American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-8, Issue-10, pp-230-236
www.ajer.org
Research Paper Open Access

Solution Checks By Mohr's Curvature System in Uni-axial Plate Buckling

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ABSTRACT *:. The widely used "XX-SC,YY-CC" and "XX-SC,YY-SC" plates compressed on "X-X" which have no direct recorded solutions are investigated ,among others, and the results found are accurately justified. For a solution, the relative-curvature/deflection ratio in the deformation must be a scalar. The X- and Y-curvatures when applied to the Mohr's diagram lead to a first and second loading-curvature-circles in axial compression, "$\mathcal{X}_x$" and "$(\mathcal{X}_x+\mathcal{X}_y)/2$" ; the least buckling load is sought . When "$\mathcal{X}_x/\mathcal{X}_y < \mu$", uniqueness of uni-axial X-compression is lost . A simpler solution of all rectangular plate buckling in axial compression is sought within little percentage discrepancy of known literature values ;this was achieved .Some designers desire independent check of results; this present method will come in useful.*
Keywords: *rectangular plates; buckling ; uni-axial compression; Mohr's loading-curvature-circles*

Date of Submission: 20-10-2019 Date of acceptance: 03-11-2019

I. INTRODUCTION

The buckling of axially compressed rectangular plates have been in examination for over eight decades; Bryan, 1891[1] demonstrated the relation $\boldsymbol{\sigma}_{cr} = k(Et^2/b^2)/(12(1-\mu^2))$ as critically relevant for rectangular plates in axial compression .Studies in plates are readily justified by their extensive deployment in assembly of vessels-ships, aircraft, cans, cylinders. Detailed experimental support are widely available, Richard Pride and George Heimeri,1949[2], Nishino et-al, Lehigh Preserve,1966[3] . A striking aspect of the latter publication is that theoretical prediction of strength exceeded experiment by 30 to 50 percent in simply supported plates. In many instances the "ssss" plate is tested by compressing rectangular tubes whose sides behave as simply-supported rectangular plates,Schfer,2009[4]; also [2]. In most of this period reliable reference solutions for plates in which axially compressed opposite ends have different boundary conditions were hardly found; they are not passed down in contemporary texts and research papers. Perhaps, they are not unique but there is scant discussion. The graphs of many classical buckling solutions will be found "https//ocw.mit.edu/courses/2013,[5]" ,also in Pope[6] and David Rees[7] .

An immediate observation is that the one-axis compression buckling equation, Eq.1, is bi-axial on the reaction side but uni-axial on the load side; this is unlike the bending problem in which any transverse-load, 'q', which replaces $(N_x.\partial^2 w/ \partial x^2)$ is also bi-axial in X,Y. This will pose solution problems in some cases**.**

In alternate solutions [5,6], and in order to encourage lower-bound results and also to bring theory and experiments closer, the unloaded boundaries of plates in axial compression are ,sometimes, forced to remain straight and in that way bring into play Poisson's effect contributing additional axial trust in the unloaded Y-direction . Additionally, elasto-plastic theory of plate buckling also leads to lower and safer values of the buckling load, at the cost of closed-form solutions, [2,8] .

The present study adopts the relative-curvature/deflection resonance buckling criterion, Johnarry[9] where for a solution, the relative-curvature/deflection ratio must be a scalar .Buckling deflection functions must meet this requirement a-priori . The X- and Y- curvatures when applied to the Mohr's diagram leads to a first and second loading circles under axial compression , "$\mathcal{X}_x$" and "$(\mathcal{X}_x+\mathcal{X}_y)/2$" ; the minimum buckling load is sought .

Nomenclature

a,b rectangular plate dimensions in X,Y

$\boldsymbol{\delta}$b change in width of plate …Poisson's effect

s* aspect ratio ,a/b
E Young's modulus of elasticity
$\rho = D\pi^2/b^2$
t thickness of plate
D flexural rigidity of plate, (isotropic); $D=Et^3/12(1-\mu)^2$.
μ poisson's ratio
w deflection symbol
$w_{,xx-r}$: relative curvature in X-direction
$w_{,yy-r}$: relative curvature in Y-direction
$w_{,xx-r}/w$: relative-curvature /deflection ratio ;must be a scalar for any solution
XX-SC,YY-CC ; plate simply and clamped on X-X; and clamped-clamped on Y-Y
r_{cap} . capacity ratio of axes as, $(\partial^4 w/\partial x^4)/(\partial^4 w/\partial y^4)$
H $= (\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4) = H_{xx} + H_{xy} + H_{yy}$.
$\mathcal{X}$ curvature
σ stress symbol

II. APPLICABLE EQUATIONS

The existing uni-axial buckling equation, Eq.1, is firs t stated. Fig.1 shows plates, axes and typical boundary conditions in some cases; The uni-axial analysis equation,

$(\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4) = (N_x/D)^2 w/\partial x^2$ ….. (1)

$D\{\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4\} = [N_x\,\partial^2 w/\partial x^2 + N_y\,\partial^2 w/\partial y^2]$ (2)

is generally axially unbalanced ; it is bi-axial on the reactive "LHS" and uni-axial on the load , "RHS" side . So the load does not apply uniformly over the plate ; Eq.2 is a balanced "RHS" and should be investigated first for limiting values of Eq.1.

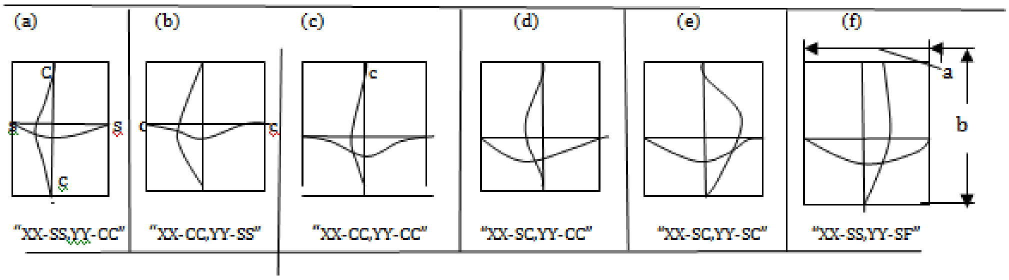

Fig.1 Various plates, boundary conditions and buckling-modes

Also the shear loading equation, Eq.3 is balanced .

$\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4 = (N_{xy}/D)(2\partial^2 w/\partial x\partial y)$ (3)

Eq.1 is summarized as ,

$H_{xx} + H_{xy} + H_{yy} = H = (N_x/D)\,\partial^2 w/\partial x^2$. (4)

Everything is known and the solution is rapidly concluded.

For solutions , all differentials are given finite values; for example,

$$\frac{\partial^4 w/(\partial x4)}{(1)} = \frac{\int\int w(\partial^4 w/\partial x^4)\partial x\partial y}{\iint w\partial x\partial y} = H_{xx}. \quad (5)$$

$$\frac{(\partial^4 w)/(\partial y^4)}{(1)} = \frac{\int\int w(\partial^4 w/\partial y^4)\partial x\partial y}{\iint w\partial x\partial y} = H_{yy}. \quad (6)$$

$$\frac{(2\partial^4 w)/(\partial x^2\partial y^2)}{(1)} = \frac{\int\int 2w(\partial^4 w/\partial x^2\partial y^2)\partial x\partial y}{\iint w\partial x\partial y} = H_{xy}. \quad (7)$$

$$\frac{(\partial^2 w)/(\partial x^2)}{(1)} = \frac{\int\int w(\partial^2 w/\partial x^2)\partial x\partial y}{\iint w\partial x\partial y} = \mathcal{X}_x \quad (8)$$

$$\frac{(\partial^2 w)/(\partial y^2)}{(1)} = \frac{\int\int w(\partial^2 w/\partial y^2)\partial x\partial y}{\iint w\partial x\partial y} = \mathcal{X}_y \quad (9)$$

$\mathcal{X}_{1,2} = (\mathcal{X}_x + \mathcal{X}_y)/2 \pm \sqrt{[\{(\mathcal{X}_x - \mathcal{X}_y)^2/2\}^2 + \{(\mathcal{X}_{xy})/2\}^2]}$;principal curvatures; (10a)

Or by reference to the Mohr's circle,

$\mathcal{X}_{1,2} = (\mathcal{X}_x + \mathcal{X}_y)/2 \pm R$; R= Mohr's circle radius. (10b)

These integrals are the outcomes of criterion of buckling as relative-curvature/deflection resonance. A typical buckling resistance integral is ,

$$\frac{(\partial^4 w)/(\partial x^4)}{(1)} = C_{xd4}\,(w,_{xx\text{-}r}/w) = C_{xd4}\,(R_{xcd}) \quad (11)$$

The ratio, "($w,_{xx\text{-}r}$ /w) = R_{xcd}." must always be a scalar constant or else the function is inadmissible ; indeed this is the inbuilt buckling criteria . Buckling deflection function ,w, must be chosen as to make the ratio, ($w,_{xx\text{-}r}$/w), a scalar. The domain compliant factor at resonance, C_{xd4}, is what is left to be found. Multiply both sides of Eq.11 and integrate to find it.

$$C_{xd4}.R_{xcd} = [\,\frac{(\partial^4 w/\partial x^4)}{1}\,] = [\,\frac{\int\int w(\partial^4 w/\partial x^4)\partial x\partial y}{\iint w\partial x\partial y}\,] \quad (12)$$

2.1 Buckling Potential Limits

Three possibilities are identified relative to X- and Y-axes in emulation with the reactive potentials" ∂^4w/ ∂x^4 ;2∂^4w/ $\partial x^2\partial y^2$; ∂^4w/ ∂y^4)" . No curvature "$\mathcal{X}_i$" is ever applied in practice but "$\mathcal{X}_x$" and "$\mathcal{X}_y$" act jointly .

(i) $\boldsymbol{\sigma}_x\,\mathcal{X}_x$.

This is first in contention in uni-axial X-compression; this case easily solves Eq.1 .

(ii) $\boldsymbol{\sigma}_y\,\mathcal{X}_y$.

This is out of contention when no load is applied in the Y-axis, whatever the value of "$\mathcal{X}_y$."

(iii) ($\boldsymbol{\sigma}_x\,\mathcal{X}_{av}$.)

This "average loading-curvature" situation will always happen and also in contention. (i) and (iii) are identified in the Mohr's diagram ,Fig.2

In effect, two curvature-loading circles ($\mathcal{X}_x$, $\mathcal{X}_{av}$) are operative and the larger circle gives the required solution for "N_x". This process softens the stiff constraint that the wave numbers ",m ,n", must be whole numbers.

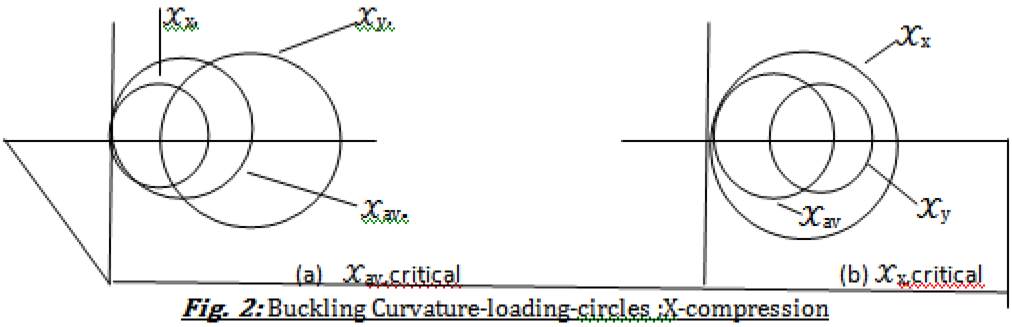

Fig. 2: Buckling Curvature-loading-circles ;X-compression

3.1a; Square-plate: xx-ss, yy-cc ,X-thrust ;Fig.1a

- check bi-axial solution ; "$N_{biaxial}$", for clue …Eq.1-10

Deflection Function and Solution

w= (Sin m$\boldsymbol{\pi}$x/a) (Cos n$\boldsymbol{\pi}$y/b -1) ; m =1,2,3..;n=2,4,6…

a/b=1 ;H_{xx} =114.76; H_{xy}=305.99;H_{yy}=612

($N_{biaxial}$/D) = (114.76 + 305.99 + 612.0)/(11.627+15.5) =10 32.79/27.127

=30.0766=3.856ρ. ;cf,"3.85ρ" in Maarefdoust and Kadkhodayan[8]

This solution in "($N_{biaxial}$/D)"is exact; this is for comparison and also testing the deflection function "w".

Also the maximum uni-axial buckling load cannot be greater than "(2.0)($N_{biaxial}$)" so as to exhaust the capacity of the square plate which is constant as the "LHS"

3.1b Solve for "(N_x)" ,X-compression;

($w,_{xx}$) = (11.627); ($w,_{yy}$) =15.5 ; ($w,_{xx}$) < ($w,_{yy}$) ; Check ($w'',_{av}$)

Average curvature loading-circle-curvature : (w''_{av}) =($\mathcal{X}_{av}$) =(11.637+15.5)/2 =13.56

(N_x /D)=(1032.79)/(13.56) =7.716ρ. S3.1-b

The difference from the literature-exact,7.7ρ,[7] is 0.2-percent . Table-2 gives more details

3.2: Square-plate; xx-cc,yy-ss ;X-thrust ,Fig.1b

a/b=1 ;H_{xx}= 612, H_{xy}=305.99, H_{yy}=114.76; ($w,_{xx}$) =15.5 ;

($w,_{yy}$) =11.627 ; $\mathcal{X}_1$=13.127;r_{cap} = H_{xx}/H_{yy} =5.3

H=(612+305.99+114.76)=1032.75;

($\mathcal{X}_{cr}$) = $\mathcal{X}_x$=15.5…Fig.2b; **;confirm "** $\mathcal{X}_y$=11.67"

(N_x/D)=1032.75/15.5=66.63=6.75ρ. "Ref-[5]=6.75ρ"

3.3: "CCCC" square plate, uni-axial compression ;Fig.1c
a/b=1 ;H_{xx}=1168.8 ;H_{xy}=779.2; H_{yy}=1168.8 ; and ($\mathcal{X}_x$)=($\mathcal{X}_y$)=29.61 =$\mathcal{X}_{cr}$.
(N_x /D)= 3116.8/29.61 =105.26 =10.66ρ
Taylor,G.I[10]=10.66ρ; Maulbetsch, J. L.[11] =10.48ρ;Ref.[5]=10.35ρ ; Levy, Samuel[12]=10.07ρ. The references show that small differences in results come with differences in the exactness of the deflection functions . The "Ref.[5]=10.35ρ" appears the most quoted solution. Table-1 gives more details. Polynomial deflection function will always yield upper-bound result,10.94ρ,,Ada,E.I ,et al [13];this is about the best expected from such functions .Buckling cannot depart from Euler's discovery of "sine" function in 1756.

3.4: square plate; xx-sc,yy-cc ,X-thrust;Fig.1d
It is noted that the end boundary conditions are different on the compressed X-axis ; this is not covered in published classical solutions ; exact strip functions for the case
W_{-x}=[Sin S_cX/a + AX/a] ;S_c=4.5 , A=0.977 …these values solve the case of the bar exactly.
W_{-y}= [Cos n$\boldsymbol{\pi}$y/b -1] ;n=2,4,6…
W= [Sin S_cX/a + AX/a][Cos n$\boldsymbol{\pi}$y/b -1] ;n=2,4,6…
a/b=1 ;H_{xx}= 365.37, H_{xy}=474.89, H_{yy}=790.1,
$N_{biaxial}$ = (365.37+474.89+790.1)/(18.05+19.96) =1630.36/38.01 = 4.346ρ
$\mathcal{X}_x$.= (18.05) ; $\mathcal{X}_y$.= (19.96) ;
($\mathcal{X}_{av}$.)= (18.05 + 19.96) /2 =19.005
($w''_{,av}$) = (19.005) ; this is the relevant Mohr's failure -$\mathcal{X}$
N_x /D= 1630.36/(19.005) = 8.69ρ
There is no reference direct solution anywhere to compare with but the "CCCC" plate at (a/b=4/3) is equivalent and offering a result ≅ 8.6ρ; from graph,[5]

3.5: Square plate; XX-SC,YY-SC ;X-thrust ;Fig.1e
W=(Sin K.x/a +A.x/a)(Sin K.y/b +A.y/b) ;K=4.5 ,A=0.977
a/b=1 ;H_{xx}=379.87,H_{xy}=533.90,H_{yy}=379.87 ;H=1293.64
($w_{,xx}$)= 18.76; ($w_{,yy}$)=18.76 ; OK
$N_{biaxial}$ = 1293.64/(18.76+18.76) = 34.479ρ =3.493ρ
N_x /D= 1293.64/18.76 =68.957=6.987ρ; Ref[5]=6.99ρ;
This plate was left out in the extensive literature solutions; [5,6,7] .However, a comparison will be found in XX-SS,YY-SC plate at "a/b=0.67", that is ,N_x≅7.0ρ,[5]

4. Effects of Aspect-ratios , a/b = s*, on strength
The full cycle cosine/sine curve ,s1,s2,s3,s4,Fig 3, explains everything about buckling ; the clamped-clamped starts with an acceleration and simply-supported starts with a velocity .
"s1,s2,s3,s4" solves the "CCCC" at s*=1;
"s2,s3" ≡"XX-SS,YY-CC" at s*=1 solves the "CCCC" at s*=2
s1s3 ≡ "XX-SS,YY-CC" at s*=1 solves "CCCC" at s*=4/3
In this way independent checks are easily found for some aspect ratio investigations.

Infinitely Long Rectangular Plates
Infinitely long plates may be accounted for by augmenting Eq.1 with shear waves as demonstrated in Fig.4ii to give Eq.13 ;for s*≤1 plate the augmentation is zero ."{..}/N_x" is considered as extra average X-curvature .

$$D(\partial^4 w/ \partial x^4+2\partial^4 w/ \partial x^2\partial y^2+\partial^4 w/ \partial y^4)=N_x\boldsymbol{\partial}^2 w/\partial x^2+(-\boldsymbol{\mu} N_x \mathrm{Cos}45)\mathrm{Cos}45(-\boldsymbol{\mu}\mathcal{X}_y)2/\boldsymbol{\pi}(a-b)/a \;\ldots\ldots(13)$$

4.1 : Constrained uni-axial X-Compression :
By Ref.[5] and Fig.4i, for "SSSS" ,Eq.14 evolves .

$$H=D\boldsymbol{\pi}^4[(m/a)^2+(n/b)^2)]^2=(N_x)(\mathcal{X}_x)+(\mu N_x)(\mathcal{X}_y) \quad ; \quad (\mu N_x)=N_y. \qquad (14)$$

For long plates only the compression load-length ,b, can be kept constant and away from the local load zone the Y-width ,b, can increase with the external-constraints removed ,Fig.4ii;

4.2 Onset of a Long Plate Dimension
The strength of a long plate is virtually invariant with aspect ratio; so there is only need to make a few calculations ,s*=1,a_{-long},and "0.5a_{-long}" for graphing when s*>1

$$\iint w\boldsymbol{\partial}^2 w/\boldsymbol{\partial}x^2 =\boldsymbol{\mu}\iint w\boldsymbol{\partial}^2 w/\boldsymbol{\partial}y^2. \qquad (15)$$

From Eq.15, find starting point of a long plate, "a_{-long}".

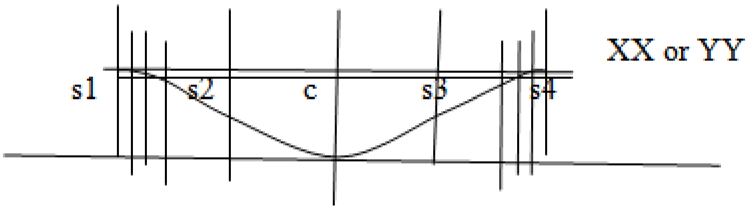

Fig. **3:** Aspects of "CCCC" plate modes

Table-1; "CCCC" at s* =1 to 9 ;$N_x=\rho K_{cr}$.

s*	m	n	K_{cr} (Ref.5)	$\mathcal{X}_x/\mathcal{X}_y$
1	2	2	10.66# (10.35)	1
1.83	2	2	8.0# (8..1)	0.28 ≅0.3
2	2	2	7.86 (7.85)	
2.38^^	2	2	7.80# (7.75)	0.3
5.48	6	2	7.66 (7.4)	0.3
9.13	10	2	7.34 (7.3)	0.3

$N_x \cong$ "7.8ρ"≅minimum at s*≥2.4 for $\mathcal{X}_x/\mathcal{X}_y \geq 0.3$; #plotpoint
At "$\mathcal{X}_x/\mathcal{X}_y$" <μ, "$\mathcal{X}_x$" is inseparable from residuals;^^,long-plate onset

Table-2 ; "XX-SS,YY-CC" at s* =1 to 14 ; $N_x=\rho K_{cr}$.

s*=a/b	m	n	K_{cr} (Ref.5)	$\mathcal{X}_x/\mathcal{X}_y$
0.5	1	2	8.0# (8.0)	>1
0.0.65	1	2	7.3# (7.3)	>1
0.82	1	2	7.74# (7.74)	>1
1	1	2	7.71# (7.7)	0.75
1.58^^	1	2	7.55# (7.4)	0.3
4.74	3	2	7.30 (7.1)	0.3
14.2	9	2	7.02 (6.97)	0.3

$N_x \cong$ "7.55ρ" ≅minimum at s*≥1.6 for $\mathcal{X}_x/\mathcal{X}_y \geq 0.3$;
At "$\mathcal{X}_x/\mathcal{X}_y$" <μ ,"$\mathcal{X}_x$" is inseparable from residuals;#plotpoint
^^,long-plate onset

4.4 The Simply-Supported Plate, "SSSS"

The huge irregularity in the variation of the strength of plates with aspect ratio, where the compression direction has simple supports at both ends,[5], cannot recommend the relevant curves to design.

The all-simply supported plate is solved by the deflection function "w= (Sin mπx/a)(Sin nπy/b)" with the solution $N_x/D=\pi^2a^2/(m^2)[(m/a)^2+(n/b)^2)]^2$.;"Timoshenko and Krieger, [14]".

At "s*"=1,2,3… ,$N_x/D=4\rho$;problems arise if "s*" is not a whole number as generally encountered in design . If s*=√2 is isolated for design and with no reference to any other ,$N_x/D=4.5\rho$ for m,n=1 ;this result is held with suspect because a longer plate is now much stronger than a shorter one. Also the plate is long before "s*"=2 and so in the realm of frequently used dimension.

Controlled "National-Advisory-Committee-For-Aeronautics(NACA)"-laboratory test results presented by Pride and Heimeri,[2] showed that the strength of "a/b=2 plate" was consistently over 12-percent weaker than "a/b=1,plate" for the same slenderness; continuing, over 22-percent weaker for "a/b=5 plate". It was also revealed in the same Report[2] and also by Nishino, et al[3]that the theoretical was 25 to 40-percent higher than experiment for "b/t" ranging from 50 to 70.

The use of the intermediate-loading-curvature,"$\mathcal{X}_{av}$", is a new input in this study and it predicts decline of strength with "s*" > 1 . Table-3 summarizes the "SSSS" plate by the present method.

Table-3 ; "SSSS-plate ,s*=1 to 11 ; $N_x=\rho K_{cr}$

s*=a/b	m	n	K_{cr} (Ref.5)	$\mathcal{X}_x/\mathcal{X}_y$
0.25	1	1	18.1# (18.1)	>1
1	1	1	4# (4)	1.00
√2	1	1	3# (4.5)	>0.3
1.83^^	1	1	2.6# (>4)	0.30
5.48	3	1	2.50 (4)	0.3
10.95	6	1	2.45 (≅ 4)	0.3

"N_x≅2.57ρ ≅minimum at $s^*\geq1.83$ for $\mathcal{X}_x/\mathcal{X}_y>\mu$; #plotpoint
At "$\mathcal{X}_x/\mathcal{X}_y$" <μ ,"$\mathcal{X}_x$" is inseparable from residuals;^^long-plate onset

Table-4 -.-"XX-SS,YY-SC" at s* =1 to 5 ; $N_x=\rho K_{cr}$.

s*=a/b	m	s_c	(new);K_{cr} (Ref-5)	$\mathcal{X}_x/\mathcal{X}_y$
1	1	4.5	5.35# (5.85)	>0.3
1.1	1	4.5	4.98# (6.3;so high ?)	
1.67^^	1	4.5	4.32# (5.6)	0.3
5	3	4.5	4.25# (5.5)	0.3

N_x ≅ "4.32ρ" ≅minimum at $s^*\geq1.67$ for $\mathcal{X}_x/\mathcal{X}_y\geq0.3$;
At "$\mathcal{X}_x/\mathcal{X}_y$" <μ ,"$\mathcal{X}_x$" is inseparable from residuals;#plotpoint
^^,long-plate onset ;Reference variation too wild.

4.5 : Square plate; XX-SS,YY-SC ; X-thrust
a/b=1 ; H_{xx}=76.5; H_{xy}=186.6;H_{yy}=191.4;H=454 ;$\mathcal{X}_x$=7.87 ; $\mathcal{X}_y$=9.46; $\mathcal{X}_{av}$ =8.66
(a) One solution is ;
$N_x/D=H/\mathcal{X}_x$ =57.7=5.84ρ. exactly as Ref.[5] .
Now "$\mathcal{X}_x<\mathcal{X}_y$." ;so this quoted result cannot be exact . "$\mathcal{X}_x$" has to be greater than "$\mathcal{X}_y$" for this to happen, Fig.2a.

(b)By present method :
$N_x/D=H/\mathcal{X}_{av}$ =5.31ρ ;this is a new and more exact result. A useful comparison is found in the result,5.756ρ at s*=1 of Adah,E.I., et-al[13] . More details are found in Table-4 .For strength versus aspect-ratio, the literature [5], variation is too wild to be acceptable ;for example N=5.83ρ at s*=1 and N=6.3ρ at s*=1.1 ,then down to N=5.5ρ at s*=1.45. The present solution in Table-4 gives a more acceptable variation.

4.6 The "XX-SS,YY-SF" plate : Fig.1f
This is a frequently met case in construction . The "YY" axis plate is independently unstable and so enjoys poisson's existence from "XX" . By Ref.[5] approximate solution is preferred ,

$$K_{cr} = 0.456 +(b/a)^2.$$

Further insight is offered from the "w-function"

$$W= (\text{Sin } m\pi x/a)(\text{Sin } \pi y/cb) \quad ;m=1;c=?$$

Find "c" such that,
$(\mathcal{X}_y)_{domain}=(\mu\mathcal{X}_x)_{domain}$;; (μ=0.707μ =0.212 for a sine variation) . In this way the exact free-edge boundary condition is approximated .
(i) "a=b" ;c=2.17;H_x=60.083;H_{xy}=25.5;H_y=2.7;H=88.28;$\mathcal{X}_x$=6.0875; $\mathcal{X}_y$=1.29; N_x=1.469ρ
(ii)"a=2b" ; c=4.34;H_x=3.755;H_{xy}=1.59,H_y=0.169;H=5.5;N_x=0.37ρ

5: Wave numbers in buckling
Undue emphasis is not placed on wave numbers in buckling; vibration and buckling are inseparable and vibration is the preferred mode of resistance and failure .In the "SSSS" plate a wave number in "m=4" for s*=7.5 was tried as shown in Table3 .In the situation, the fundamental and harmonics can act together. Until a plate becomes long the fundamental numbers are sufficient. In very long plates additional shear compression from harmonics can reduce strength slightly calling for slight adjustment in the present method.

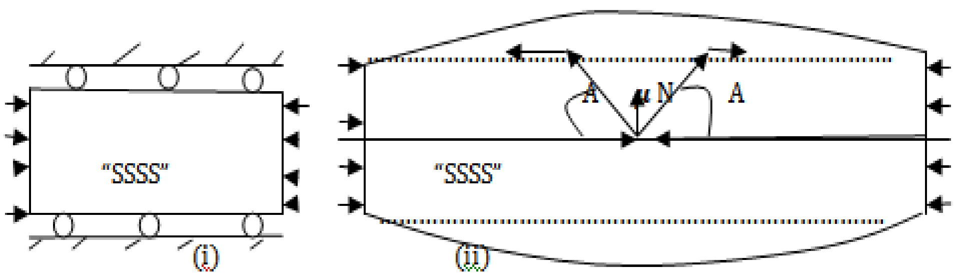

Fig.4;i:X-sides constrained; ii: X-sides bow-out,away from load ;angle-A=45

III.CONCLUSIONS

1. Many referral solutions of the uni-axial plate buckling are not listed when the ends of the compression axis have differing boundary conditions; these were successfully handled in this study.
2. The relative-curvature/deflection resonance buckling criterion was adopted and very accurate results found here are verified against those in the literature. Problems skipped in the literature are covered here.
3. Two competing loading-curvatures are identified with reference to the Mohr's curvature loading-circles , "$\mathcal{X}_x$" and "$(\mathcal{X}_x+\mathcal{X}_y)/2$" ;the one which leads to the smaller buckling load is the relevant .
4. The new method predicts a result of "$N_x/D=8.67\boldsymbol{\pi}^2$" for a square-plate "XX-SC,YY-CC", compressed on 'XX' .There is no direct literature solution but the result is adjudged correct because it approximately matches that of "CCCC plate' at "a/b=4/3", as it should.
5. By this study a plate becomes long after the computable "$\mathcal{X}_x/\mathcal{X}_y$" ratio is less than the Poisson's ratio in uni-axial X-compression; there is then no longer unique uni-axial action .A "CCCC" plate is short up to s*=2.38 and 1.83 for "SSSS" .
6. Contrary to existing literature this study concludes that the "SSSS" plate reduces in strength after "a/b">1 ,reducing from "4.0ρ"in square- to "2.6ρ" at "a/b"=1.83
7. A more satisfactory solution of the "XX-SS,YY-SC" at "a=b" is found as "5.31ρ" ,much less than the literature[5] value "5.85ρ" ;the checking ability of the present method is in focus. The wild literature fluctuation with "s*" is attenuated by the present method.
8. Beyond the length-description of a short plate, the strength reduction remains marginal , similar to typical strength versus slenderness relation in one-way buckling plates .

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