

## A Study of Joint Inversion of Self-Potential (SP) and Magnetic Anomalies in the Geophysical Prospecting

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**ABSTRACT:** The joint inversion of various geophysical prospecting data has been proved to reduce uncertainty and achieves a high resolution in the interpretation of geophysical prospecting. In this study, a joint inversion method of self-potential (SP) and magnetic anomalies generated simultaneously by a geological source was proposed. The joint inversion was effectively performed by reasonably arranging the elements of two parameter vectors representing two different fields (the parameter vectors of magnetic body and SP body) and by normalizing the elements of parameter vectors with different dimensions. The properties of the joint inversion were analyzed throughout model experiment on sphere and infinite horizontal dike. Finally, the proposed joint inversion method was proved to reduce the uncertainty of inversion and improve the convergence property.

**KEYWORDS:** Joint inversion, Self-potential (SP) anomaly, Magnetic anomaly

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### I. INTRODUCTION

The combined application of several geophysical prospecting methods including electrical and magnetic prospectings is very important and widely-used in the mineral exploration, the environmental survey and the geological engineering prospecting due to its capability of reducing the uncertainty of target object's properties. Moreover, the joint inversion of various geophysical fields measured from the combined application of several prospecting methods is considered as an important way to enhance the effectiveness of geophysical prospecting and also one of the recent development trends of geophysical prospecting technology.

Several studies on the combined application of various prospecting methods reported that the joint inversion of various geophysical prospecting data reduces the uncertainty and achieves a high resolution in the interpretation of geophysical prospecting. Hering et al. (1995) conducted the joint inversion of both dispersive Love and Rayleigh waves on one hand and the joint inversion of dipole-dipole, Schlumberger and two electrode sounding data on the other hand to achieve a better resolution and a finer details of subsurface. Gallardo-delgado. L. A, et al. (2003) carried out the joint inversion of 3D gravity and magnetic data to show a great reduction of uncertainty in the interpretation of the gravity and magnetic data. Dobroka et al. (2001) claimed that the joint inversion of magneto telluric and DC resistivity data achieves a better resolution of subsurface. Vozoff and Jupp (1975) and Verma and Sharma (1993) reported that joint inversion of magneto telluric and direct current resistivity data can not only resolve thin bed with greater degree of clarity but also effectively solve the problems of equivalence and suppression in the geoelectrical model. Benech et al. (2002) have claimed that the joint inversion of electromagnetic and magnetic data makes it possible to obtain the magnetic susceptibility image for the depth geological structures of the investigated area. H.V. Ram Babu (2003) derived the relationship between the self-potential (SP) anomaly and the gravity and magnetic anomaly. On the basis of the theoretical relationship illustrated from the measured magnetic and self-potential data, he has first interpreted the magnetic anomaly, and then interpreted the self-potential anomaly by using the information from the magnetic anomaly interpretation. According to his demonstration, when the sphere body is vertically polarized, the normalized gravity and self-potential anomalies,  $\Delta g$  and  $\Delta V$  respectively, are identical except the sign. Also, when the inclinations of magnetic and electrical polarizations,  $I$  and  $\alpha$  respectively, are the same, the horizontal components of magnetic anomaly  $\Delta H$  and the horizontal gradient of self-potential anomaly  $\Delta V_x$  are also

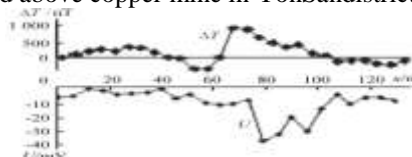
identical except the sign. In addition, he generalized the relationship for the objects with arbitrary shapes and reported that the relationships can be used to compute the pseudo-SP anomalies from the observed gravity and magnetic anomalies and to formulate the joint inversion schemes.

In this study, a joint inversion method of the self-potential and magnetic anomalies that are simultaneously generated by a geological source was proposed. The joint inversion was effectively implemented by reasonably arranging the elements of two parameter vectors of a source generating two different fields and by normalizing the elements of model parameter vectors with different dimensions. The proposed algorithm is similar to quasi Gauss-Newton algorithm. We developed a joint inversion software of the self-potential and magnetic anomalies for the sphere and infinite horizontal dike and estimated the properties through model experiment. The proposed joint inversion method obviously reduced uncertainty and improved convergence property. In addition, a field example of the joint inversion of self-potential and magnetic anomalies measured in a certain region was shown.

## II. PRINCIPLE OF THE METHOD

If we jointly invert geophysical anomalies which are essentially different, the target to be found must generate simultaneously those anomalies.

SP and magnetic anomalies measured above copper mine in YonSandistrict, D.P.R.Korea is seen in Fig 1.



**Fig1. SP anomaly (top) and magnetic anomaly (bottom) measured above copper mine in YonSandistrict.**

This region in the Proterozoic era is covered with thin overburden. There are some copper and gold mine in the region, where minerals of ore dike are copper pyrites, natural gold, pyrites, pyrrhotite, arsenopyrites and etc. Country rocks in the region are mainly limestone, dolomite and schist. Mineral with ferromagnetism among sulfide ores is only pyrrhotite  $\text{FeS}_{1+x}$  ( $0.11 \leq x \leq 0.15$ ). Magnetic susceptibility of pyrrhotite is about  $0.02 \sim 0.15\text{SI}$ , whereas those of copper pyrites and pyrites is about  $0.002\text{SI}$  and that of arsenopyrites is lower than  $0.006\text{SI}$ . Because pyrrhotite are minerals formed always in different mineralization steps of magma origin, both sp and magnetic anomalies are generated over ores including copper pyrites and pyrites together with pyrrhotite. Because ores in the region are sulfide ores and include pyrrhotite with high magnetism than country rocks (average magnetic susceptibility of country rocks is lower than  $0.0005\text{SI}$ ), both strong SP and magnetic anomalies are generated.

Amplitude of anomaly on the magnetic anomaly map in the region was about  $200 \sim 400\text{nT}$  at height from on the earth's surface and that of SP measured on the same survey line about  $-50 \sim -70\text{mV}$ .

Object generating both SP and magnetic anomalies also is coal layer. As well known, strong SP anomaly is generated in coal layer. Also, it is demonstrated by many investigators that in our country magnetic susceptibility of coal usually is lower than that of country rocks and negative anomaly of  $10 \sim 30\text{nT}$  is generated by that.

There are four problems important in completion of joint inversion.

First problem is to reasonably arrange elements of two vectors of parameters (one vector of model parameter is vector of model parameter of magnetic body and other vector is vector representing SP polarization body.) of the same model for convenience of construction of joint inversion.

When one body generates both SP and magnetic anomalies, let's that magnetic body can represent by  $m_t$  parameters and SP polarization body by  $m_s$  parameters, and suppose that  $m_r$  among  $m_t$  and  $m_s$  is repeated. Since the more number of repeated parameters is, the more these parameters are simultaneously constrained by magnetic and SP anomalies, it is obvious that effectiveness of joint inversion is more and more good.

Let total number of parameters to estimate to  $m$  and express with  $P = \{P_1, P_2, \dots, P_m\}$ . And expressing parameter of magnetic body as  $P^{(t)}$  and parameter of SP polarization body as  $P^{(s)}$ , relationships of  $P^{(t)} \cup P^{(s)} = P$ ,  $m > m_t$  and  $m > m_s$  are established.

In order to reasonably arrange elements of vectors of parameters of magnetic body and SP polarization body for convenience of construction of joint inversion, we set number of parameters to estimate to  $m$  and in the case of magnetic body assign zero to elements of parameters of greater number than  $m_t$  and in the case of SP polarization body assign zero to those of smaller number than  $m_s$  from end of array of parameter elements and

arrange repeated elements at the middle place of array of parameter elements. That is, we respectively expressed parameters of magnetic body and parameters of SP polarization body as follows.

$$P^{(t)} = \{P_1^{(t)}, P_2^{(t)}, \dots, P_m^{(t)}, 0, \dots, 0\} \quad (1)$$

$$P^{(s)} = \{0, \dots, 0, P_1^{(s)}, P_2^{(s)}, \dots, P_m^{(s)}\} \quad (2)$$

Since

$$P^{(t)} = \{P_1^{(t)}, P_2^{(t)}, \dots, P_{m_0+1}^{(t)}, \dots, P_{m_t}^{(t)}, 0, \dots, 0\} \\ + P^{(s)} = \{0, \dots, 0, \dots, 0, P_1^{(s)}, P_2^{(s)}, \dots, P_{m_s}^{(s)}, \dots, P_{m_s}^{(s)}\} \quad (3)$$

$$P = \{P_1, P_2, \dots, P_{m_0+1}, \dots, P_{m_t}, \dots, P_m\}$$

following relationship is established.

$$P = P^{(t)} \cup P^{(s)} = \{P_1, P_2, \dots, P_m\} \quad (4)$$

Above relationships, top subscript  $t$  and  $s$  express respectively magnetic anomaly and SP anomaly,  $m = m_t + m_s - m_r$  is number of total parameter elements of magnetic body and SP polarization body,  $m_0$  number of zero elements of parameter of SP polarization body,  $m_e$  number of zero elements of parameter of magnetic body and relationship  $m_r = m_e - m_0$  is established.

Second problem in completion of joint inversion is problem how one constructs object function using two fields with different dimension. Dimension of SP anomaly is  $mV$ , whereas dimension of magnetic anomaly is  $nT$ . Therefore, using sum of square of difference between measured field and theoretical field as object function, that has dimension of  $V^2 + (A/m)^2$ , which is contradiction.

Third problem in completion of joint inversion is that dimension of elements of model parameter vectors differ also. For example, in the case of sphere, dimension of its depth and horizontal position is meter (m), whereas that of its inclination of electrical polarization is degree( $^\circ$ ) and dimensions of dipole moments of SP polarization body and magnetic body also differ to each other. Therefore dimensions of sensitivities of elements also differ to each other, which is problem in constructing sensitivity matrix.

Fourth problem is that sensitivity matrix of joint inversion is very large compared that of single inversion.

To overcome these problems, We constructed as following object function.

$$\Phi(P) = \|W^{(s)}(U_{Obs} - U(P))\|^2 + \|W^{(m)}(\Delta T_{Obs} - \Delta T(P))\|^2 + \alpha \|W^{(P)}(P - P_0)\|^2 \quad (5)$$

Then, problem of finding information  $P$  of magnetic and SP polarization body from measurements of full magnetic anomaly and SP anomaly results in problem finding model parameter  $P$  minimizing object function(5). Where  $U(P)$  and  $\Delta T(P)$  are respectively theoretical SP anomaly and magnetic anomaly corresponding to model parameter  $P$ ,  $W^{(s)}$  and  $W^{(m)}$  respectively normalized matrixes for SP anomaly and magnetic anomaly and  $W^{(P)}$  weight matrix for model parameters and those are presented as follow.

$$W_{ij}^{(s)} = C_{ij}^{(s)} / U_j^{(Obs)} \\ W_{ii}^{(m)} = C_{ij}^{(m)} / \Delta T_j^{(Obs)}, \quad (6)$$

$$W_{kk}^{(P)} = C_{kk}^{(P)} / P_{kk}, \quad j = \overline{1, N_s}, \quad i = \overline{1, N_t}, \quad k = \overline{1, m}$$

Where  $N_s$  and  $N_t$  are respectively numbers of measurements of SP anomaly and magnetic anomaly,  $C^{(m)}$ ,  $C^{(s)}$  and  $C^{(P)}$  respectively covariance matrixes for magnetic anomaly, SP anomaly and model and in the case when these are not known we take these as unit.

As normalized and weight matrixes for SP and magnetic anomalies and model parameters expressed by equation (6) are incorporated, object function becomes that of abstract number with no dimension and as error terms of SP and magnetic anomalies are incorporated to object function as relative errors, portion of those of two anomalies are so equal that phenomena for only one anomaly to crucially effect on solution of inverse problem are overcome.

$m_r$  parameters among parameters of anomalous body are common in formulas of SP and its magnetic anomalies. Therefore, in step of inversion to find information of anomalous body from two measurements, these parameters are simultaneously constrained by both anomalies.

Expanding formulas of SP and magnetic anomalies Tyler series around initial approximate vector  $P_0$ , truncating high order terms for parameter perturbation and minimizing object function  $\Phi(P)$  without respect to  $\Delta P_k$ , following equation is obtained.

$$(A_k^T W^{(s)T} W^{(s)} A_k + B_k^T W^{(t)T} W^{(t)} B_k + \alpha_k W^{(P)T} W^{(P)} I) \Delta P_k = A_k^T W^{(s)T} W^{(s)} d_k^{(s)} + B_k^T W^{(t)T} W^{(t)} d_k^{(t)} \quad (7)$$

Where subscripts  $s$  and  $t$  in brackets present respectively SP and magnetic anomalies, subscript  $k$  iterative number, top subscript  $T$  transpose of matrix,  $\alpha$  control parameter,  $\Delta P_k$  parameter perturbation,  $I$  unit matrix,  $d_k^{(s)}$  and  $d_k^{(t)}$  respectively data misfit vector between theoretical and measured SP anomalies and that between theoretical and measured magnetic anomalies in  $k$ 's iteration,  $A$  and  $B$  are matrixes of sensitivities. After solving equation (7) and finding parameter perturbation  $\Delta P_k$ , solution in next step is found by,

$$P_{k+1} = P_k + \Delta P_k \quad (8)$$

This iteration is repeated until object function  $\Phi(P)$  converges to given error limit.

### III. THE METHOD OF JOINT INVERSION OF SP AND MAGNETIC ANOMALIES FOR SOME MODELS.

Silver deposition is carried out in an electron beam evaporator equipped with a real time thickness detector and the heat treatment is carried out in a heat treatment furnace capable of automatically controlling the temperature in the range of 250 ° C.

On the basis of the geophysical principle of joint inversion of SP and magnetic anomalies as above mentioned, I established the joint inversion for some simple models. Although this joint inversion is applied to simple models, this method is basis of joint inversion for arbitrary complex geological section and has also practical significance since there are many objects which can be approximate as these models in nature.

#### 1) Sphere model.

SP anomaly  $U(x)$  and magnetic anomaly  $H(x)$ ,  $Z(x)$  and  $\Delta T(x)$  over the sphere as seen in Fig. 2 are respectively expressed.

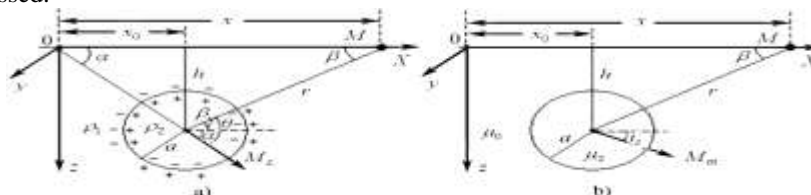


Fig 2. SP polarization body (a) and magnetic body (b) of sphere shape.

$$U(x) = 2M_s [(x - x_0) \cos \theta - h \sin \theta] / (x^2 + h^2)^{3/2} \quad (9)$$

$$H_x(x) = \frac{\mu_0 M_m}{4\pi [(x - x_0)^2 + h^2]^{5/2}} \{ [2(x - x_0)^2 - h^2] \cos i_s - 3h(x - x_0) \sin i_s \} \quad (10)$$

$$H_y(x) = \frac{-\mu_0 M_m}{4\pi [(x - x_0)^2 + h^2]^{5/2}} \{ [(x - x_0)^2 - h^2] \sin i_s \cot I \sin A' \cdot \cos \lambda \}$$

$$Z(x) = \frac{\mu_0 M_m}{4\pi [(x - x_0)^2 + h^2]^{5/2}} \{ [2h^2 - (x - x_0)^2] \sin i_s - 3h(x - x_0) \cos i_s \} \quad (11)$$

$$\Delta T(x) = \frac{\mu_0 M_m}{4\pi [(x - x_0)^2 + h^2]^{5/2}} \{ [2h^2 - (x - x_0)^2] \sin i_s \sin I + (2(x - x_0)^2 - h^2) \cdot \cos i_s \cos I \cos A' - 3h(x - x_0) 2 \sin i_s \cos I \cos A' - ((x - x_0)^2 + h^2) \cdot \sin i_s \cot I \cos I \sin^2 A' \} \quad (12)$$

Where  $M_s$  represented as  $M_s = \epsilon_0 a^2 \frac{\rho_1}{\rho_1 + 2\rho_2}$  is dipole moment of sphere ( $\rho_1$  and  $\rho_2$  are respectively resistivities of country rocks and the sphere,  $\epsilon_0$  polarization electromotive force,  $a$  radius of the sphere),  $x$  horizontal position of measurement point,  $h$  and  $x_0$  depth and horizontal position of the sphere,  $\theta$  inclination angle,  $\mu_0$  absolute permittivity of vacuum,  $I$  the inclination of the earth's magnetic field,  $i_s$  effective magnetic inclination,  $M_m$  magnetic moment of the sphere,  $A'$  magnetic azimuth angle of survey line,  $\lambda = \arccos(\sin I / \sin i_s)$ .

If representing parameter of the magnetic sphere as  $P^{(m)} = \{M_m, h, x_0, 0, 0\}$  and parameter of the SP polarization sphere as  $P^{(s)} = \{0, h, x_0, \theta, M_s\}$ , parameter of the magnetic and SP polarization sphere  $P$  is  $P = \{M_m, h, x_0, \theta, M_s\} = \{P_1, P_2, P_3, P_4, P_5\}$ .

After finding parameter perturbation  $\Delta P_k$  by calculating sensitivity matrix, constructing equations system (7) and solving that, update of the parameter is carried out by equation (8).

Iteration is repeated until relative error

$$\varepsilon = \sum_{i=1}^{N_s} (U_{obsi} - U_i(P)) / |U_{obsi}| + \sum_{i=1}^{N_m} |(\Delta T_{obsi} - \Delta T_i(P)) / \Delta T_{obsi}| \quad (13)$$

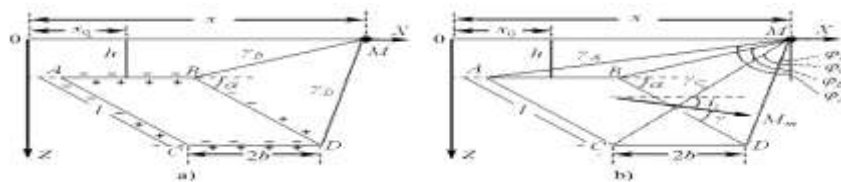
is achieved at given limit and control parameter is set as follows.

$$a_k = \begin{cases} K_0 \times a_{k-1}, & k = 1, 2, 3 \dots \\ 0.001, & \text{if } K_0 \times a_{k-1} < 0.001 \end{cases} \quad (14)$$

where  $a_0$  is initial control parameter  $a_0 = 10$  and  $K_0 = 0.5$ .

**2) Infinite horizontal dike.**

In the cases of following infinite horizontal dike also inversion method is equal to that of the sphere except sensitivity matrix.SP anomaly  $U(x)$  and magnetic anomaly  $H(x), Z(x)$  and  $\Delta T(x)$  over the infinite horizontal 2D dike as seen in Fig. 3 are respectively expressed.



**Fig 3. The infinite horizontal SP polarization dike (a) and the infinite horizontal magnetic dike (b).**

$$U(x) = M_s \{ (h + l \sin \alpha)(\phi_c - \phi_D) + (x - x_0 - l \cos \alpha - b) \ln r_c - (x - x_0 - l \cos \alpha + b) \ln r_D - h(\phi_B - \phi_A) - (x - x_0 - b) \ln r_B + (x - x_0 + b) \ln r_A \} \quad (15)$$

$$H(x) = \frac{\mu_0 M_m \sin \alpha}{2\pi} \left\{ \cos \gamma \cdot \ln \frac{r_B \cdot r_C}{r_A \cdot r_D} + \sin \gamma [(\phi_A - \phi_B) - (\phi_C - \phi_D)] \right\} \quad (16)$$

$$Z(x) = \frac{\mu_0 M_m \sin \alpha}{2\pi} \left\{ \sin \gamma \cdot \ln \frac{r_B \cdot r_C}{r_A \cdot r_D} + \cos \gamma [(\phi_A - \phi_B) - (\phi_C - \phi_D)] \right\} \quad (17)$$

$$\Delta T(x) = \frac{\mu_0 M_m \sin \alpha}{2\pi} \cdot \frac{\sin I}{\sin i_s} \left\{ \cos(\alpha - 2i_s) \ln \frac{r_B \cdot r_C}{r_A \cdot r_D} - \sin(\alpha - 2i_s)(\phi_A - \phi_B - \phi_C + \phi_D) \right\} \quad (18)$$

Where  $x$  is horizontal position of measurement point,  $M_s$  dipole moment of SP polarization body,  $h$  and  $x_0$  respectively depth and horizontal position of the dike,  $b$  half thick of the dike,  $l$  the inclination of the earth's magnetic field,  $i_s$  effective magnetic inclination,  $\gamma = \alpha - i_s$ ,  $M_m$  magnetic moment of the magnetic dike,  $r_A, r_B, r_C, r_D$  represented as

$$\left. \begin{aligned} r_A &= [(x - x_0 + b)^2 + h^2]^{1/2} \\ r_B &= [(x - x_0 - b)^2 + h^2]^{1/2} \\ r_C &= [(x - x_0 - l \cos \alpha + b)^2 + (h + l \sin \alpha)^2]^{1/2} \\ r_D &= [(x - x_0 - l \cos \alpha - b)^2 + (h + l \sin \alpha)^2]^{1/2} \end{aligned} \right\} \quad (19)$$

respectively distances between vertical line being gone down measurement point M and points A, B, C and D,  $\phi_A, \phi_B, \phi_C$  and  $\phi_D$  angles between vertical line being gone down measurement point M and sight line looking points A, B, C and D(Fig 3).

If representing parameter of the magnetic dike as  $P^{(m)} = \{M_m, h, x_0, b, l, \alpha, 0\}$  and parameter of the SP polarization dike as  $P^{(s)} = \{0, h, x_0, b, l, \alpha, M_s\}$  parameter of the magnetic and SP polarization dike  $P$  is  $P = \{M_m, h, x_0, b, l, \alpha, M_s\} = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ .

**IV. MODEL EXPERIMENTS**

A method of easily attaching the metal nanoparticles was proposed in order to increase the light absorption efficiency in the crystalline silicon solar cell. The light absorption of solar cell is enhanced by the scattering of incident light on the metal nanoparticles by the surface plasmonic resonance. The silver nanoparticles have been deposited by the

deposition of the silver thin film by using the thermal evaporation and the thermal process. And then the antireflection coating on it has been conducted and the reflection spectrum has been measured. The measurement demonstrates, the best absorption has been realized in the case of the evaporation film of 18nm, the thermal process time of 50min and the thermal process temperature of 200°C. And the absorption efficiency of light has been increased by 1.17 times.

We developed software for joint inversion of SP and magnetic anomalies due to the sphere and infinite horizontal dike on the basis of algorithm which we proposed and carried out model experiments for examining the properties.

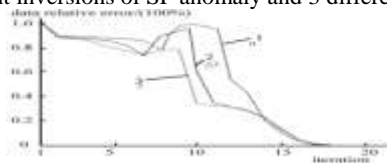
Contents of model experiments includes property of joint between SP anomaly and different magnetic anomalies, estimation of noise stability.

In all model experiments, interval between measurement points is 5m and the numbers of those are respectively 30 and therefore total numbers of those (sum of SP measurements and magnetic measurements) are 60. And the inclination of the earth's magnetic field is 60° and magnetic azimuth angle of survey line varies from 0° to 90°.

**1) Properties of joint between SP anomaly and different magnetic anomalies.**

In order to estimate what how components of magnetic anomaly is jointed with SP anomaly is good, we carried out joint inversions of SP anomaly and full magnetic anomaly  $\Delta T$ , of that and magnetic vertical component  $Z$  and of that and magnetic horizontal component  $H$ .

Result of model experiments for the sphere is seen in Fig 4. As seen in Fig 4, accuracy and properties of convergence are almost similar for all joint inversions of SP anomaly and 3 different components of magnetic anomaly.



**Fig 4. Properties of convergence of inversions of SP anomaly and components  $\Delta T$  (2),  $Z$  (3),  $H$  (1) of magnetic anomaly for the sphere.**

**2) Noise effect.**

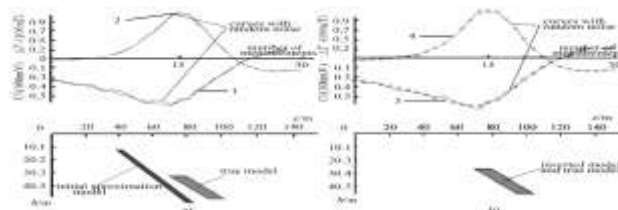
If one applies practically the joint inversion to the interpretation of field data, the joint inversion must be able to overcome the effects of random noise. To simulate field environment, we added random noises corresponding to 1%, 5% and 10% of anomaly value in each measurement points to theoretical SP and magnetic anomalies for dike model and estimated stability of random noises of the method. The results are seen in the Table 1.

**Table 1. Noise stability for the dike**

	1 (%)			5 (%)			10 (%)		
	true	approx.	estimated	true	approx.	estimated	true	approx.	estimated
$M_m / (10^{-9} \text{ A} \cdot \text{m}^{-1})$	5	2	5.0	5	2	5.0	5	2	4.7
$h/m$	27	6	27.0	27	6	27.1	27	6	26.7
$X_0/m$	75	40	75.0	75	40	75.2	75	40	74.6
$b \times 2$	5	2	5.0	5	2	5.10	5	2	5.2
$\alpha / (^\circ)$	38	45	38.0	38	45	38.0	38	45	37.9
$l/m$	30	60	30.1	30	60	29.4	30	60	31.2
$M_s / (10^{-3} \text{ V} \cdot \text{m}^2)$	10	4	10.0	10	4	9.9	10	4	9.3
Model relative error/%			0.047			0.81			3.27
Data relative error/%			0.52			2.26			4.83

As seen table 1, the noise stabilities for the anomalous dike is fairly better, in which when random noises are 1% data and model relative errors are respectively 0.52% and 0.047%, when those are 5% those are respectively 2.26% and 0.81% and when those are 10% those are respectively 4.84% and 3.27%.

When random noise is 10%, the result of joint inversion for the dike is seen in Fig5. Curves at which random noise added, theoretical curves, initial approximation model and true model before the joint inversion are seen in Fig 5(a) and those after that are seen in Fig 5(b).



**Fig5. Result of joint inversion for dike 1—SP anomaly, 2—magnetic anomaly, 3—inverted SP anomaly, 4—inverted magnetic anomaly**

**FIELD EXAMPLE**

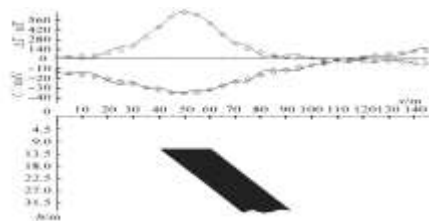
In investigation region-YonSandistrict, the rocks of SaDangu group (BanChon, OBongSan and NaDong Formations) and JikHyon group (HaeChang and MulGumSan Formations) in Mesoproterozoic era, the ProterozoicEon and MukChongroup (MaChon and YonSan Formations) in NeoproterozoicEra, the ProterozoicEon are developed.

The composition of minerals of the ore bodies in the investigation region are mainly copper pyrites, natural gold, pyrites, pyrrhotine and arsenopyrite.

For us to elucidate the properties of development of the ore bodies in the investigation region, we performed both SP and magnetic field measurements setting two survey networks. Taking account of strike of the geological formations, in the first survey network the strike of survey line set as North-East 30°, whereas in the second network that set as North-East 35°. In two survey networks the length of survey line was 150m, interval between lines 20m and distance between measurement points 5m. In the first network total number of survey lines was 26 and in the second network that was 15.

The strikes of the SP and magnetic anomalies is coincided with each other, which are about North-West 305~310° and lengths of horizontal extension of anomalies are about 1200m.

Fig6 shows SP and magnetic anomalies(solid lines) measured on No.5 survey line in the second survey network and inverted results(Fig. 6), the thickness, depth, depth extension, horizontal center position and inclination angle of the dike were respectively 9.8m, 11.9m, 184.6m, 50.7m and 45°, which were verified by means of drilling.



**Fig6. SP and magnetic anomalies (solid line) measured on the No.5 survey line, inverted anomalies (circles) and inverted dike.**

**V. CONCLUSIONS AND DISCUSSIONS**

In this paper, a joint inversion method of the self-potential (SP) anomaly and magnetic prospecting anomaly that are effective in mineral exploration was proposed. The proposed algorithm is similar to quasi Gauss-Newton algorithm. The joint inversion was effectively performed by reasonably arranging the elements of two parameter vectors representing two different fields (the parameter vectors of magnetic body and SP body) and by normalizing the elements of model parameter vectors with different dimensions. The proposed method can be applied to the exploration of objects with such shapes as sphere and dike. This method only used some simple models, but in case of complex geological subsurface, it can be used as the basis to jointly invert the SP and magnetic anomalies. The model experiment shows that the joint inversion of SP and magnetic anomalies has an obvious advantages compared to the single inversion in which only individual anomaly is inverted. That is, the joint inversion method has a great convergence radius, a little of uncertainty and a high accuracy of estimation. In the future, this joint inversion method must be extended to the joint imaging method for the inversion of arbitrary complex geological subsurface. When the number of estimation parameters is large (to avoid the calculation of sensitivity matrix) or it is difficult to reasonably determine the initial approximation, it is better to use the global optimization algorithms including GA, SA and NA as joint inversion algorithm.

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