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Identification of the thermo-physical parameters of an anisotropic material by inverse problem

H. Arjdal[a], J. Chaoufi[a], J.C. Dupre[b], A. Germaneau[b], S. Ouhni[b]
a. Laboratory of electronics, Signal Processing and Physical Modeling (LESPPM,) University Ibn Zohr, Agadir Morocco.
b. University of Poitiers, Institute P ', UPR 3346 CNRS, France.
Corresponding Author : H. Arjdal

ABSTRACT: The purpose of this work is to determine the thermo-physical parameters of an anisotropic material. The method consists in looking for these parameters from the knowledge of the temperature field. The resolution of the problem is based on the finite element method. The direct problem has yielded convincing results. The latter thus found are in agreement with the experimental results. Subsequently, we approach the opposite problem without apprehension by proposing an optimization method based on the conjugate gradient algorithm.

KEYWORDS: Identification; Thermophysical parameters; Inverse problem; Finite Element Method; Infrared Camera; Infrared Thermography; Projected Conjugate Gradient Method

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I. INTRODUCTION

The good knowledge of the thermophysical properties of the materials, made it possible to predict thermal phenomena. In the field of mechanical and design knowledge, the thermal properties of materials are needed for more realistic modeling. It is also crucial for the design of photovoltaic cells.

The exploitation of experimental temperature fields constitutes the basis of the approach framing the present study. A rectangular plate is heated [1, 2] on one side and the temperature field is captured by an infrared camera. The Inverse Problem is solved by crossing back the equations obtained by the Finite Element Method for solving the Direct Problem. Doing so, we elaborate efficient algorithms able to accurately identify the thermal parameters of polymethylmethacrylate.

II. DIRECT PROBLEM

2.1 Position of the problem

In our endeavour to identify the thermal characteristics of a material we will undertake a procedure that is both experimental and numerical. We consider a rectangular thin solid plate of length L, width h and thickness e. The plate is homogeneous; while its thickness is small, its length is very close to its width. We will assume that the temperature distribution is two-dimensional.

2.2 Problem formulation

In order to extend the description to strongly anisotropic materials where the conductivity matrix is solid, an earlier study was carried out for the diagonal conductivity matrix [10]. The problem to be solved is identical to the previous one except that the number of parameters to be determined is greater.

This part is devoted to solving a problem of heat transfer [5]. Let us consider that a rectangular homogeneous solid plate, with length l, width h and of very small thickness e in front of its other dimensions. Suppose then that the plate occupies the interval $[0, \mathrm{L}]$ of the, Ox axis, $[0, \mathrm{h}]$ of the, Oy axis and that at time t = 0, the distribution of the temperature is known to all M (x, y) of the field, and equal to, $T(x, y, t)$. A constant heat flux φ_1 is imposed on the side bounded by $x = 0$ (denoted Γ_1), and a constant φ_2 is imposed on the side bounded by $y = 0$ (denoted Γ_2). The other sides (denoted Γ_3 and Γ_4) are well protected against any convective, radiative or conductive currents [2, 4].

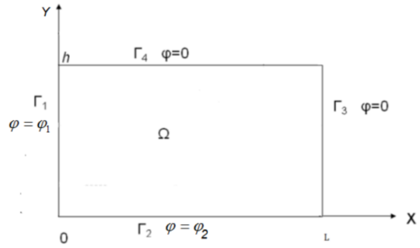

FIG. 1 – Rectangular plate and boundary conditions

Let ρc be the heat capacity per unit volume (ρ being the specific mass and c the heat capacity per unit mass). Let $\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12} & \lambda_{22} \end{bmatrix}$ be the thermal conductivity tensor.

The direct problem for calculating the temperature $T(x, y, t)$ is therefore defined by the following Partial Differential Equation (PDE) system:

$$\rho c \frac{\partial T}{\partial t} = div(\overline{\overline{\lambda}}.\overrightarrow{gradT}) \quad in \;\; \Omega \qquad (1)$$

The above boundary and initial conditions read as follows:

$$-\left(\overline{\overline{\lambda}} \cdot \overrightarrow{gradT}\right) \cdot \vec{n} = \left(\lambda_{11} \frac{\partial T}{\partial x} + \lambda_{12} \frac{\partial T}{\partial y}\right)_{x=0} = \varphi_s \text{ on } \Gamma_1 \qquad (2a)$$

$$-\left(\overline{\overline{\lambda}} \cdot \overrightarrow{gradT}\right) \cdot \vec{n} = \left(\lambda_{12} \frac{\partial T}{\partial x} + \lambda_{22} \frac{\partial T}{\partial y}\right)_{y=0} = \varphi_2 \qquad on\; \Gamma_2 \qquad (2b)$$

$$\lambda_{11} \frac{\partial T}{\partial x} + \lambda_{12} \frac{\partial T}{\partial y} = 0 \qquad on\, \Gamma_3 \qquad (2c)$$

$$\lambda_{12} \frac{\partial T}{\partial x} + \lambda_{22} \frac{\partial T}{\partial y} = 0 \qquad on\;\; \Gamma_4 \qquad (2d)$$

$$T(x, y, 0) = T_0(x, y) \quad (2e)$$

Equations (1), (2a), (2b), (2c), (2d) and (2e) defining the direct problem can be solved numerically by the finite element method.

2.3 Solving the direct problem by the finite element method

With the finite element approach, we will develop a calculation code with quantitative information on the thermo-physical properties of the materials, the boundary conditions and heat fluxes applied, as well as the characteristics of the chosen discretization (type of element, mesh size).

we proceed to the discretization of the domain Ω in sub domains Ωe called elements. The geometry of these elements is quadrangular element with linear interpolation functions $N_i^e(x, y)$. The total number of nodes is N, while the total number of elements is Nt.

2.3.1 Local representation

In this study, the approximate solution is the temperature function T(x,y,t) with the form:

$$T^e(x, y, t) = \sum_{i=1}^{4} T_i^e(t)\, N_i^e(x, y)$$

As interpolation functions: $N_i^e(x_j, y_j) = \delta_{ij} = \begin{cases} 1 & if\;\; i = j \\ 0 & else \end{cases}$

For each element domain e, After simplification of the calculations, we get:

$$\left[C^e\right]\left\{\frac{dT}{dt}\right\} + \left[K^e\right]\{T\} = \left\{F^e\right\} \qquad (4)$$

where $\lfloor K^e \rfloor$, $\lfloor C^e \rfloor$ and $\{F^e\}$ are respectively conductance matrix, the capacitance matrix and the heat load vector per element :

$$\left[K^e\right] = \int_{\Omega_e} (\overline{grad}\, N^e)\, \overline{\overline{\lambda}}\, (\overline{grad}\, N^e)^t\, dxdy$$
$$\left[C^e\right] = \rho c \int_{\Omega_e} (N^e)^t (N^e)\, dxdy \qquad (5)$$
$$\left\{F^e\right\} = \int_{\partial\Omega_e} [N^e]^t\, \Phi\, d\Gamma$$

2.3.2 Global representation and assembly

We deploy w to refer to the cross sectional area and [J] to stand for the Jacobian of the geometric transformation by adopting an isoparmetric element. Integrating the Equations (5).
The assemblage system equation takes the matrix form

$$[C]\left\{\frac{dT}{dt}\right\} + [K]\{T\} = \{F\} \qquad (6)$$

where the conductance matrix, the capacitance matrix and the heat load vector are respectively :

$$[K] = \sum_{e=1}^{n^e} \left[A^e\right]^t \left[K^e\right]\left[A^e\right],\ [C] = \sum_{e=1}^{n^e} \left[A^e\right]^t \left[C^e\right]\left[A^e\right]\ and\ \{F\} = \sum_{e=1}^{n^e} \left[A^e\right]^t \left\{F^e\right\}$$

In solving a problem of transient conduction, we are guided into solving a system of first order differential equation with respect to time t (Equation (6)) for which the initial condition is: $\{T_0\}^T = \{T_1(0) \quad T_2(0) \quad \dots \quad T_N(0)\}$
The determination of the temperature field in the material returns to find the temperature values at the nodes over time. The numerical solution of the previous system determines the evolution of the temperature in the material for thermo-physical parameters imposed.

2.4 Experimental device

The goal is to create a temperature field, we will have to heat the sample studied to identify variations in temperature. A heater is confined between two plates, and the electrical power is supplied by a generator. Under these conditions, it is assumed that the imposed flow is divided equally between the two plates. The heater is controlled by a voltmeter and an ammeter. It requires very low current intensities. To access the temperature fields of the material, we use an infrared thermography. Finally, a video monitor connected to it can follow the evolution of the thermal mapping

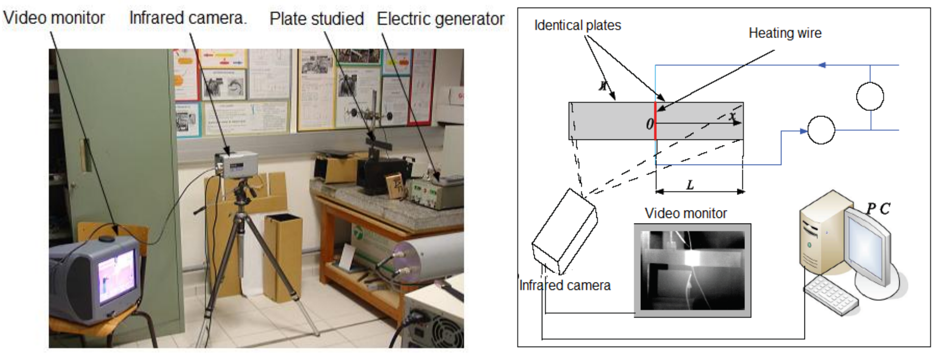

FIG. 2 – Fitting the experimental measurement of the temperature.

Figure 3 below illustates the evolution of experimental or simulated temperatures of the plate, with respect to time

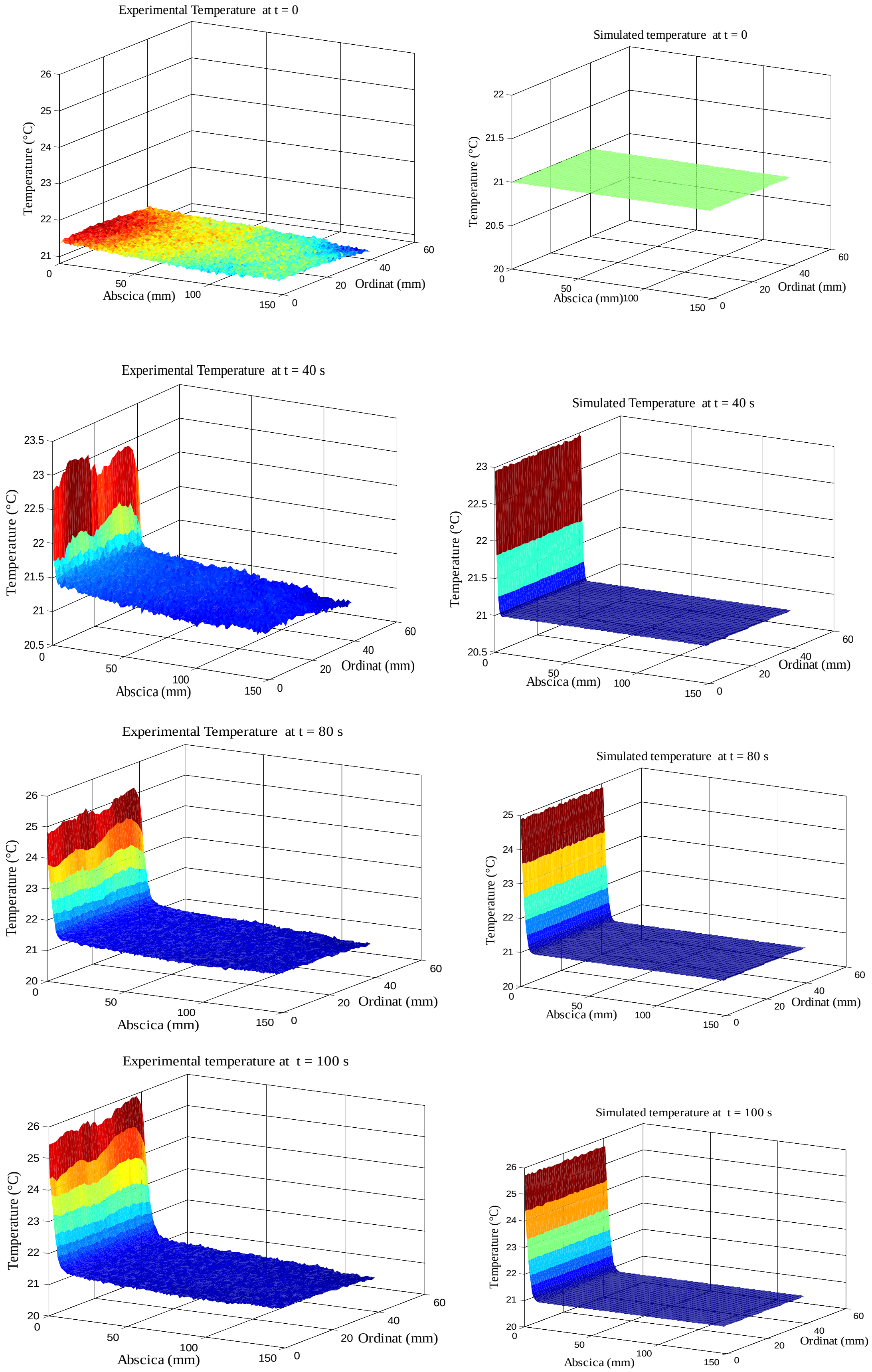

FIG. 3 Temperature distribution of the plate with respect to time

Note that the progression of both temperatures, i.e., numerically calculated and experimental, are very close. This comparison also validates the direct approach developed.
Next, in the inverse problem, and to undertake the experimental conditions, a noise with standard deviation $\sigma = 0.02°C$ is imposed on the temperature according to the precision current infrared cameras

III. INVERSE PROBLEM

The experiments were conducted at the Institute P' of the University of Poitiers From the measured temperature fields, we try to identify the thermal conductivity tensor $\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12} & \lambda_{22} \end{bmatrix}$ and the specific heat ρc of an anisotropic material. Once the temperature function T(x, y, t) is recorded in a series of points on the surface of the plate, at several times, we apply the least squares method for estimating the thermophysical parameters. We made m experiments indexed from $i = 1\, to\, m$ .The time duration of the i^{th} experiment is referred to by t_i . The least squares method brings about the constrained optimization process: Minimize the objective function

$$J(\rho c, \lambda) = \frac{1}{2}\sum_{i=1}^{m}\int_0^{t_i}\left\| [C]\left\{\frac{dT^i}{dt}\right\} + \sum_e \left[B^e A^e\right]\lambda\left[B^e A^e\right]^t \left\{T^i\right\} - \left\{F^i\right\} \right\|^2 dt \quad (7)$$

under the positive constraint for ρc , and the symmetric positive-definite constraints for λ .
To minimize this function by a steepest descent method, we need to express its gradient with respect to the conductivity tensor λ :
To minimize this function, we calculate for the conductivity tensor

$$J\left(\bar{\bar{\lambda}} + \delta\bar{\bar{\lambda}}, \rho c\right) - J\left(\bar{\bar{\lambda}}, \rho c\right) = \int_0^{t_f}\left\langle \delta\bar{\bar{\lambda}}, \sum_e \left[B^e A^e\right] U \{T\}^t \left[B^e A^e\right]^t \right\rangle dt$$

with

$$U^i = [C]\left\{\frac{dT^i}{dt}\right\} + [K]\left\{T^i\right\} - \left\{F^i\right\}.$$

Since $\delta\bar{\bar{\lambda}}$ is symmetric, the gradient R of the of the cost function $J\left(\bar{\bar{\lambda}}, \rho c\right)$ versus $\bar{\bar{\lambda}}$ is equal to the matrix zeros implies:

$$R = \int_0^{t_f} \sum_e \left[B^e A^e\right]\left(U\{T\}^t + \{T\}U^t\right)\left[B^e A^e\right]^t dt = 0$$

We will also need its derivative $r = \dfrac{\partial J}{\partial(\rho c)}$.

3.1. Identification algorithm

The conjugate gradient algorithm is introduced in detail and applied in thermophysical parameters identification. As the steepest descent method to minimize the function $J(\rho c, \lambda)$, we implement the projected conjugate gradient method, which consists in constructing iteratively a sequence converging to the minimum.
The algorithm of this method can be summarized as follows

1. Initialize λ by λ_0 and ρc by $(\rho c)_0$,

Deduce the initial values r_0 and R_0 of r and R ,
Initialize a sequence of scalars d_i by $d_0 = -r_0$ and a sequence of directions D_i by $D_0 = -R_0$,

2. At iteration i

calculate μ_i and ν_i which minimize $J((\rho c)_i + \mu d_i, \lambda_i + \nu D_i)$ with respect to μ and ν

$(\rho c)_{i+1} = (\rho c)_i + \mu_i d_i$

$\lambda_{i+1} = \lambda_i + \nu_i D_i$

if $\lambda_{i+1} \leq 0$, $\lambda_{i+1} = proj\lambda_{i+1}$

3. if $\|r_{i+1}\| < \varepsilon$ and $\|R_{i+1}\| < \varepsilon$ stop, otherwise

$d_{i+1} = -r_{i+1}$

$\beta_i = \dfrac{(R_{i+1} - R_i)^t R_i}{R_i^t R_i}$

$D_{i+1} = -R_{i+1} + \beta_i D_i$

$i = i + 1$ and return to step 2.

The abbreviation (Proj) is a subroutine that executes the projection algorithm of Higham [8]. Indeed, the projection of a not positive symmetric matrix takes place orthogonally to the edge of the positive cone matrix.

3.2. Identification Results

3.2.1. Simulations for anisotropic materials

The Projected Conjugate Gradient method developed in the last section is applied to the simulated temperature fields obtained by solving equation (6) by FEM. The material is supposed to be anisotropic. The results from our identification algorithm are shown in the table below.

Parameters	**Values used in the simulation**	**identified Values**	
λ_{11}(W/m/°C)	0.45	0.4309	±0.0170
λ_{12}(W/m/°C)	0.2	0.2134	±0.0121
λ_{22}(W/m/°C)	0.17	0.1638	±0.0140
ρc(J/m3/°C)	1.666 .10^6	1.6573 .10^6	±0.027 .10^6

Table 1: Identified Values for an anisotropic material from simulated temperature fields

3.2.2. Experiments for isotropic materials

The experimental device was applied to an isotropic polymer (polymethylmethacrylate, PMMA), with thermophysical parameters [1]:

$\rho = 1190\, Kg/m^3$

$c = 1400\, J/Kg/°C$

$\lambda = 0{,}17\, W/m/°C$

The results from our identification algorithm are shown in the table below.

Parameters	λ_{11}(W/m/°C)	ρc(J/m^3/°C)
Manufacturer values [1]	0.17	1.666. 10^6
Values identified	0.1786	1.6797 .10^6

Table 2: Identified Values for PMMA from experimental temperature fields

IV. CONCLUSION

The finite element method meets the requirements imposed by the sample geometry and the boundary conditions. Its application on a homogeneous anisotropic material enabled us to transform the Fourier's heat conduction equation in a first order ordinary differential equation. Therefore, the resolution of the direct problem needs solely a time integration algorithm. The developed algorithm allows us to simulate the temperature field in the bidimensional case. The accuracy of the simulations ensured the validity of our approach. Moreover, our code proved to be fast handling both for varied geometric dimensions and for varied boundary and initial conditions.

The identification algorithm is based on the Projected Conjugate Gradient method. It allows characterizing the thermal conductivity tensor and the specific heat of polymers. The identification results are demonstrated to be in good agreement with the manufacturer values.

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