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Matlab Based Buckling Analysis of Thin Rectangular Flat Plates

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ABSTRACT:*One of the major problems of rectangular platebuckling under in-plane load is the rigorous approach use in its analysis. In this study, the problem of buckling was addressed by developing a Matlab based computer program for ease of analysis of rectangular plates for critical buckling load, which is needed for safe design.The plates were assumed to be loaded axially along the x-axis, and polynomial shape functions used in Ritz energy equation to formulate a general solutionwhich is computer user-friendly. The critical buckling load coefficients' 'n' values obtained from this program were compared with thoseavailablein scholarly literaturesso as to demonstrate theirvalidity. These values were found to be very close to existing values in literature. It therefore implies that, this general computer program for buckling analysis of rectangular plates is a better and quicker means of obtaining the critical buckling load of rectangular thin isotropic plates.*

KEY WORDS: *Computer Program, Critical buckling load, Polynomial Shape Function, Rectangular Plates.*

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I. INTRODUCTION

Previous studies on critical buckling load werebased on classical thin plate theory,in which Fourier series was applied in both classical and approximate methods. These methods are very rigorous, and the uncertainty of assuming correctly the shape function makes it most difficult to use. Scholars who have used such approaches are [1], [2],[3],[4], [5]and [6].

The use of polynomial shape functionsto ease the uncertainty of the previous trigonometric based shape function became evident recently in the works of [7], [8],[9], and [10]. This method no doubt have given some element of relief,though buckling of plates are still characterized by lengthy and time consuming derivations and computations.

The application of computer programs in analysis of some specific rectangularplates namely SSSS and CSCS plates based on the polynomial shape functions was carried out as seen in [11]. The results of which were comparable with available research works based on manual computation. The present research work is concerned with the development of a general computer based program forbuckling analysis ofrectangularthin isotropic plates.

II. DERIVATION OF THE CRITCAL BUCKLING LOAD EQUATION

[12], gave the general polynomial deflected shape function 'w' for rectangular plates as:

$$w = A(c_1R^{f1}-d_1R^{m1}+e_1R^{n1})\,(c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2}) \quad (1)$$

where, $c_1,c_2,f^1,f^2,d_1,d_2,m^1,m^2,e_1,e_2$ and n^1,n^2 are coefficients which may assumed 0.5, 1,2,..as applicable.

A is the amplitude, R& Q are non-dimensional parameters in x-and y- directions respectively.

let $k = (c_1R^{f1}-d_1R^{m1}+e_1R^{n1})\,(c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2})$ (2)

Substituting (2) into (1), yields:

Thus, $w = Ak$ (3)

The summary of the expressions of shape functions,w, and parameter k, for rectangular plate of 12 different boundary conditions are presented in Table 1.

Thetotal potential energy functional, Π_x, for a rectangular thin Isotropic plate subject to in-plane load in x-direction was given by [7] as (4):

$$\Pi_x = \frac{D}{2b^2}\iint \left[\frac{b^3}{a^3}(w''^{R})^2+\frac{2b}{a}(w''^{RQ})^2+\frac{a}{b}(w''^{Q})^2\right]\partial R\partial Q-\frac{bN_x}{2a}\iint (w'^{R})^2\partial R\partial Q \quad (4)$$

where, D = Flexural Rigidity, W = Deflected shape function; N_x = Critical buckling load in x -direction, $w''^R = \frac{\partial^2 w}{\partial R^2}$; $w''^Q = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R\,\partial Q}$; $w'^R = \frac{\partial w}{\partial R}$;
Now if the aspect ratio is given by (5).

$$s = b/a, \tag{5}$$

then, substituting (5) into (4), yields (6):

$$\Pi_x = \frac{D}{2a^2}\iint \left[s(w''^R)^2 + \frac{2}{s}(w''^{RQ})^2 + \frac{1}{s^3}(w''^Q)^2\right]\partial R\partial Q - \frac{N_x}{2}\iint s(w'^R)^2\partial R\partial Q \tag{6}$$

Substituting (3) into (6), gives the total potential energy functional, Π_x, as (7):

$$\Pi_x = \frac{DA^2}{2a^2}\iint \left[S(k''^R)^2 + \frac{2}{S}(k''^{RQ})^2 + \frac{1}{S^3}(k''^Q)^2\right]\partial R\partial Q - \frac{N_x A^2}{2}\iint S(k'^R)^2\partial R\partial Q \tag{7}$$

where k is define by (2), and $k''^R = \frac{\partial^2 k}{\partial R^2}$; $k''^Q = \frac{\partial^2 k}{\partial Q^2}$; $k''^{RQ} = \frac{\partial^2 k}{\partial R\,\partial Q}$; $k'^R = \frac{\partial k}{\partial R}$;

Minimizing (7) and making N_x the subject of the formula yields the equation of the critical buckling load, N_x as(8):

$$N_x = \frac{D\int\int\left[(k''^R)^2 + \frac{2}{s^2}(k''^{RQ})^2 + \frac{1}{s^4}(k''^R)^2\right]\partial R\,\partial Q}{a^2\int\int(k''^R)^2 dR.dQ} \tag{8}$$

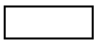
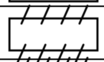
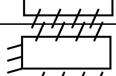
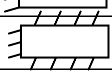
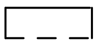
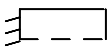
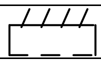
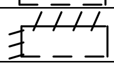
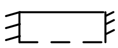
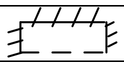
$$\text{let}\, n_x = \frac{\int\int\left[(k''^R)^2 + \frac{2}{s^2}(k''^{RQ})^2 + \frac{1}{s^4}(k''^R)^2\right]\partial R\,\partial Q}{\int\int(k'^R)^2\,\partial R\,\partial Q} \tag{9}$$

substituting (9) into (8) yields (10):

$$\text{Thus, } N_x = n_x\frac{D}{a^2} \tag{10}$$

Where n_x is the critical buckling load factor or coefficient in x- direction,

Table 1: Polynomial Formulated Shape Functions for various types of Rectangular Plates

S/N	Types of Plates	Plate Sketch	Shape Function W = Ak_i (where k_i =k)	Shape Parameter k_i= U*V (where i = 1,2,3, ..., 12)
1	SSSS		W =A k_1	k_1= $(R-2R^3+R^4)(Q-2Q^3+Q^4)$
2	CCCC		W =A k_2	$K_2 = (R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
3	CSSS		W =A k_3	$K_3 = (R-2R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
4	CSCS		W =A k_4	$K_4= (R-2R^3+R^4)(Q^2-2Q^3+Q^4)$
5	CCSS		W =A k_5	$K_5 = (1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
6	CCCS		W =A k_6	$K_6 = (1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
7	SSFS		W =A k_7	$K_7 = (R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
8	SCFS		W =A k_8	$K_8 = (1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
9	CSFS		W =A k_9	$K_9 = (R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
10	CCFS		W =A k_{10}	$K_{10} = (1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
11	SCFC		W =A k_{11}	$k_{11} = (R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
12	CCFC		W =A k_{12}	$k_{12} = (R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

where S-simply Supported edge, C- Clamped edge, F-Free edge
The values of the parameter, $k_i$givenin Table 1 can be evaluated and substitutedinto (8) or (9) to obtain the values of critical load factor 'n_x'.

III. COMPUTER APPLICATION

Due to the complexity of expressions for shape functions, w, and shape parameter, k, it is very rigorous and stressful to evaluate shape function, critical buckling load factor, n_x and critical buckling load N_x, of plates. Computer programbased on the shape functions presented in Table 1 and the other expressions derived weredeveloped for the determination of the critical buckling load of these plates as presented in appendix. The flow chart is as shown in Fig.1:

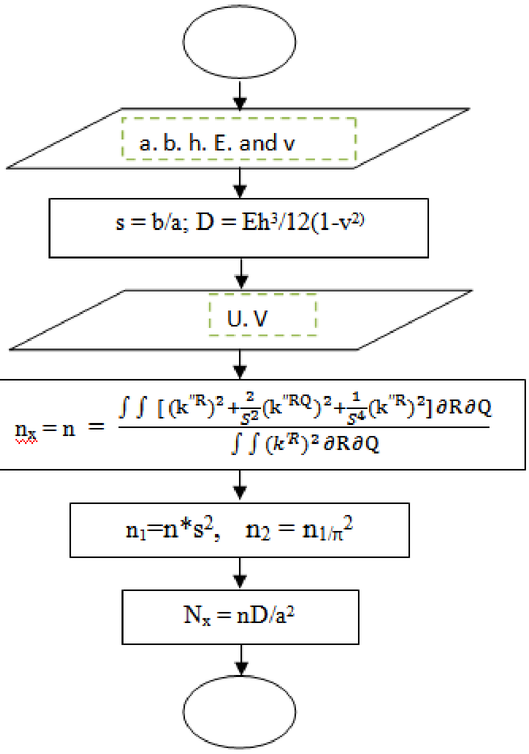


Figure. 2: Flowchart

The program is easy to understand and follow. The user is expected to respond to on screen messages by inputting the required data. The values of 'n_x' obtained from the executed program for some of the plates are presented on Table2.

IV. RESULTS AND DISCUSSIONS

In order to validate the results of this work a comparison of the results obtained from the computer program for CCCC, CSSS, and SCFC plates are made with those of [13], who used the same polynomial functions. The comparison is presented on Table 3. For the CCCC plate, the maximum percentage difference is -0.692% foraspect ratios of 2.0, while the minimum is -0.609% for aspect ratio of 1.0. These percentage differences are very insignificant. This indicates that the computer results agree with the results obtained by [13] for all round clamped plate. In the case of CSCS plate, the percentage differences, range from 0.00% for aspect ratio, b/a of1.2, 1.5 and 1.6 to 0.017% for aspect ratio, b/a of 1.0. Again, the percentage differences being less than 1.0% in all cases are negligible. And for the SCFC plate, Table 4 reveals that the maximum percentage difference between the computer results and those obtained by [13] is4.681% for aspect ratio, b/a of 1.0 and minimum percentage difference is 1.235% for aspect ratio of 2. These differences are below 5% and are considered insignificant.

Table 2: Values of 'n_x' from present study for SSSS Plate

Aspect Ratio S = b/a	CCCC $N_x = n_x\pi^2\frac{D}{b^2}$ n_x	CSSS $N_x = n_x\pi^2\frac{D}{b^2}$ n_x	CCSS $N_x = n_x\pi^2\frac{D}{b^2}$ n_x	SCFC $N_x = n_x\pi^2\frac{D}{b^2}$ n_x	CCFC $N_x = n_x\pi^2\frac{D}{b^2}$ n_x
1.0	10.943	5.756	6.559	4.808	5.139
1.2	11.515	5.447	6.845	6.668	6.912
1.5	13.898	5.646	8.037	10.106	10.279
1.6	14.988	5.824	8.582	11.423	11.580
2.0	20.518	6.922	11.347	17.545	17.663

Table 3: Values of 'n' from present study for SSSS Plate with those of Ibearugbulem et al (2014)

Aspect Ratio S = b/a	Present Study (CCCC) n_1	Ibearugbulem et al.(2014) n_2	% difference $100(n_1 - n_2)/ n_2$	Present Study (CSSS) n_3	Ibearugbulem et al.(2014) n_4	% difference $100(n_3 - n_4)/ n_4$
1.0	10.943	11.010	-0.609	5.756	5.755	0.017
1.2	11.515	11.587	-0.621	5.447	5.447	0.000
1.5	13.898	13.989	-0.651	5.646	5.646	0.000
1.6	14.988	15.088	-0.663	5.824	5.824	0.000
2.0	20.518	20.661	-0.692	6.922	6.921	0.014

Table 4: Values of'n' from present study for SCFC plate with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a S	Present Study (SCFC) n_1	Ibearugbulem et al.(2014) n_2	% difference $100(n_1 - n_2)/ n_2$
1.0	4.808	4.593	4.681
1.2	6.668	6.454	3.316
1.5	10.106	9.891	2.174
1.6	11.423	11.208	1.918
2.0	17.545	17.331	1.235

V. CONCLUSION

From the three sample plates resultsobtained and compared with [13], it is seen that the percentage difference are very little or insignificant and thusthe results obtained are satisfactory. Also, this computer program is faster and simpler. Thus, it can be concluded that, this general computer program for buckling analysis of rectangular plates is a better and safer means of obtaining the critical buckling load of rectangular thin isotropic plates.

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APPENDIX

```
clear
%Program for analysis of rectangular plates
a = input('Enter plate dimension  along x-axis -length- a(m):');
b = input('Enter plate dimension  along y-axis -width- b(m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
```

```
v = input('Enter value of poission ratio v:');
echo on
s = b/a
echo off
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
%Deflected shape function w = Ak; k = U*V
syms r q
U = input('Enter U:');
V = input('Enter V:');
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,5)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),6)
```

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