momentum densities; T_{01}, T_{02}, T_{03} denote energy flows; T_{21}, T_{31}, T_{32} represent moment flows; T_{12}, T_{13}, T_{23} symbolize terms of viscosity; T_{11}, T_{22}, T_{33} denote terms associated to pressure.

The matrix formula of the electromagnetic tensor is given by

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -c\,B_z & c\,B_y \\ E_y & c\,B_z & 0 & -c\,B_x \\ E_z & -c\,B_y & cB_x & 0 \end{pmatrix}, \quad (9)$$

where E_x, E_y , E_z represent the components of electric field vector; B_x , B_y , B_z symbolize the components of magnetic field vector and c is the speed of light.

I. FINAL CONSIDERATIONS

This article is an attempt to report the first notions about tensors. To cite an illustrative case, we can say that the mathematical description of physical laws, to be valid, must be independent of the coordinate system employed: mathematical equations that express the laws of nature must be invariant in their form under changes of coordinates. It is exactly the fulfillment of this requirement that leads physicists to the study of tensor calculation, of capital importance in the General Theory of Relativity and very useful in several other branches of physics. Moreover tensors are important because they provide a concise mathematical structure for formulating and solving physics problems in areas such as elasticity, fluid mechanics, electromagnetism, general relativity, and so forth. Tensors are extremely useful tools, particularly when describing phenomena in higher dimensions. Other important considerations from the point of view of concepts and applications can be found in SOKOLNIKOFF [8].

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