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Bipolar Fuzzy Quasi Prime Ideals and Weakly Bipolar Fuzzy Quasi Prime Ideals in Left Almost Rings

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ABSTRACT :*In this article we introduced bipolar fuzzy quasi prime and weakly bipolar fuzzy quasi prime ideals of LA-rings and their properties. We find practical way to prove that a bipolar fuzzy left ideal is a bipolar fuzzy quasi prime ideal that is by observe their membership values. But we can do this if this bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ hold additional properties, such that if $max\{f_B^+(x), f_B^+(y)\} = f_B^+(x)$ then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(x)$ or if $max\{f_B^+(x), f_B^+(y)\} = f_B^+(y)$then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(y)$.*

KEYWORDS *LA-rings, bipolar fuzzy sets, bipolar fuzzy quasi prime ideals, bipolar fuzzy weakly quasi prime ideals.*

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I. INTRODUCTION

A fuzzy set is a function from a non empty set X to [0,1][18]. Fuzzy ideal is one of the topics discussed in fuzzy theories. Many researchers have studied fuzzy ideals of various algebraic structures such as Liu in [6] that studied fuzzy ideals in rings and further, Mukherjee &Senin [10] studied fuzzy prime ideals in rings. Xie in [13] also studied fuzzy quasi prime ideals in semigroups, and Mandal in [8] studied fuzzy quasi ideals in ordered semirings.

The concepts of LA-rings have been introduced in 2006. Studies about ideals, prime ideals, bi-ideals, and quasi ideals in LA-rings have been conducted. In fuzzy theories, LA-rings also take part in the researchers' attention. Further, Yiarayongin [16] introduced fuzzy quasi prime ideals and weakly fuzzy quasi prime ideals of LA-rings.

A fuzzy set can be extended on its codomain. One of that is to be a bipolar fuzzy whose codomain expanded from [0,1] to $[-1,1]$[19]. Some researchers have studied bipolar fuzzy ideals of various algebraic structures such as Majumder in [9] that studied bipolar fuzzy ideals in Γ-semigroups and Yaqoob in [14] that studied about bipolar fuzzy (ideals, bi-ideals, and interior ideal) in LA-semigroups. Further, Mahmood& Hayat in [7] introduced bipolar fuzzy h-quasi ideals by intrinsic product in hemirings, also Subbian&Kamaraj[12] introduced bipolar fuzzy ideals and bipolar fuzzy ideal extensions in subrings.

There is no study that discuss about bipolar fuzzy quasi prime ideals in LA-rings. Hence we motivated to developed [16] into bipolar fuzzy. So in this paper we study bipolar fuzzy quasi prime and weakly bipolar fuzzy quasi prime ideals in LA-rings.

II. PRELIMINARIES

In this section the basic definitions and theorems needed in study the bipolar fuzzy quasi prime ideals and weakly bipolar fuzzy quasi prime ideals in left almost rings are given.

Definition 2.1.[4]A grupoid $(G, \cdot)$ is called an LA-semigroup (Left Almost-semigroup) $(G, \cdot)$ if the left invertive law hold, such that $(a \cdot b) \cdot c = (c \cdot b) \cdot a$for all $a, b, c \in G$.

Proposition 2.2.[4]If G is an LA-semigroup then the Medial law hold, such that
$(a \cdot b)(c \cdot d) = (a \cdot c)(b \cdot d)$,for all $a, b, c, d \in G$

Proof: By the left invertive law we have
$(a \cdot b)(c \cdot d) = \big((c \cdot d) \cdot b\big) \cdot a = \big((b \cdot d) \cdot c\big) \cdot a = (a \cdot c)(b \cdot d).$

Proposition 2.3.[15]If G is an LA-semigroup with left identity then the Paramedial law hold, such that $(a \cdot b)(c \cdot d) = (d \cdot c)(b \cdot a)$,for all $a, b, c, d \in G$

Proof: By using the left invertive law and the property of left identity then we have

$$\begin{aligned}(a \cdot b)(c \cdot d) &= \big((e \cdot a) \cdot b\big)\big((e \cdot c) \cdot d\big)\\ &= \big((b \cdot a) \cdot e\big)\big((d \cdot c) \cdot e\big)\\ &= \Big(\big((d \cdot c) \cdot e\big) \cdot e\Big) \cdot (b \cdot a)\\ &= \big((e \cdot e) \cdot (d \cdot c)\big) \cdot (b \cdot a)\\ &= (d \cdot c)(b \cdot a)\end{aligned}$$

Definition 2.4.[4] in [3] An LA-semigroup$(G, \cdot)$ is called an LA-group if these following properties hold
1. Has a left identity element e, such that $e \cdot a = a$for all $a \in G$,
2. For all$a \in G$ there is a' such that $a' \cdot a = e$.

Definition 2.5.[17] in [16] A non empty set R with two binary operations $(+,\cdot)$ is called an LA-ring if these following properties hold
1. $(R, +)$ is an LA-group,
2. $(R,\cdot)$ is an LA-semigroup, and
3. Distributive laws of $\cdot$over $+$ hold, such that for all $a, b, c \in R$, $(a + b) \cdot c = a \cdot c + b \cdot c$and
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

If $e \in R$ such that $e \cdot a = a$for all $a \in R$ then R is called an LA-ring with left identity. A non empty subset of LA-ring R is called a Left Almost-subring (LA-subring) of R if it is an LA-ring under binary operation of R[16].

Definition 2.6.[11] in [16] If for all r element of LA-ring R and for any $a$element of LA-subring S of R hold $ra \in S(ar \in S)$ then S is called left (right) ideal of R. Further, if S is left and right ideal of R then S is an ideal of R.

Definition 2.7.[11]in [16] A left ideal S is called quasi prime (weakly quasi prime) ideal of R if and only if $AB \subseteq P(\{0\} \neq AB \subseteq P)$implies $A \subseteq P$ or $B \subseteq P$, for A, B ideals of R.

Definition 2.8.[19] A bipolar fuzzy set of LA-ring R is defined by $A = (f_A^+, f_A^-)$with$f_A^+: R \to [0,1]$ and $f_A^-: R \to [-1,0]$.
An LA-ring Rcan be regarded as a bipolar fuzzy set by define $\Gamma = (R_\Gamma^+, R_\Gamma^-)$ with $R_\Gamma^+: R \to 1$ and$R_\Gamma^-: R \to -1$.

Definition 2.9.[2]Let$N$ be a subset of LA-ring $R$and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$. $t'A_N = \big(t^+A_N, t^-A_N\big)$as defined bellowis a bipolar fuzzy set of R

$$t^+A_N(x) = \begin{cases} t^+, & \text{if} \quad x \in N \\ 0\,, & \text{if} \quad \text{otherwise} \end{cases} \text{and} \; t^-A_N(x) = \begin{cases} t^-, & \text{if} \quad x \in N \\ 0\,, & \text{if} \quad \text{otherwise} \end{cases}$$

Definition 2.10.[16]Let βbe an index and $\{A_\alpha = \big(f_{A_\alpha}^+, f_{A_\alpha}^-\big): \alpha \in \beta\}$be a family of bipolar fuzzy subsets of R, then we have

$$\bigcap_{\alpha\in\beta} f_{A_\alpha}^-(x) = \inf\{f_{A_\alpha}^-(x): \alpha \in \beta\}$$
$$\bigcup_{\alpha\in\beta} f_{A_\alpha}^+(x) = \sup\{f_{A_\alpha}^+(x): \alpha \in \beta\}$$

Definition 2.11.[1]The Cartesian product of bipolar fuzzy sets $A = (f_A^+, f_A^-)$ and $B = (f_B^+, f_B^-)$ of sets I and J respectively is defined as follows

$$A \times B = \big((x, y), (f_A^+ \times f_B^+)(x, y), (f_A^- \times f_B^-)(x, y)\big)$$

with
$(f_A^+ \times f_B^+)(x, y) = \min\{f_A^+(x), f_B^+(y)\}and(f_A^- \times f_B^-)(x, y) = \max\{f_A^-(x), f_B^-(y)\}$, for all$(x, y) \in I \times J$.

Definisi 2.12.[14]Theproduct of bipolar fuzzy sets $A=(f_A^+,f_A^-)$ and $B=(f_B^+,f_B^-)$ of a non empty set X is defined as follows

$$(AoB)(x)=\left((f_A^+ of_B^+)(x),(f_A^- of_B^-)(x)\right)$$

with

$$(f_A^+ of_B^+)(x)=\begin{cases}\bigcup_{x=yz}\min\{f_A^+(y),f_B^+(z)\}, & \text{if} \quad \exists y,z\in R\ni x=yz\\ 0, & \text{if} \quad \text{otherwise}\end{cases}$$

$$(f_A^- of_B^-)(x)=\begin{cases}\bigcap_{x=yz}\max\{f_A^-(y),f_B^-(z)\}, & \text{if} \quad \exists y,z\in R\ni x=yz\\ 0, & \text{if} \quad \text{otherwise}\end{cases}$$

Definition 2.13.[2]Ifa bipolar fuzzy set $A=(f_A^+,f_A^-)$ of an LA-ring R hold these properties
1. $f_A^+(x-y)\geq\min\{f_A^+(x),f_A^+(y)\}$
2. $f_A^-(x-y)\leq\max\{f_A^-(x),f_A^-(y)\}$
3. $f_A^+(xy)\geq\min\{f_A^+(x),f_A^+(y)\}$
4. $f_A^-(xy)\leq\max\{f_A^-(x),f_A^-(y)\}$

then it is called a bipolar fuzzy LA-subring of R.

Definition 2.14.[2]Ifa bipolar fuzzy set $A=(f_A^+,f_A^-)$ of an LA-ring R hold these properties
1. Bis a bipolar fuzzy LA-subring of R
2. $f_B^+(xy)\geq f_B^+(y)\left(f_B^+(xy)\geq f_B^+(x)\right)$
3. $f_B^-(xy)\leq f_B^-(y)\left(f_B^-(xy)\leq f_B^-(x)\right)$

then it is called a bipolar fuzzy left (right) ideal of R.

Definition 2.15.[2]Ifa bipolar fuzzy set $A=(f_A^+,f_A^-)$ of an LA-ring R hold these properties
1. Bis a bipolar fuzzy LA-subring of R
2. $f_B^+(xy)\geq\max\{f_B^+(x),f_B^+(y)\}$
3. $f_B^-(xy)\leq\min\{f_B^-(x),f_B^-(y)\}$

then it is called a bipolar fuzzy ideal of R.

III. RESULT AND DISCUSSION

In this section, we give the results of this study. We use the notation of R to abbreviate LA-ring.

Lemma 3.1 If $A=(f_A^+,f_A^-),B=(g_B^+,g_B^-),C=(h_C^+,h_C^-)$, and $D=(l_D^+,l_D^-)$ are bipolar fuzzy sets of R then these following properties are hold
1. $(AoB)oC=(CoB)oA$
2. $Ao(BoC)=Bo(AoC)$

Proof:
1. Let x be any element of R. If $x\neq yz$, for all $y,z\in R$, then

$\left((f_A^+ og_B^+)oh_C^+\right)(x)=0=\left((h_C^+ og_B^+)of_A^+\right)(x)$and $\left((f_A^- og_B^-)oh_C^-\right)(x)=0=\left((h_C^- og_B^-)of_A^-\right)(x)$.

If there are $y,z\in R$such that $x=yz$ then

$$\begin{aligned}\left((f_A^+ og_B^+)oh_C^+\right)(x)&=\bigcup_{x=yz}\min\{(f_A^+ og_B^+)(y),h_C^+(z)\}\\&=\bigcup_{x=yz}\min\left\{\left(\bigcup_{y=pq}\min\{f_A^+(p),g_B^+(q)\}\right),h_C^+(z)\right\}\\&=\bigcup_{x=(pq)z}\min\{f_A^+(p),g_B^+(q),h_C^+(z)\}\\&=\bigcup_{x=(zq)p}\min\{h_C^+(z),g_B^+(q),f_A^+(p)\}\\&=\bigcup_{x=wp}\min\left\{\left(\bigcup_{w=zq}\min\{h_C^+(z),g_B^+(q)\}\right),f_A^+(p)\right\}\end{aligned}$$

$$= \bigcup_{x=wp} \min\{(h_C^+ og_B^+)(w), f_A^+(p)\} = \left((h_C^+ og_B^+)of_A^+\right)(x)$$

Hence $\left((f_A^+ og_B^+)oh_C^+\right)(x) = \left((h_C^+ og_B^+)of_A^+\right)(x)$. And we have

$$\left((f_A^- og_B^-)oh_C^-\right)(x) = \bigcap_{x=yz} \max\{(f_A^- og_B^-)(y), h_C^-(z)\}$$

$$= \bigcap_{x=yz} \max\left\{\left(\bigcap_{y=pq} \max\{f_A^-(p), g_B^-(q)\}\right), h_C^-(z)\right\}$$

$$= \bigcap_{x=(pq)z} \max\{\{f_A^-(p), g_B^-(q)\}, h_C^-(z)\}$$

$$= \bigcap_{x=(zq)p} \max\{h_C^-(z), g_B^-(q), f_A^-(p)\}$$

$$= \bigcap_{x=wp} \max\left\{\left(\bigcap_{w=zq} \max\{h_C^-(z), g_B^-(q)\}\right), f_A^-(p)\right\}$$

$$= \bigcap_{x=wp} \max\{(h_C^- og_B^-)(w), f_A^-(p)\} = \left((h_C^+ og_B^-)of_A^-\right)(x)$$

Hence $\left((f_A^- og_B^-)oh_C^-\right)(x) = \left((h_C^- og_B^-)of_A^-\right)(x)$.

If $y \neq pq$, for all $p, q \in R$, then $\left((f_A^+ og_B^+)oh_C^+\right)(x) = 0 = \left((h_C^+ og_B^+)of_A^+\right)(x)$ and $\left((f_A^- og_B^-)oh_C^-\right)(x) = 0 = \left((h_C^- og_B^-)of_A^-\right)(x)$.

Hence for allx $\in$ R, $\left((f_A^+ og_B^+)oh_C^+\right)(x) = \left((h_C^+ og_B^+)of_A^+\right)(x)$ and $\left((f_A^- og_B^-)oh_C^-\right)(x) = \left((h_C^- og_B^-)of_A^-\right)(x)$.Thus $(AoB)oC = (CoB)oA$.

2. It can be proved by the same way of point 1.

Corollary 3.2. LetBF(R) be the set of all bipolar fuzzy sets of R. Based on Lemma 3.1.(BF(R), o)hold the left invertive law, hence (BF(R), o) is an LA-semigroup.

Definition 3.3. Let$x \in$ Rand $t' = (t^+, t^-) \in (0,1] \times [-1,0)$. Bipolar fuzzy point $x_{t'} = (x_{t^+}, x_{t^-})$of R is a bipolar fuzzy set defined as follows

$$x_{t^+}(\mathbf{y}) = \begin{cases} t^+, & \text{if} \quad x = y \\ 0, & \text{if} \quad \text{otherwise} \end{cases} \text{and} x_{t^-}(\mathbf{y}) = \begin{cases} t^-, & \text{if} \quad x = y \\ 0, & \text{if} \quad \text{otherwise} \end{cases}$$

$x_{t'} \in B$if$B(x) \geq t'$ that is if $f_B^+(x) \geq t^+$ and $f_B^-(x) \leq t^-$.

Proposition 3. 4. If $x_{t'}$ and $y_{t'}$ are bipolar fuzzy points of R as defined in Definition 3.3. then

$$x_{t'} o y_{t'} = ((xy)_{t^+}, (xy)_{t^-})$$

Proof: Take any$z \in R$. If there are $x, y \in R$such that $z = xy$, then

$$(x_{t^+} o y_{t^+})(z) = \bigcup_{z=mn} min\{x_{t^+}(m), y_{t^+}(n)\}$$
$$= min\{x_{t^+}(x), y_{t^+}(y)\}$$
$$= min\{t^+, t^+\}$$
$$= t^+$$
$$= (xy)_{t^+}(z).$$

and

$$(x_{t^-} o y_{t^-})(z) = \bigcap_{z=mn} max\{x_{t^-}(m), y_{t^-}(n)\}$$
$$= max\{x_{t^-}(x), y_{t^-}(y)\}$$
$$= max\{t^-, t^-\}$$
$$= t^-$$
$$= (xy)_{t^-}(z).$$

If $z \neq xy$for all $x, y \in R$, then $(x_{t^+} o y_{t^+})(z) = 0 = (xy)_{t^+}(z)and(x_{t^-} o y_{r^-})(z) = 0 = (xy)_{t^-}(z)$.

Hence for all $z \in R$, $(x_{t^+} o y_{r^+})(z) = (xy)_{t^+}(z)$ and $(x_{t^-} o y_{r^-})(z) = (xy)_{t^-}(z)$.

Thus$x_{t'} o y_{r'} = ((xy)_{t^+}, (xy)_{t^-})$.

Lemma 3.5. Let M and N be non empty subsets of R. If $t'A_{\boldsymbol{M}}$ and $t'A_N$ are bipolar fuzzy sets of R as in Definition 2.9. then these following statements are true

1. $t'A_{\boldsymbol{M}}ot'A_N = t'A_{MN}$
2. $t'A_{\boldsymbol{M}} = (\bigcup_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{+}, \bigcap_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{-})$
3. $Rot'A_M = t'A_{RM},\ t'A_M oR = t'A_{MR}$

Proof: Take any $x \in R$

1. If $x \in MN$ then there are $y \in M, z \in N$ such that $x = yz$, hence we have

$$(t^{+}A_M ot^{+}A_N)(x) = \bigcup_{x=yz} min\{t^{+}A_M(y), t^{+}A_N(z)\}$$
$$= min\{t^{+}, t^{+}\} = t^{+}A_{MN}(x)$$

and

$$(t^{-}A_M ot^{-}A_N)(x) = \bigcap_{x=yz} max\{t^{-}A_M(y), t^{-}A_N(z)\}$$
$$= max\{t^{-}, t^{-}\} = t^{-}A_{MN}(x)$$

if$x \notin MN$ then $x \neq yz$for all $y \in M, z \in N$, so we have

$$(t^{+}A_M ot^{+}A_N)(x) = \bigcup_{x=yz} min\{t^{+}A_M(y), t^{+}A_N(z)\}$$
$$= min\{0,0\} = 0 = t^{+}A_{MN}(x)$$

and

$$(t^{-}A_M ot^{-}A_N)(x) = \bigcap_{x=yz} max\{t^{-}A_M(y), t^{-}A_N(z)\}$$
$$= max\{0,0\} = 0 = t^{-}A_{MN}(x)$$

Thus $t'A_{\boldsymbol{M}}ot'A_N = t'A_{MN}$.

2. If $x \in M$then $\bigcup_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{+}(\boldsymbol{x}) = t^{+} = t^{+}A_M(\boldsymbol{x})$and $\bigcap_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{-}(\boldsymbol{x}) = t^{-} = t^{-}A_M(\boldsymbol{x})$**.** If $x \notin M$then $\bigcup_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{+}(\boldsymbol{x}) = 0 = t^{+}A_M(\boldsymbol{x})$and $\bigcap_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{-}(\boldsymbol{x}) = 0 = t^{-}A_M(\boldsymbol{x})$**.** Thus $t'A_{\boldsymbol{M}} = (\bigcup_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{+}, \bigcap_{a\in M} \boldsymbol{a}_{\boldsymbol{t}}^{-})$**.**
3. If $x \in RM$ then there are $y \in R, z \in M$ such that $x = yz$, so we have

$$(R_{\Gamma}^{+}ot^{+}A_M)(x) = \bigcup_{x=yz} min\{R_{\Gamma}^{+}(y), t^{+}A_M(z)\}$$
$$= min\{1, t^{+}\} = t^{+} = t^{+}A_{RM}(x)$$

and

$$(R_{\Gamma}^{-}ot^{-}A_M)(x) = \bigcap_{x=yz} max\{R_{\Gamma}^{-}(y), t^{-}A_M(z)\}$$
$$= max\{-1, t^{-}\} = t^{-} = t^{-}A_{RM}(x)$$

if$x \notin RM$ then $x \neq yz, forally \in R, z \in M$, so we have

$$(R_{\Gamma}^{+}ot^{+}A_M)(x) = \bigcup_{x=yz} min\{R_{\Gamma}^{+}(y), t^{+}A_M(z)\}$$
$$= min\{1,0\} = 0 = t^{+}A_{RM}(x)$$

and

$$(R_{\Gamma}^{-}ot^{-}A_N)(x) = \bigcap_{x=yz} max\{R_{\Gamma}^{-}(y), t^{-}A_M(z)\}$$
$$= max\{-1,0\} = 0 = t^{-}A_{RM}(x)$$

Hence for all $x \in R$, $(Rot'A_M)(x) = t'A_{RM}(x)$. By the same way, we can prove that $t'A_M oR = t'A_{MR}$.

Definition 3.6. Let $A = (f_A^{+}, f_A^{-})$ and $B = (f_B^{+}, f_B^{-})$ be bipolar fuzzy sets of R. $A \subseteq B$if and only if $f_A^{+} \subseteq f_B^{+} and f_A^{-} \supseteq f_B^{-}$that is $f_A^{-}(x) \leq f_B^{-}(x)$and $f_A^{-}(x) \geq f_B^{-}(x)$, for all$x \in R$.

Lemma 3.7. If A is a bipolar fuzzy LA-subring of R then A is a bipolar fuzzy left ideal of R if and only if $R_{\Gamma}^{+}of_A^{+} \subseteq f_A^{+}$and $R_{\Gamma}^{-}of_A^{-} \supseteq f_A^{-}$.

Proof:

($\Longrightarrow$) Since $A$is a bipolar fuzzy left ideal of Rthen we have $f_A^{+}(yz) \geq f_A^{+}(z)$and $f_A^{-}(yz) \leq f_A^{-}(z)$. Take any $x \in R$, if there are y, zsuch that $x = yz$ then

$$(R_\Gamma^+ o f_A^+)(x) = \bigcup_{x=yz} min\{R_\Gamma^+(y), f_A^+(z)\}$$
$$\leq \bigcup_{x=yz} min\{1, f_A^+(yz)\} = \bigcup_{x=yz} min\{1, f_A^+(x)\} = f_A^+(x)$$
$$(R_\Gamma^- o f_A^-)(x) = \bigcap_{x=yz} max\{R_\Gamma^-(y), f_A^-(z)\}$$
$$\geq \bigcap_{x=yz} max\{-1, f_A^-(yz)\} = \bigcap_{x=yz} max\{-1, f_A^-(x)\} = f_A^-(x)$$

If $x \neq yz, forally, x \in R, then (R_\Gamma^+ o f_A^+)(x) = 0 \leq f_A^+(x)$ and $(R_\Gamma^- o f_A^-)(x) = 0 \geq f_A^-(x)$. Hence for all $x \in R, R_\Gamma^+ o f_A^+ \subseteq f_A^+$and $R_\Gamma^- o f_A^- \supseteq f_A^-$.
($\Longleftarrow$)Let $yz = x$ then we have

$$f_A^+(yz) = f_A^+(x) \geq (R_\Gamma^+ o f_A^+)(x) = \bigcup_{x=yz} min\{1, f_A^+(z)\} \geq f_A^+(z)$$
$$f_A^-(yz) = f_A^-(x) \leq (R_\Gamma^- o f_A^-)(x) = \bigcap_{x=yz} max\{-1, f_A^-(z)\} \leq f_A^-(z)$$

Since $f_A^+(yz) \geq f_A^+(z)$ and $f_A^-(yz) \leq f_A^-(z)$, then A is a bipolar fuzzy left ideal of R.

Lemma 3.8. If $A = (f_A^+, f_A^-)$is a bipolar fuzzy left ideal of R with left identity then $RoA = A$.

Proof: Based onLemma 3.7. we have $R_\Gamma^+ o f_A^+ \subseteq f_A^+$and $R_\Gamma^- o f_A^- \supseteq f_A^-$. We will show that $f_A^+ \subseteq R_\Gamma^+ o f_A^+$ and $f_A^- \supseteq R_\Gamma^- o f_A^-$. Since for all $x \in R$, $x = ex$, then we have

$$(R_\Gamma^+ o f_A^+)(x) = \bigcup_{x=yz} min\{R_\Gamma^+(y), f_A^+(z)\} \geq min\{R_\Gamma^+(e), f_A^+(x)\} = f_A^+(x)$$

and

$$(R_\Gamma^- o f_A^-)(x) = \bigcap_{x=yz} min\{R_\Gamma^+(y), f_A^+(z)\} \leq min\{R_\Gamma^-(e), f_A^-(x)\} = f_A^-(x).$$

Thus $f_A^+ \subseteq R_\Gamma^+ o f_A^+$ and $f_A^- \supseteq R_\Gamma^- o f_A^-$. Since $R_\Gamma^+ o f_A^+ \subseteq f_A^+$, $R_\Gamma^- o f_A^- \supseteq f_A^-$, $f_A^+ \subseteq R_\Gamma^+ o f_A^+$, and $f_A^- \supseteq R_\Gamma^- o f_A^-$ then $R_\Gamma^+ o f_A^+ = f_A^+$and $R_\Gamma^- o f_A^- = f_A^-$.

Definition 3.9.A bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ of R is called bipolar fuzzy quasi prime ideal if $t'A_{\boldsymbol{M}} o t'A_N \subseteq B$ implies $t'A_{\boldsymbol{M}} \subseteq B$or $t'A_N \subseteq B$for left ideals M, N of R and for all
$t' = (t^+, t^-) \in (0,1] \times [-1,0)$.

Definition 3.10.A bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ of R is called weakly bipolar fuzzy quasi prime ideal if $0_{t'} \neq t'A_{\boldsymbol{M}} o t'A_N \subseteq B$impliest$'A_{\boldsymbol{M}} \subseteq B$ or $t'A_N \subseteq B$for left ideals M, N of R and for all
$t' = (t^+, t^-) \in (0,1] \times [-1,0)$.

Theorem 3.11. Let$B = (f_B^+, f_B^-)$ be a bipolar fuzzy left ideal of R with left identity. These following statements are equivalent

1. B is a bipolar fuzzy quasi prime ideal of R
2. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $x_{t'} o(Roy_{t'}) \subseteq B$implies $x_{t'} \in B$or $y_{t'} \in B$.
3. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $t'A_{\boldsymbol{x}} o t'A_{\boldsymbol{y}} \subseteq B$implies $x_{t'} \in B$ or
$y_{t'} \in B$.
4. If M, N are left ideals of R such that $t'A_{\boldsymbol{M}} o t'A_{\boldsymbol{N}} \subseteq B$ then $t'A_{\boldsymbol{M}} \subseteq B$or $t'A_{\boldsymbol{N}} \subseteq B$.

Proof:
(**1** $\Longrightarrow$ **2**)Let$B$ be a bipolar fuzzy quasi prime ideal of R. For any $x, y \in R$ and$t' = (t^+, t^-) \in (0,1] \times [-1,0)$if $x_{t'} o(Roy_{t'}) \subseteq B$ then
$t'A_{(xe)R} o t'A_{(ye)R} = (t'A_{(xe)} oR) o (t'A_{(ye)} oR)$(Lemma 3.5.)
$= (t'A_{(xe)} o t'A_{(ye)}) o(RoR)$(Proposition 2.2.)
$= ((t'A_{(x)} o t'A_{(e)}) o (t'A_{(y)} o t'A_{(e)})) o(RoR) = ((t'A_{(x)} o t'A_{(y)}) o (t'A_{(e)} o t'A_{(e)})) o(RoR)$
$= ((t'A_{(e)} o t'A_{(e)}) o (t'A_{(y)} o t'A_{(x)})) o(RoR)$(Proposition 2.3.)

$= \left(t'A_{(ee)}o\left(t'A_{(y)}ot'A_{(x)}\right)\right)o(RoR)$
$= \left(t'A_{(y)}o\left(t'A_{(e)}ot'A_{(x)}\right)\right)o(RoR)$(Lemma 3.1.)
$= \left(t'A_{(y)}ot'A_{(ex)}\right)o(RoR)$
$= (RoR)o\left(t'A_{(x)}ot'A_{(y)}\right)$
$= Ro\left(t'A_{(x)}ot'A_{(y)}\right)$
$= t'A_{(x)}o\left(Rot'A_{(y)}\right)$
$= x_{t'}o\left(Roy_{t'}\right) \subseteq B$
Since B is a bipolar fuzzy quasi prime ideal, then we have
$x_{t'} = t'A_{(x)} = t'A_{(ee)x} = t'A_{(xe)e} \subseteq t'A_{(xe)R} \subseteq B$ or
$y_{t'} = t'A_{(y)} = t'A_{(ee)y} = t'A_{(ye)e} \subseteq t'A_{(ye)R} \subseteq B$. Thus $x_{t'} \in B$or $y_{t'} \in B$.
(**2** $\Longrightarrow$ **3**)Let$x, y \in R$, $t' = (t^+, t^-) \in (0,1] \times [-1,0)$. If $t'A_{(x)}ot'A_{(y)} \subseteq B$ then $x_{t'}o\left(Roy_{t'}\right) = t'A_{(x)}o\left(Rot'A_{(y)}\right) = Ro\left(t'A_{(x)}ot'A_{(y)}\right) \subseteq RoB = B$. Thus, by point 2 we have $x_{t'} \subseteq B$ or $y_{t'} \subseteq B$.
(**3** $\Longrightarrow$ **4**)Assume that $t'A_Mot'A_N \subseteq B$ and $t'A_M \not\subseteq B$ then there is $x \in M$ such that $x_{t'} \notin B$. For any $y \in N$we have $t'A_{(x)}ot'A_{(y)} = t'A_{(xy)} \subseteq t'A_{MN} = t'A_Mot'A_N \subseteq B$. Since $x_{t'} \notin B$then by the hypothesis we have $y_{t'} \in B$. Further, based on Lemma 3.5.we have $t'A_N = \bigcup_{y \in N} y_{t'} \subseteq B$.
(**4** $\Longrightarrow$ **1**)Obviously from Definition3.9.

Theorem 3.12. Let$B = (f_B^+, f_B^-)$ be a bipolar fuzzy left ideal of R with left identity. These following statements are equivalent

1. B is a weakly bipolar fuzzy quasi prime ideal of R
2. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $0_{t'} \neq x_{t'}o\left(Roy_{t'}\right) \subseteq B$implies $x_{t'} \in B$or $y_{t'} \in B$.
3. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $0_{t'} \neq t'A_xot'A_y \subseteq B$implies $x_{t'} \in B$or $y_{t'} \in B$.
4. If M, N are left ideals of R such that $0_{t'} \neq t'A_{\boldsymbol{M}}ot'A_{\boldsymbol{N}} \subseteq B$ then $t'A_{\boldsymbol{M}} \subseteq B$or $t'A_{\boldsymbol{N}} \subseteq B$.

Proof:
(**1** $\Longrightarrow$ **2**)Let$B$ be a bipolar fuzzy quasi prime ideal of R. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$if $0_{t'} \neq x_{t'}o\left(Roy_{t'}\right) \subseteq B$ then
$t'A_{(xe)R}ot'A_{(ye)R} = \left(t'A_{(xe)}oR\right)o\left(t'A_{(ye)}oR\right)$
$= \left(t'A_{(xe)}ot'A_{(ye)}\right)o(RoR)$
$= \left(\left(t'A_{(x)}ot'A_{(e)}\right)o\left(t'A_{(y)}ot'A_{(e)}\right)\right)o(RoR)$
$= \left(\left(t'A_{(x)}ot'A_{(y)}\right)o\left(t'A_{(e)}ot'A_{(e)}\right)\right)o(RoR)$
$= \left(\left(t'A_{(e)}ot'A_{(e)}\right)o\left(t'A_{(y)}ot'A_{(x)}\right)\right)o(RoR)$
$= \left(t'A_{(ee)}o\left(t'A_{(y)}ot'A_{(x)}\right)\right)o(RoR)$
$= \left(t'A_{(y)}o\left(t'A_{(e)}ot'A_{(x)}\right)\right)o(RoR)$
$= \left(t'A_{(y)}ot'A_{(ex)}\right)o(RoR)$
$= (RoR)o\left(t'A_{(x)}ot'A_{(y)}\right)$
$= Ro\left(t'A_{(x)}ot'A_{(y)}\right)$
$= t'A_{(x)}o\left(Rot'A_{(y)}\right)$
$= x_{t'}o\left(Roy_{t'}\right) \subseteq B$
Since B is a bipolar fuzzy quasi prime ideal, then we have
$x_{t'} = t'A_{(x)} = t'A_{(ee)x} = t'A_{(xe)e} \subseteq t'A_{(xe)R} \subseteq B$or
$y_{t'} = t'A_{(y)} = t'A_{(ee)y} = t'A_{(ye)e} \subseteq t'A_{(ye)R} \subseteq B$. Thus $x_{t'} \in B$or $y_{t'} \in B$.
(**2** $\Longrightarrow$ **3**)Let$x, y \in R$, $t' = (t^+, t^-) \in (0,1] \times [-1,0)$. If $0_{t'} \neq t'A_{(x)}ot'A_{(y)} \subseteq B$ then $0_{t'} \neq x_{t'}o\left(Roy_{t'}\right) = t'A_{(x)}o\left(Rot'A_{(y)}\right) = Ro\left(t'A_{(x)}ot'A_{(y)}\right) \subseteq RoB = B$. Thus, by point 2 we have $x_{t'} \subseteq B$or $y_{t'} \subseteq B$.
(**3** $\Longrightarrow$ **4**)Assume that $0_{t'} \neq t'A_Mot'A_N \subseteq B$ and $t'A_M \not\subseteq B$ then there is $x \in M$ such that $x_{t'} \notin B$. For any

$y \in N$ we have $t'A_{(x)}ot'A_{(y)} = t'A_{(xy)} \subseteq t'A_{MN} = t'A_{M}ot'A_{N} \subseteq B$. Since $x_{t'} \notin B$, then by the hypothesis we have $y_{t'} \in B$. Further, based on Lemma 3.5.we have $t'A_{N} = \bigcup_{y \in N} y_{t'} \subseteq B$.
(**4** $\Longrightarrow$ **1**)Obviously from Definition3.10.

Corollary 3.13. Let$B = (f_B^+, f_B^-)$ be a bipolar fuzzy left ideal of R with left identity. These following statements are equivalent
1. $B$is a quasi prime ideal of R.
2. For any $x, y \in R$and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $x_{t'}oy_{t'} \in B$, implies $x_{t'} \in B$or $y_{t'} \in B$.

Proof: Obviously from Theorem3.11. (statement1 $\Longrightarrow$ 3 and 3 $\Longrightarrow$ 1).

Corollary 3.14. Let$B = (f_B^+, f_B^-)$ be a bipolar fuzzy left ideal of R with left identity. These following statements are equivalent
1. $B$is a weakly quasi prime ideal of R.
2. For any $x, y \in R$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ such that $0_{t'} \neq x_{t'}oy_{t'} \in B$, implies $x_{t'} \in B$ or $y_{t'} \in B$.

Proof: Obviously from Theorem3.12. (statement1 $\Longrightarrow$ 3 and 3 $\Longrightarrow$ 1).

Theorem 3.15. LetR_1, R_2 be LA-rings with left identity. A bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ is a bipolar fuzzy quasi prime ideal of R_1 if and only if $B \times R_2$ is a bipolar fuzzy quasi prime ideal of $R_1 \times R_2$.

Proof:
($\Longrightarrow$) Let B be a bipolar fuzzy quasi prime ideal of R_1. Let $(a,b),(c,d) \in R_1 \times R_2 such that$
$(ac,bd)_{t'} = (a,b)_{t'}o(c,d)_{t'} \in B \times R_2$. We will show that $(a,b)_{t'} \in B \times R_2$ or $(c,d)_{t'} \in B \times R_2$. Notice that $(ac,bd)_{t'} \in B \times R_2 \Longrightarrow (f_B^+ \times \Gamma_{R_2}^+)(ac,bd) \geq t^+$and $(f_B^- \times \Gamma_{R_2}^-)(ac,bd) \leq t^-$.
Then we have $f_B^+(ac) = min\{f_B^+(ac),1\} = min\{f_B^+(ac), \Gamma_{R_2}^+(bd)\} = (f_B^+ \times \Gamma_{R_2}^+)(ac,bd) \geq t^+$and
$f_B^-(ac) = max\{f_B^-(ac),-1\} = max\{f_B^-(ac), \Gamma_{R_2}^-(bd)\} = (f_B^- \times \Gamma_{R_2}^-)(ac,bd) \leq t^-$. Hence $f_B^+(ac) \geq t^+$ and $f_B^-(ac) \leq t^-$, thus $(ac)_{t'} = a_{t'}oc_{t'} \in B$.
Since B is a bipolar fuzzy quasi prime ideal then $a_{t'} \in B$or $c_{t'} \in B$. Thus $f_B^+(a) \geq t^+$ and $f_B^-(a) \leq t^-$ or $f_B^+(c) \geq t^+$ and $f_B^-(c) \leq t^-$. Notice that if $f_B^+(a) \geq t^+$ and $f_B^-(a) \leq t^-$ then
$(f_B^+ \times \Gamma_{R_2}^+)(a,b) = min\{f_B^+(a), \Gamma_{R_2}^+(b)\} \geq min\{t^+,1\} = t^+$and
$(f_B^- \times \Gamma_{R_2}^-)(a,b) = max\{f_B^-(a), \Gamma_{R_2}^-(b)\} \leq max\{t^-,-1\} = t^-$. Thus $(f_B^+ \times \Gamma_{R_2}^+)(a,b) \geq t^+$ and $(f_B^- \times \Gamma_{R_2}^-)(a,b) \leq t^-$which implies $(a,b)_{t'} \in B \times R_2$. Or if $f_B^+(c) \geq t^+$ and $f_B^-(c) \leq t^-$ then
$(f_B^+ \times \Gamma_{R_2}^+)(c,d) = min\{f_B^+(c), \Gamma_{R_2}^+(d)\} \geq min\{t^+,1\} = t^+$and
$(f_B^- \times \Gamma_{R_2}^-)(c,d) = max\{f_B^-(c), \Gamma_{R_2}^-(d)\} \leq max\{t^-,-1\} = t^-$. Thus $(f_B^+ \times \Gamma_{R_2}^+)(c,d) \geq t^+$ and $(f_B^- \times \Gamma_{R_2}^-)(c,d) \leq t^-$which implies $(c,d)_{t'} \in B \times R_2$.
Hence if $(a,b)_{t'}o(c,d)_{t'} \in B \times R_2$ then $(a,b)_{t'} \in B \times R_2$or $(c,d)_{t'} \in B \times R_2$, thus from Corollary 3.13. we have $B \times R_2$ is a bipolar fuzzy quasi prime ideal of $R_1 \times R_2$.
($\Longleftarrow$) Let $B \times R_2$ be a bipolar fuzzy quasi prime ideal of $R_1 \times R_2$. Take any $b, d \in R_2$ and $a, c \in R_1 \ni (ac)_{t'} = a_{t'}oc_{t'} \in B$. We will show that $a_{t'} \in B$or $c_{t'} \in B$.
If $(ac)_{t'} \in B$ then $t^+ \leq f_B^+(ac)$and $t^- \geq f_B^-(ac)$. Notice that

$$t^+ \leq f_B^+(ac) = min\{f_B^+(ac),1\} = min\{f_B^+(ac), \Gamma_{R_2}^+(bd)\} = (f_B^+ \times \Gamma_{R_2}^+)(ac,bd).$$

Hence $(f_B^+ \times \Gamma_{R_2}^+)(ac,bd) \geq t^+$, and

$$t^- \geq f_B^-(ac) = max\{f_B^-(ac),-1\} = max\{f_B^-(ac), \Gamma_{R_2}^-(bd)\} = (f_B^- \times \Gamma_{R_2}^-)(ac,bd).$$

Hence $(f_B^- \times \Gamma_{R_2}^-)(ac,bd) \leq t^-$.
Since $(f_B^+ \times \Gamma_{R_2}^+)(ac,bd) \geq t^+$ and $(f_B^- \times \Gamma_{R_2}^-)(ac,bd) \leq t^-$ then $(ac,bd)_{t'} = (a,b)_{t'}o(c,d)_{t'} \in B \times R_2$. Since $B \times R_2$ is bipolar fuzzy quasi prime ideal of $R_1 \times R_2$ then $(a,b)_{t'} \in B \times R_2$or $(c,d)_{t'} \in B \times R_2$. Hence we have $(f_B^+ \times \Gamma_{R_2}^+)(a,b) \geq t^+$ and $(f_B^- \times \Gamma_{R_2}^-)(a,b) \leq t^-$ or $(f_B^- \times \Gamma_{R_2}^-)(c,d) \geq t^+$ and $(f_B^- \times \Gamma_{R_2}^-)(c,d) \leq t^-$.
Notice that if $(a,b)_{t'} \in B \times R_2$ then we have
$f_B^+(a) = min\{f_B^+(a),1\} = min\{f_B^+(a), \Gamma_{R_2}^+(b)\} = (f_B^+ \times \Gamma_{R_2}^+)(a,b) \geq t^+$and
$f_B^-(a) = max\{f_B^-(a),-1\} = max\{f_B^-(a), \Gamma_{R_2}^-(b)\} = (f_B^- \times \Gamma_{R_2}^-)(a,b) \leq t^-$.
Since $f_B^+(a) \geq t^+$and $f_B^-(a) \leq t^-$, then $a_{t'} \in B$.

Or if $(c,d)_{t'} \in B \times R_2$ then $f_B^+(c) = min\{f_B^+(c), 1\} = min\{f_B^+(c), \Gamma_{R_2}^+(d)\} = (f_B^+ \times \Gamma_{R_2}^+)(c,d) \geq t^+$and $f_B^-(c) = max\{f_B^-(c), -1\} = max\{f_B^-(c), \Gamma_{R_2}^-(d)\} = (f_B^- \times \Gamma_{R_2}^-)(c,d) \leq t^-$. Since $f_B^+(c) \geq t^+$ and $f_B^-(c) \leq t^-$ then $c_{t'} \in B$.
Since if $a_{t'} o c_{t'} \in B$ implies $a_{t'} \in B$or $c_{t'} \in B, then B$ is a bipolar fuzzy quasi prime ideal of R_1.

Corollary 3.16. Let $R_1$and R_2 be LA-rings with left identity. Bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ is a bipolar fuzzy quasi prime ideal of R_2 if and only if$R_1 \times B$ is a bipolar fuzzy quasi prime ideal of $R_1 \times R_2$.

Proof: It can be proved analog to Theorem 3.15.

Theorem 3.17. Let R be an LA-$r$ing with left identity. If a bipolar fuzzy left ideal $B = (f_B^+, f_B^-)$ of R is a bipolar fuzzy quasi prime ideal then $max\{f_B^+(x), f_B^+(y)\} = f_B^+(xy)$and $min\{f_B^-(x), f_B^-(y)\} = f_B^-(xy)$ for all $x, y \in R$.

Proof: If B is a bipolar fuzzy ideal of R then we have $f_B^+(xy) \geq max\{f_B^+(x), f_B^+(y)\}$ and $f_B^-(xy) \leq min\{f_B^-(x), f_B^-(y)\}$. Assume that $f_B^+(xy) > max\{f_B^+(x), f_B^+(y)\}$ and $f_B^-(xy) < min\{f_B^-(x), f_B^-(y)\}$ then there is $(t^+, t^-) \in (0,1] \times [-1,0)$ such that $f_B^+(xy) > t^+ > max\{f_B^+(x), f_B^+(y)\}andf_B^-(xy) < t^- < min\{f_B^-(x), f_B^-(y)\}$.
Hence for all $x, y \in R x_{t'} o(R o y_{t'}) = R o(x_{t'} o y_{t'}) = R o(xy_{t'}) \in R o B \subseteq B$ (3.17.1)
Since B is a bipolar fuzzy quasi prime ideal then it must implies $x_{t'} \in B$or $y_{t'} \in B$. But if $x_{t'} \in B$ or $y_{t'} \in B$ then we have $f_B^+(x) \geq t^+$ and $f_B^-(x) \leq t^-$ or $f_B^+(y) \geq t^+$ and $f_B^-(y) \leq t^-$ which implies $max\{f_B^+(x), f_B^+(y)\} \geq t^+$and $min\{f_B^-(x), f_B^-(y)\} \leq t^-$. This is contradiction with 3.17.1, hence it must be $max\{f_B^+(x), f_B^+(y)\} = f_B^+(xy)$and $min\{f_B^-(x), f_B^-(y)\} = f_B^-(xy)$ for all $x, y \in R$.

Theorem 3.18. Let R be an LA-$r$ing with left identity and B is a bipolar fuzzy ideal of R that hold these properties

1. If $max\{f_B^+(x), f_B^+(y)\} = f_B^+(x)$ then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(x)$, or
2. If $max\{f_B^+(x), f_B^+(y)\} = f_B^+(y)$ then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(y)$

$B$is a bipolar fuzzy quasi prime ideal if and only if $max\{f_B^+(x), f_B^+(y)\} = f_B^+(xy)$and $min\{f_B^-(x), f_B^-(y)\} = f_B^-(xy)$ for all $x, y \in R$.

Proof: ($\Longrightarrow$)It is proved inTheorem3.17.
($\Longleftarrow$) take any $x_{t'}, y_{t'}$ such that $x_{t'} o(R o y_{t'}) \subseteq B$ then $R o(x_{t'} o y_{t'}) = R o(xy_{t'}) \subseteq B$ thus $B(xy) \geq t'$ that is $f_B^+(xy) \geq t^+$ and $f_B^-(xy) \leq t^-$. Since $max\{f_B^+(x), f_B^+(y)\} = f_B^+(xy)$and $min\{f_B^-(x), f_B^-(y)\} = f_B^-(xy)$ then $max\{f_B^+(x), f_B^+(y)\} \geq t^+$and $min\{f_B^-(x), f_B^-(y)\} \leq t^-$. Hence $f_B^+(x) \geq t^+$ or $f_B^+(y) \geq t^+$and $f_B^-(x) \leq t^-$ or $f_B^-(y) \leq t^-$. If $max\{f_B^+(x), f_B^+(y)\} = f_B^+(x) \geq t^+$ then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(x) \leq t^-$ hence $f_B^+(x) \geq t^+$ and $f_B^-(x) \leq t^-$ which implies $x_{t'} \in B$. Or if $max\{f_B^+(x), f_B^+(y)\} = f_B^+(y) \geq t^+$then $min\{f_B^-(x), f_B^-(y)\} = f_B^-(y) \leq t^-$ hence $f_B^+(y) \geq t^+$ and $f_B^-(y) \leq t^-$ which implies $y_{t'} \in B$. Thus $x_{t'}$ o (R o $y_{t'}$) $\subseteq$ B implies $x_{t'} \in$ Bor $y_{t'} \in$ B. Hence based on Corollary 3.13.we have,B is a bipolar fuzzy quasi prime ideal of R.

IV. CONCLUSION

In this paper we give the definitions andproperties of bipolar fuzzy quasi prime ideals and weakly bipolar fuzzy quasi prime ideals of LA-rings. We find that a bipolar fuzzy left ideal of $R_1$is quasi prime ideal if its product with R_2 is a bipolar fuzzy quasi prime ideal of $R_1 \times R_2$. In the study of [16] stated that a practical way to prove that a fuzzy left ideal is a fuzzy quasi prime ideal is by observe their membership values. This way can be used in bipolar fuzzy if the bipolar fuzzy left ideal hold the properties mentioned in the last theorem. Further study can be conducted to identify that property and to obtain others properties of bipolar fuzzy quasi prime ideals in LA-rings or in the others algebraic structures.

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