

## Investigating the Impact of Equations of State on Predicting Liquid Loadings in Gas Wells

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**ABSTRACT:** Pressure and temperature variations of natural gas flow in a pipeline may cause partial condensation of liquids. Fluid phase behavior and prevailing conditions often make liquid appearance inevitable, which subjects the pipe flow to a higher pressure loss and eventual loss of high quality liquid hydrocarbons. Liquid formation is a major concern for transmission companies due to the significant decrease in gas flow capacity, inaccuracy in metering and potential damage to instrumentation and equipment. This paper investigates the impact of cubic equations of state in accurately predicting liquid loading in gas wells. The model involves coupling of hydrodynamic models, thermodynamic model, and associated constitutive equations that incorporates mass transfer for predicting liquid accumulations in natural gas and condensate transmission pipelines. Selecting the most appropriate equation of state model gives invaluable information for locating and sizing liquid collection tanks and the design of cost effective pigging schedules. The results of the numerical model showed that the Peng-Robinson EOS predicted the highest condensate dropout compared to SRK and Modified SRK EOS. The results from Peng-Robinson EOS are in good agreement with that obtained using commercial software, NIST REFPROP

**KEYWORDS:** Liquid loading, equation of state, gas well, phase behavior, pressure drop, condensate

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### I. INTRODUCTION

Liquid loading in gas wells is a major flow assurance problem that requires urgent attention to prevent early abandonment or outright untimely killing of gas wells. It involves the accumulation of heavier hydrocarbon fractions or water or both in the wellbore of gas wells. Liquid loading is peculiar to gas wells and most prevalent in wells producing from wet gas and condensate reservoirs. It can also occur in dry gas reservoir howbeit, only when an adjacent aquifer is the source of the liquid.

During the transportation of produced gas through pipelines, liquids also drop out from the gas stream due to pressure drop in the direction of flow. The condensation of the heavier fractions can cause serious flow assurance problems such as hydrate formation which could block flow lines and thus, prevent gas from being transported to designated locations.

The problem of partial condensation in gas pipelines leads to multiphase flow in pipes. Multiphase fluid flow in pipes has been studied by many researchers for many years. More sophisticated methods of analysis were initiated by the nuclear industry, where two-phase flow occurs in reactor cooling systems and affects heat transfer [1].

Optimization and successful operation of two-phase pipeline systems require knowledge of the phase behavior and characteristics of such flows. Various studies have shown that at present no single theory or correlation can satisfactorily predict the characteristics of two-phase gas-liquid in pipe over a wide range of conditions [2].

Several correlations exist for predicting pressure drop and liquid holdup in horizontal, vertical, and inclined pipelines. One of the recent improvements in this field has been the development of compositional multiphase hydrodynamic models for analyzing gas condensate pipelines [3,4]. These models couple a two-fluid hydrodynamic two-phase flow model with a phase behavior model for single pipelines. However, gas condensation in pipelines commonly occurs owing to the multicomponent nature of the transmitted natural gas and its associated phase behavior, as well as the inevitable temperature and pressure variations that occur along the pipeline. Condensation of heavier fractions subjects the gas pipeline to two-phase transportation.

The hydrodynamic behaviour of two-phase system is radically different from that of single-phase systems. The two-phase systems not only exhibit higher pressure loss, but are also subjected to interphase forces and mass-generation-induced forces, all of which are completely absent in single-phase systems [5]. Other system variables needed to define a two-phase system include liquid hold-up, phase velocity, phase thermo-physical properties, and the flow regimes.

In order to model the mechanism of liquid accumulation, equations of state plays a major role in appropriately describing the thermodynamics of the condensation. However, several equations of state exist, hence there is need to know which is best suitable to adequately describe the mechanism of liquid loading. In this paper, a numerical simulation using MATLAB is performed for a two-phase flow in pipe to compare some renowned equations of state for adequately predicting liquid accumulations in pipelines.

## II. METHOD

The method used in this paper involves coupling hydrodynamic and thermodynamic equations along with appropriate constitutive equations. Thereafter, a MATLAB program was developed to perform numerical simulations for two-phase gas-liquid flow in pipe. Basically the three most commonly used equations of state; Soave Redlich Kwong [6], the modified Soave Redlich Kwong [7] and the Peng-Robinson [8] equations of state, were used for the comparative analysis during the numerical simulation.

### (a) Hydrodynamic Equations for Two-Phase Flow

The formulation of the governing equations for two-phase flow in pipes consists of solving five ordinary differential-equations (ODE) simultaneously at steady-state condition. The system of equations consists of two mass conservation equations, two momentum conservation equations and a combined energy conservation equation respectively. These set of equations completely describes the behavior of this type of flow.

#### Mass conservation equation

The continuity equation for the liquid and vapour phases to describe the mass conservation can be expressed as [9]:

$$\text{Vapour phase: } \rho_g v_g \frac{d\alpha_g}{dx} + \left[ \alpha_g v_g \left( \frac{\partial \rho_g}{\partial P} \right)_T - G_m \left( \frac{\partial f_{mg}}{\partial P} \right)_T \right] \frac{dP}{dx} + \left[ \alpha_g v_g \left( \frac{\partial \rho_g}{\partial T} \right)_P - G_m \left( \frac{\partial f_{mg}}{\partial T} \right)_P \right] \frac{dT}{dx} + \alpha_g \rho_g \frac{dv_g}{dx} = 0 \quad (1)$$

$$\text{Liquid phase: } \rho_l v_l \frac{d\alpha_l}{dx} + \left[ \alpha_l v_l \left( \frac{\partial \rho_l}{\partial P} \right)_T - G_m \left( \frac{\partial f_{ml}}{\partial P} \right)_T \right] \frac{dP}{dx} + \left[ \alpha_l v_l \left( \frac{\partial \rho_l}{\partial T} \right)_P - G_m \left( \frac{\partial f_{ml}}{\partial T} \right)_P \right] \frac{dT}{dx} + \alpha_l \rho_l \frac{dv_l}{dx} = 0 \quad (2)$$

#### Momentum Conservation equation

The momentum conservation equation for the vapour and liquid phases can also be expressed as [9]

$$\text{Vapour phase: } \left[ \alpha_g v_g^2 \left( \frac{\partial \rho_g}{\partial P} \right) + v_g^2 \right] \frac{dP}{dx} + 2\alpha_g v_g \rho_g \frac{dv_g}{dx} + \rho_g v_g^2 \frac{d\alpha_g}{dx} + \alpha_g v_g^2 \left( \frac{\partial \rho_g}{\partial T} \right)_P \frac{dT}{dx} = -F_{tg} \quad (3)$$

$$\text{Liquid phase: } \left[ \alpha_l v_l^2 \left( \frac{\partial \rho_l}{\partial P} \right) + v_l^2 \right] \frac{dP}{dx} + 2\alpha_l v_l \rho_l \frac{dv_l}{dx} + \rho_l v_l^2 \frac{d\alpha_l}{dx} + \alpha_l v_l^2 \left( \frac{\partial \rho_l}{\partial T} \right)_P \frac{dT}{dx} = -F_{tl} \quad (4)$$

#### Energy conservation equation

The combined energy conservation equation for the vapour and liquid phases is given as:

$$\begin{aligned} & \left\{ \alpha_g v_g \left[ H_g + \frac{v_g^2}{2} \right] \left( \frac{\partial \rho_g}{\partial P} \right)_T + \alpha_l v_l \left[ H_l + \frac{v_l^2}{2} \right] \left( \frac{\partial \rho_l}{\partial P} \right)_T - \alpha_g v_g \rho_g \eta_g C_{pg} - \alpha_l v_l \rho_l \eta_l C_{pl} \right\} \frac{dP}{dx} \\ & + \left\{ \alpha_g v_g \left[ H_g + \frac{v_g^2}{2} \right] \left( \frac{\partial \rho_g}{\partial T} \right)_P + \alpha_l v_l \left[ H_l + \frac{v_l^2}{2} \right] \left( \frac{\partial \rho_l}{\partial T} \right)_P - \alpha_g v_g \rho_g \eta_g C_{pg} - \alpha_l v_l \rho_l \eta_l C_{pl} \right\} \frac{dT}{dx} \\ & + \alpha_g \rho_g \left[ H_g + \frac{3v_g^2}{2} \right] \frac{dv_g}{dx} + \alpha_l \rho_l \left[ H_l + \frac{3v_l^2}{2} \right] \frac{dv_l}{dx} + \rho_g v_g \left[ H_g + \frac{v_g^2}{2} \right] \frac{d\alpha_g}{dx} + \rho_l v_l \left[ H_l + \frac{v_l^2}{2} \right] \frac{d\alpha_l}{dx} \\ & = Q_t - v_g F_{tg} - v_l F_{tl}, \end{aligned} \quad (5)$$

### (b) Closure Relationships

The closure relationships for the different parameters used in this paper are given as follows:

#### Wall Friction Force

The wall friction is defined as [10]:

$$F_{wa} = A_{wa} f_{wa} \frac{\rho_a |v_a| v_a}{2g_c} \quad (6)$$

where:  $F_{wa}$  = Forces between fluid “a” and the pipe walls,  $f_{wa}$  = Fanning friction factor,  $A_{wa}$  = Pipe surface wetted by fluid “a” per unit volume,  $\rho_a$  = Density of fluid “a”,  $v_a$  = Velocity of fluid “a”

The wetted surface area is defined using the expression[11]:

$$A_{wa} = \frac{\text{Pipesurfacewettedbyfluid "a"}}{\text{Totalvolume}}, \tag{7}$$

The Fanning friction factor ( $f_{wa}$ ) is a function of conduit roughness and Reynolds number of a fluid ( $Re_a$ ). The Reynolds number is expressed in terms of the hydraulic diameter ( $d_{ha}$ ) as:

$$Re_a = \frac{\rho_a v_a d_{ha}}{\mu_a} \tag{8}$$

where:  $d_{ha}$  = equivalent hydraulic diameter of fluid  $a$  and  $\mu_a$  = Viscosity of fluid  $a$ . The hydraulic diameter can be expressed [10] as:

$$d_{ha} = 4 \frac{\text{Volumeoffluid "a"}}{\text{Pipesurfacewettedbyfluid "a"}} \tag{9}$$

The Fanning friction factor defined in terms of laminar and turbulent flows. For laminar flow ( $Re_a < 2300$ ) this factor is approximated using the following analytical expression:

$$f_{wa} = \frac{16}{Re_a} \tag{10}$$

while for turbulent flow ( $Re_a > 4000$ ), the Fanning friction factor is estimated using the Colebrook (1939) correlation given as.

$$\frac{1}{\sqrt{f_{wa}}} = -2.0 \log \left[ \frac{(\epsilon/d_{ha})}{3.7} + \frac{2.51}{Re_a \sqrt{f_{wa}}} \right] \tag{11}$$

**Interfacial Forces**

The Interfacial friction is calculated using [10]:

$$F_{ia-b} = f_{ia-b} A_{a-b} \frac{\rho_a |v_a - v_b| (v_a - v_b)}{2g_c} \tag{12}$$

where:  $A_{a-b}$  = The total contact surface between fluids “a” and “b” per unit volume. This quantity is defined as:

$$A_{a-b} = \frac{\text{Totalcontactsurfacebetweenfluidsaandb}}{\text{TotalVolume}} \tag{13}$$

The total contact surface between fluids and the interfacial friction factors are calculated from the flow pattern geometry.

**Gravitational forces**

The gravitational force is determined using [11]

$$F_{ga} = \alpha_a \rho_a \frac{g}{g_c} \sin \theta \tag{14}$$

where:  $\alpha_a$  is Holdup of fluid “a”, and  $\theta$  is inclination angle of the phase. Since there is no flow regime-dependent-coefficients involved in the gravitational force calculation, this definition holds for any flow pattern.

**External Energy Input**

The volumetric heat exchange which lumps the effects of convection and/or conduction through the pipe from or to the environment is expressed as[12]:

$$Q_t = - \frac{U(\pi d \Delta x)(T - T_s)}{\text{Volume}} = \frac{U \pi d \Delta x (T - T_s)}{(\pi d^2 / 4) \Delta x} = \frac{4U(T_s - T)}{d} \tag{15}$$

where:  $U$  is the overall heat transfer coefficient,  $T$  is ambient temperature,  $T_s$  is the temperature of the surrounding,  $d$  is the diameter of the pipeline, and  $\Delta x$  is the length of a pipe block

### III. HERMODYNAMIC MODEL

The equations of state are commonly used to describe the thermodynamic behavior of complex mixtures. The thermodynamic model also defines transition between single phase/two phase conditions. This work compares the Soave RedlichKwong, Modified Soave RedlichKwong and the Peng-Robinson equations of state to implement the thermodynamics model to investigate their impacts on the prediction of liquid loading in gas wells. Table 1 shows the models of these EOS and their associated parameters. As can be seen in Table 1, the major difference between the SRK and the modified SRK is how the acentric factor, which is a measure of the sphericity of the droplets is defined.

**Table 1: Models of three different Cubic EOS and their respective parameters**

Equations of State	$P_{repulsion} - P_{attraction}$	$a$	$b$
Soave RedlichKwong (1972)	$P = \frac{RT}{(V-b)} - \frac{a\alpha(T)}{V(V+b)}$ where $m = 0.480 + 1.74\omega - 0.176\omega^2$	$\Omega_a \frac{R^2 T_c^2}{p_c}$	$\Omega_b \frac{RT_c}{p_c}$
Modified Soave-RedlichKwong (1978)	$P = \frac{RT}{(V-b)} - \frac{a\alpha(T)}{V(V+b)}$ where $m = 0.486508 + 1.55171\omega - 0.15613\omega^2$	$\Omega_a \frac{R^2 T_c^2}{p_c}$	$\Omega_b \frac{RT_c}{p_c}$
Peng-Robinson ( )	$P = \frac{RT}{(V-b)} - \frac{a\alpha(T)}{[V(V+b) + b(V-b)]}$	$\Omega_a \frac{R^2 T_c^2}{p_c}$	$\Omega_b \frac{RT_c}{p_c}$

To investigate the impact of the EOS in this study, it is necessary to solve systems of ordinary differential equations. A fourth/fifth-order explicit Runge-Kutta method is used in which the dependent variables of the differentials are placed on the left hand side and all other coefficients and expressions are placed on the right hand side of the equation. This arrangement, gives rise to the linear system of ordinary differential equations:

$$[A_m] \frac{d}{dx} [U] = [B_m] \quad (16)$$

where  $A_m$  represent the coefficients of the derivative of the variable U and  $B_m$  is an algebraic function of the system variables.

### IV. RESULTS AND DISCUSSION

This section presents the results obtained from the numerical simulation and the accompanying discussions. The gas composition and other data used for the numerical simulation are shown in Tables 2 and 3 for a 5.641-in horizontal pipeline. The initial pressure was assumed to be 1,595psia, while the inlet and the surrounding temperatures were set at 330°F and 310°F. The initial velocities of the vapour and liquid phases were also set to be 9.15ft/s and 2.35ft/s respectively.

To ensure that mass is conserved across the system as expected in steady state models, a material balance calculation was performed along the pipeline as shown Figure 1. Since mass flux is constant along the pipeline, then the mass conservation law is obeyed.

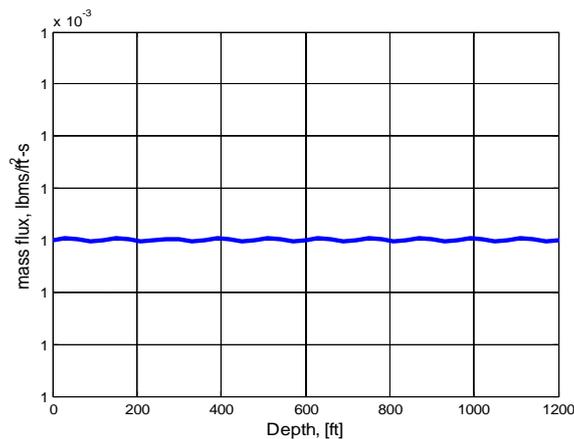
Figure 2 shows the prediction of variation of pressure along the pipeline for the three correlations considered. As can be seen in Figure 2, the pressure decreases as the fluid flows along the pipeline and all the three correlations predicted similar pressure gradient along the pipeline. A temperature gradient plot showing a decrease in the temperature as the fluid flows along the wellbore is shown in Figure 3 for all the models. However, whereas SRK and modified SRK gave identical temperature gradients along the pipeline, the temperature gradient is less steep as predicted by Peng-Robinson. Rapid cooling is predicted by SRK and Modified SRK against that of Peng-Robinson. The decrease in pressure and temperature result in the expansion of gas along the pipeline, which eventually leads to cooling and thus, liquid accumulation.

**Table 2:** Basic data for the condensate pipeline

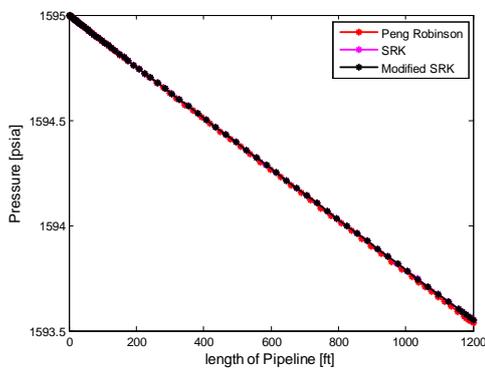
Segments	30
Length (ft)	1200
Diameter (in.)	5.461
Absolute roughness (ft.)	0.0018
Total diathermanous factor, U (Btu/°Fft <sup>2</sup> hr)	1
Ambient temperature(°F)	330
Step length (ft.)	100

**Table 3:** Compositional data used for numerical analysis

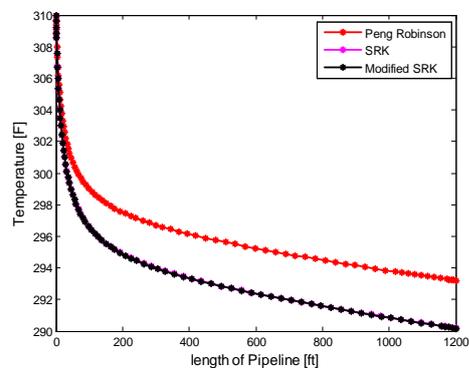
Component	Fraction
Nitrogen	0.0084520
Carbon dioxide	0.0260290
C1	0.7792670
C2	0.0700790
C3	0.0595780
i-C4	0.0117720
n-C4	0.0241040
i-C5	0.0082290
n-C5	0.0060020
n-C6	0.0030494
n-C7	0.0023357
n-C8	0.0011030



**Figure 1:** Material balance to check that mass conservation law is obeyed

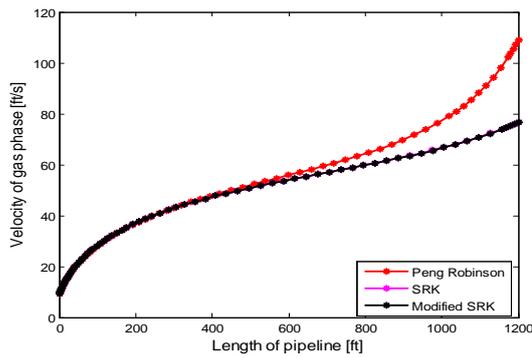


**Figure 2:** Pressure variation along a condensate along pipeline for the three cubic EOS model

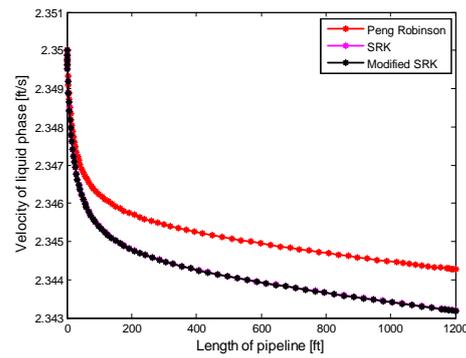


**Figure 3:** Temperature profile predicted the condensate pipeline for the EOS

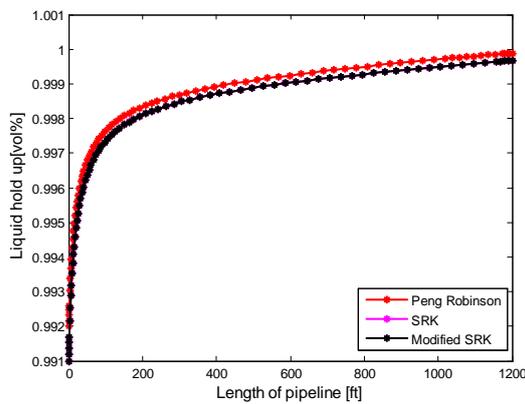
Figures 4 shows increase in the vapor phase velocity as the fluid flows along the pipeline. Again, the SRK and Modified SRK predicted identical velocity profile against the Peng-Robinson which predicted reasonably higher velocities.



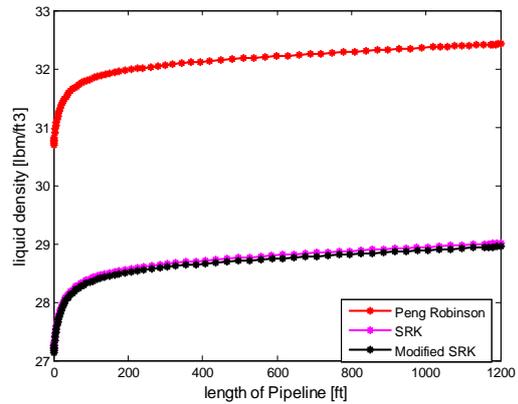
Figures 4: Velocity profile of the vapor phase



Figures 5: Velocity profile of the liquid phase



Figures 6: Liquid holdup



Figures 7: Liquid density

The liquid holdup distribution which indicates the percentage of liquid in the segment of the pipeline is shown in Figure 6. From Figure 6, the liquid holdup increases from inception and is a maximum at the end of the pipeline. This is expected as temperature and pressure gradients along the pipe decreases, triggering condensation of liquids as the fluid flows to the required destination. This is also depicted in Figure 7, where the density of the liquid phase increases along the pipeline. Whereas, the correlations predicted close liquid holdup, there is an obvious disparity in the prediction of the liquid densities; the SRK and the Modified SRK predicted approximate values while the Peng-Robinson predicted higher densities, showing its superiority in predicting liquid densities. Similar trend was also seen in the prediction of gas density along the pipeline as shown in Figure 8.

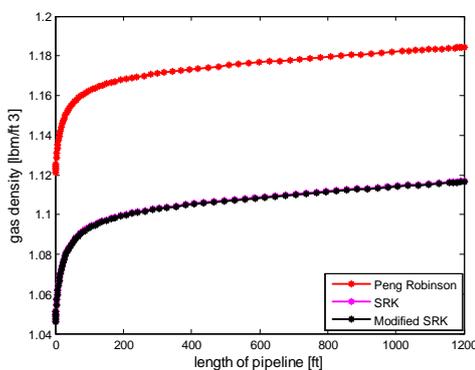


Figure 8: Gas density profile

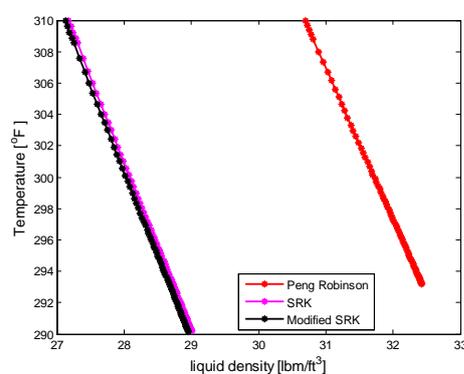


Figure 9: Plot of liquid density vs temperature

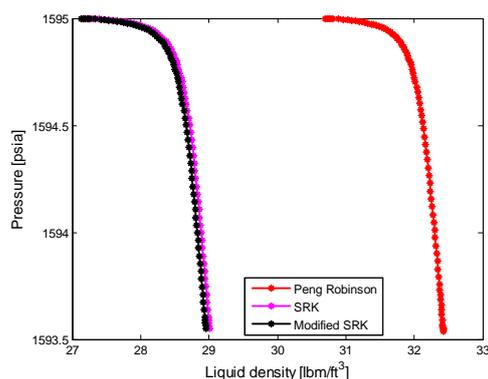


Figure 10: Plot of liquid density vs temperature

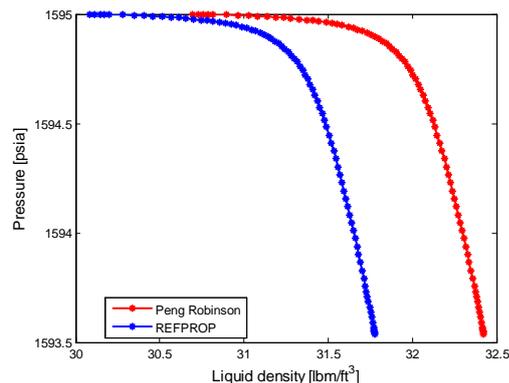


Figure 11: Validation of the numerical model

In order to investigate the impact of temperature and pressure on liquid accumulation, plots of temperature and pressure versus liquid density was explicitly determined during the numerical simulation as shown in Figures 9 and 10. From Figures 9 and 10, as the temperature and pressure decreases along the pipeline, there is a corresponding increase in the liquid accumulation, an affirmation that decrease in temperature and pressure enhances condensation of liquids from a gas. Again, from Figures 9 and 10 it can be seen that Peng-Robinson EOR predicted higher liquid dropout compared to SRK and modified SRK respectively.

The numerical simulation result is validated with data obtained from a commercial simulator, NIST RefProp as shown Figure 11. As can be seen in Figure 11, the result from the numerical simulation closely agrees with data from the commercial simulator. The slight variation in the predicted values from the commercial simulator can be attributed to the inherent shortcomings associated with EOS to effectively predict the liquid density.

## V. CONCLUSIONS

Liquid loading is predictable and not always easily recognized when it occurs in gas wells. Predicting the onset of liquid loading should have been the most effective way of managing or controlling its occurrence. Unfortunately, there is no particular model that is exhaustive in accurately predicting this problem when it eventually occurs. In order to identify models that can closely predict the occurrence of liquid loading, this work investigates the impact of using inadequate EOR models in the prediction of liquid loading since the governing principle behind the condensation of liquids is strongly a thermodynamic phenomenon. The cubic equations of state compared in this work are the Peng-Robinson, SRK and Modified SRK respectively. The Peng-Robinson shows superiority in predicting liquid condensation over the SRK and Modified SRK, and when compared with a commercial simulator, the result shows consistency and good agreement.

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**NOMENCLATURE**

$A_w$  = pipe cross sectional area ( $L^2$ )  
 $A$  = Wetted area per unit volume ( $L^2$ )  
 $A_{gt}$  = Interfacial area per unit volume ( $L^2$ )  
 $\alpha\alpha(T)$  = Generalized temperature dependent term  
 $[A_m]$  = Left-hand side square matrix of the implicit hydrodynamic formulation  
 $[B_m]$  = Right-hand side vector of the implicit hydrodynamic formulation  
 $d$  = diameter of the pipe (L)  
 $d_h$  = Hydraulic diameter (L)  
 $f_m$  = Equilibrium Mass fraction (unitless)  
 $f_w$  = Fanning friction factor (unitless)  
 $F_g$  = Gravitational force ( $m/L^2-t^2$ )  
 $F_w$  = Wall shear force ( $m/L^2-t^2$ )  
 $F_i$  = Interfacial force ( $m/L^2-t^2$ )  
 $F_t$  = Net force applied to the phase ( $m/L^2-t^2$ )  
 $g$  = acceleration of gravity ( $L/T^2$ )  
 $G_m$  = Total mass flux entering the pipe per unit area [ $M/L^2-T$ ]  
 $h_l$  = Taitel&Dukler stratified equilibrium liquid level  
 $\dot{M}_g$  = Total mass transferred into the gas phase [ $M/T-L^3$ ]  
 $\dot{M}_l$  = Total mass transferred into the liquid phase [ $M/T-L^3$ ]  
 $m$  = Temperature correction parameter  
 $w$  = acentric factor of the substance  
 $v$  = velocity of the phase (L/t)  
 $P$  = Pressure ( $m/L-t^2$ )  
 $Q_g$  = External energy input taken by liquid phase ( $m/L-t^3$ )  
 $Q_{gt}$  = Energy exchange from the liquid to the gas ( $m/L-t^3$ )  
 $Q_{lg}$  = Energy exchange from the gas to the liquid ( $m/L-t^3$ )  
 $Q_t$  = Total external energy input ( $m/L-t^3$ )  
 $R$  = Molar gas constant  
 $r$  = radius (L)  
 $T$  = Temperature (T)  
 $U$  = Overall heat transfer coefficient ( $m/t^3-T$ )  
 $U_o$  = Initial (inlet) value of any given unknown  
 $[U]$  = Vector containing all unknowns of the hydrodynamic formulation  
 $x$  = distance along the pipe length (L)  
 $W$  = total mass flow rate (m/t)  
 $W_f$  = Wetted wall fraction (unitless)

**Subscripts**

$a$  = generic phase 'a'  
 $l$  = liquid phase,  $g$  = gas phase  
 $t$  = total or net,  $w$  = wall or wet  
 $I$  = interfacial,  $S$  = surroundings

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