American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-7, Issue-7, pp-194-204
www.ajer.org
Research Paper Open Access

A Shallow Shell Triangular Finite Element For The Analysis Of Hyper Shell Dam Structure

Attia Mousa[1]and ,Majed Aldahdooh[2]
[1]*Department of Civil Engineering and Architecture, College of Engineering, University of Bahrain*
[2] *Civil and Environmental Engineering Program, West Virginia University, Bahrain Campus*
Corresponding Author: Attia Mousa

ABSTRACT:*A hyper triangular finite element is developed in this paper. The element has only five essential nodal degrees of freedom at each of the three corner node (three general external degrees of freedom and two rotations). The displacement fields of the element satisfy the exact requirements of rigid body modes of motion. Shallow shell formulation is used and the element is based on an independent strain assumption insofar as it is allowed by the compatibility equations. A simple shell problem for which a previous solution exists is first analyzed using the new element to test the efficiency of the element. Results obtained by the present element are compared with those available in the previous solution. These comparisons show that an efficient and accurate result can be obtained by using the present element. Then the new element is used to analyze a hyper shell dam. The distribution of various components of stresses is obtained. The new element is also applied to analysis cylindrical shell dam and the results show the efficiency of the element to analyze cylindrical shell problems. The various components of stresses of the hyper shell dam and cylindrical shell dam give the designer an insight to the behavior of such structures.*

KEYWORDS: *Strain-Based, hyper shell, Cylindrical Triangular element, cylindrical dam, Finite Element*

Date of Submission: 04-07-2018 Date of acceptance: 19-07-2018

I. INTRODUCTION

Considerable attention has been given applying the finite element method of analysis to curved structures. The early work on the subject was presented by Grafton and Strome (1963) who developed conical segments for the analysis of shells of revolution. Jones and Strome (1966) modified the method and used curved meridional elements which were found to lead to considerably better results for the stresses.

Further research led to the development of curved rectangular as well as cylindrical shell elements (Connor et al, 1967; Bogner et al, 1967; Cantin et al, 1968 and Sabir et al, 1972). However, to model a shell of arbitrary or triangular shape by the finite element method, a triangular shell element is needed. Thus, many authors have been occupied with the development of curved triangular shell elements and consequently many elements (Lindberg et al, 1970 and Dawe, 1975) resulting in an improvement of the accuracy of the results. However, this improvement is achieved at the expense of more computer time as well as storage to assemble the overall structure matrix.

Meanwhile, in the United Kingdom, a simple alternative approach referred to as strain based approach has been used to the development of curved elements. This approach is based on determining the exact terms representing all the rigid body modes together with the displacement functions representing the straining of the element by assuming independent strain functions insofar as it is allowed by the compatibility equations.

This approach has successfully employed in the development of curved shell elements (Ashwell et al, 1971, 1972; Sabir et al, 1975, 1982, 1983, 1987; El-Erris, 1994, 1995 and Mousa, 1991, 1994, 1998, 2012,). These elements were found to yield faster convergence with other available finite elements

The strain-based approach is employed in the present paper to develop a triangular hyper shell element having five degrees of freedom at each corner node, the five degrees of are essential external degrees of freedom and no additional degree are associated with the in-plane rotation of shell.

The element is first tested by applying it to the analysis of the partly hyper problem showing for which a previous solution exists. The work is then extended to analyze a hyper shell dam.

Results obtained by the present element are compared with those available in the previous solution. The distribution of various components of stresses of the hyper is obtained. Then the new element is applied to analysis cylindrical shell dam. The various components of stresses of the hyper shell dam and cylindrical shell dam give the designer an insight for the behavior of such structures.

II. THEORETICAL CONSIDERATIONS

II.I. *Displacement function for the new hyper shell element*

In a system of curvilinear coordinates, the simplified strain-displacement relationship for the hyper shell element shown in Figure (1) can be written as:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{r_x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} - \frac{w}{r_y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$K_x = \frac{\partial^2 w}{\partial x^2}, \quad K_y = \frac{\partial^2 w}{\partial y^2}, \quad K_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \qquad (1)$$

Where u,v and w are the displacement in the x,y and z axes, , ε_x , ε_y are the in-plane direct axial and circumferential strains and $\gamma_{xy}$is the in-plane shearing strain. AlsoK_x, Ky and K_{xy} are the mid-surface changes of curvatures and twisting curvature respectively, r_x, and r_y are the principle radii of curvature.

Equation (1) gives the relationships between the six components of the strain and the three displacements u,v and w. Hence, for such a shell there must exist three compatibility equations which can be obtained by eliminating u,v and w from equation (1).

This is done by a series of differentiations of equation (1) to yield the following compatibility equations:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{K_x}{r_y} + \frac{K_y}{r_x} = 0.0$$

$$\frac{\partial K_{xy}}{\partial x} - 2\frac{\partial K_y}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial K_{xy}}{\partial y} - 2\frac{\partial K_x}{\partial x} = 0$$

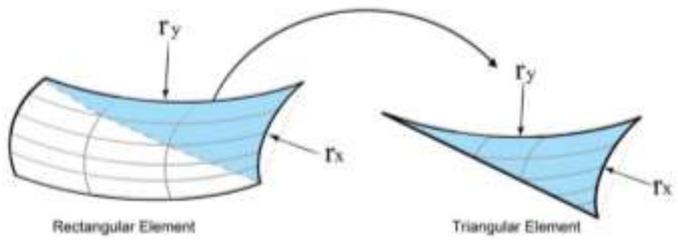


Fig.1 Geometry and finite element mesh of roof

To keep the triangular element as simple as possible, and to avoid the difficulties associated with internal and non-geometric degrees of freedom, the essential five degrees of freedom at each corner node are used, namely u, v, w, $\partial w/\partial x, \partial w/\partial y$. Thus, the triangular hyper element, to be developed has a total of fifteen degrees of freedom and 15x15 stiffness matrix.

To obtain the rigid body components of the displacement field, all the strains, as given by equations (1), are set to zero and the resulting partial differential equations are integrated. The resulting equations for u, v and w are given by:

$$u_1 = -a_1x/r_x - a_2(y^2/2r_y + x^2/2r_x) - a_3xy/r_x + a_4 + a_6y \qquad (3)$$
$$v_1 = +a_1y/r_y + a_2xy/r_y + a_3(+y^2/2r_y + x^2/2r_x) + a_5 - a_6x$$
$$w_1 = a_1 + a_2x + a_3y$$

Where u_1, v_1 and w_1 are the rigid body components of the displacement field u, v and w, respectively, and are expressed in terms of the six independent constants a_1, a_2 … a_6.

Since the element has fifteen degrees of freedom, the final displacement fields should be in terms of fifteen constants. Having used six for the representation of the rigid body modes, the remaining nine constants are available for expressing the straining deformation of the element. These nine constants can be apportioned among the strains in several ways, for the present element we take:

$$\varepsilon_x = a_7 - \frac{1}{r_x}(a_{12} y^2/2 + a_{13} x y^2/2 + a_{14} y^3/6)$$

$$\varepsilon_y = a_8 + \frac{1}{r_y}(a_{10} x^2/2 + a_{11} y x^3/6)$$

$$\gamma_{xy} = a_9 \tag{4}$$

$K_x = a_{10} + a_{11} xy$

$K_y = a_{12} + a_{13} x + a_{14} y$

$K_{xy} = a_{15} + [a_{11} x^2 + 2a_{13} y]$

In which the un bracketed independent constants terms in the above equations were first assumed. The linking bracketed terms are then added to satisfy the compatibility equation (2).

Equations (4) are then equated to the corresponding expressions, in terms of u,v and w from equations (1) and the resulting equations are integrated to obtain:

$$u_2 = a_7 x + a_9 y/2 + a_{10} x^3/6r_x + a_{11} x^4 y/24r_x + a_{13} y^4/24r_y + a_{15}(x^2 y/4r_x + y^3/12r_y)$$

$$v_2 = a_8 y + a_9 x/2 - a_{11} x^5/120r_x - a_{12} y^3/6r_y - a_{13} x y^3/6r_y - a_{14} y^4/24r_y - a_{15}(x y^2/4r_y + x^3/12r_x) \tag{5}$$

$$w_2 = -a_{10} x^2/2 - a_{11} x^3 y/6 - a_{12} y^2/2 - a_{13} x y^2/2 - a_{14} y^3/6 - a_{15}\, xy/2$$

The complete displacement functions for the element are the sum of corresponding expressions in equations (3) and (5). The rotation about the x and y-axes respectively, are given by:

$$\varphi y = -\frac{\partial w}{\partial x} = -a_2 + a_{10} x + a_{11} x^2 y/2 + a_{13} y^2/2 + a_{15} y/2$$

$$\varphi x = -\frac{\partial w}{\partial y} = -a_3 + a_{11} x^3/6 - a_{12} y - a_{13} xy - a_{14} y^2/2 - a_{15} x/2 \tag{6}$$

The stiffness matrix [K] for the shell element is calculated in the usual manner using the equation.

$$K = \left[C^{-1}\right]^T \left\{\iiint B^T D B dv\right\} \left[C^{-1}\right] \tag{7}$$

Where B and D are the strain and rigidity matrices respectively and C the matrix relating the nodal displacements to the constants a_1 to a_{15}, B can be calculated from equations (1), (3) and (5) and D is given below.

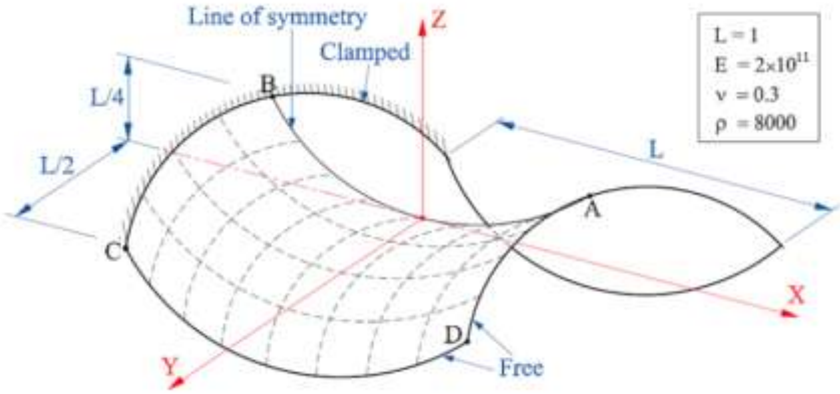
Substituting the matrices B and D into equation (5), the integrated within the bracketed terms of equation (4) are carried out explicitly and the rest of the calculations are computed to obtain the stiffness matrix [K]

$$[D] = \frac{Et}{1-v}\begin{bmatrix} 1 & v & 0 & 0 & 0 \\ v & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-v}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{t^2}{12} & \frac{vt^2}{12} \\ 0 & 0 & 0 & \frac{vt^2}{2} & \frac{t^2}{2} \end{bmatrix} \tag{8}$$

Where E is Young's modulus, $\boldsymbol{t}$ is the thickness and $\boldsymbol{v}$ is Poisson's ratio.

III. PATCH CONVERGENCE TEST

This test is to be considered which is frequently used to test the performance of the hyperbolic paraboloid shell elements is that of the partly hyperbolic paraboloid problem is a self –weighted symmetric shell ,clamped in one edge ,and free in the others.edges The problem having the geometry as shown in Fig. (2). The shell has the following dimensions and material properties: thickness, t= 0.001m, L = 1m, modulus of elasticity, E= 2x1011 N/m2, Poisson's ratio, v= 0.3, density, ρ = 8000 kg/m3 . Considering the symmetry of the problem only half of the surface needs to be considered in the analysis



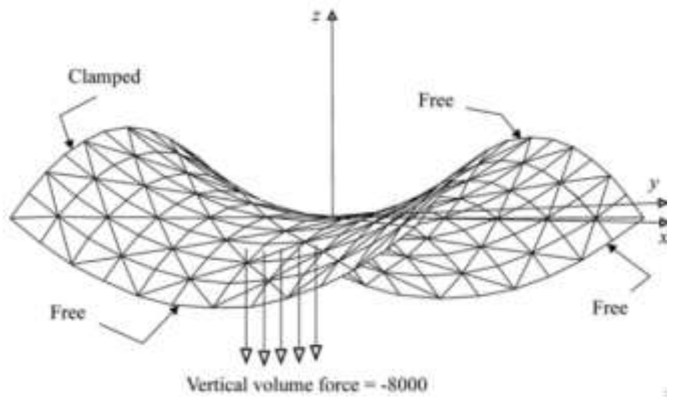


Fig.2 Geometry and meshes of partly hyperbolic parabolic Problem

The results obtained by the new present element for the vertical displacement at the midpoint A of the free edge of the roof are compared to other of shells elements (MITC element developed by Bathe, (Ref.21,22) and MISTK elements development by Nguyen-Thanh et al ,Ref.23)). For this problem there is no analytical solution,and reference value for the vertical displacement previously obtained by Bathe et al , Ref.24 In Table 1 also we have compared the displacement at point A for a regular mesh of our element to other elements in the literature . Convergence curve in Fig.3 show that the convergence of the present element faster convergence than the elements (MITC4,MIST4 ,MIST2) and almost the same accuracy with MIST1 . These results gave confidence in applying the present triangular hyper element to the practical problem of such as hyperbolic parabolic dam.

Mesh no.	MITC4 Ref.[21,21]	MIST1 Ref.[23]	MIST2 Ref.[23]	MIST4 Ref. [23]	Ref.[24] Solution	Present Element
8x4	0.004758	0.005586	0.004966	0.004847		0.005411
16x8	0.005808	0.00619	0.005929	0.5862		0.0061115
32x16	0.00619	0.006347	0.006249	0.006218		0.0063122
40x20	0.006254	0.006369	0.006298	0.006275		0.006345
48x24	0.006294	0.006383	0.006329	0.006311	0.006394	0.006357

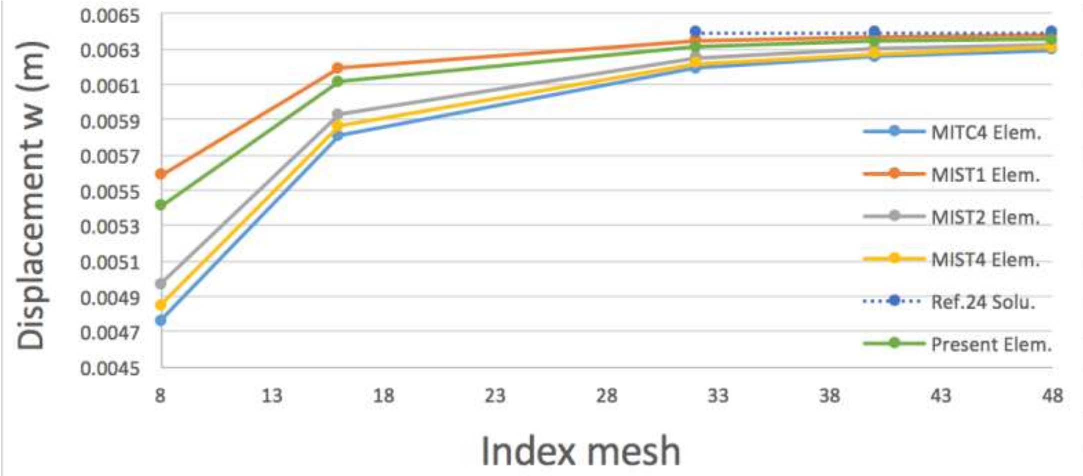


Fig.3 The Convergence of deflection at point A

IV. PROBLEM CONSIDERED

II.I. *Hyper Dam*

The problem considered is that of a hyperbolic paraboloid shell dam where he boundary nodes are supported by a rigid valley, and the external boundaries of the dam are assumed to be fixed for all the degrees of freedom at each nodal points the dam subtending a central angle of 106 degrees, 3m thick , 30m high and has a radius of r_x= 10^8 kg/m^2 and 0.15 respectively (see figure 4).×-r_y= 43.25m , young's modulus and Poisson's ratio are 20

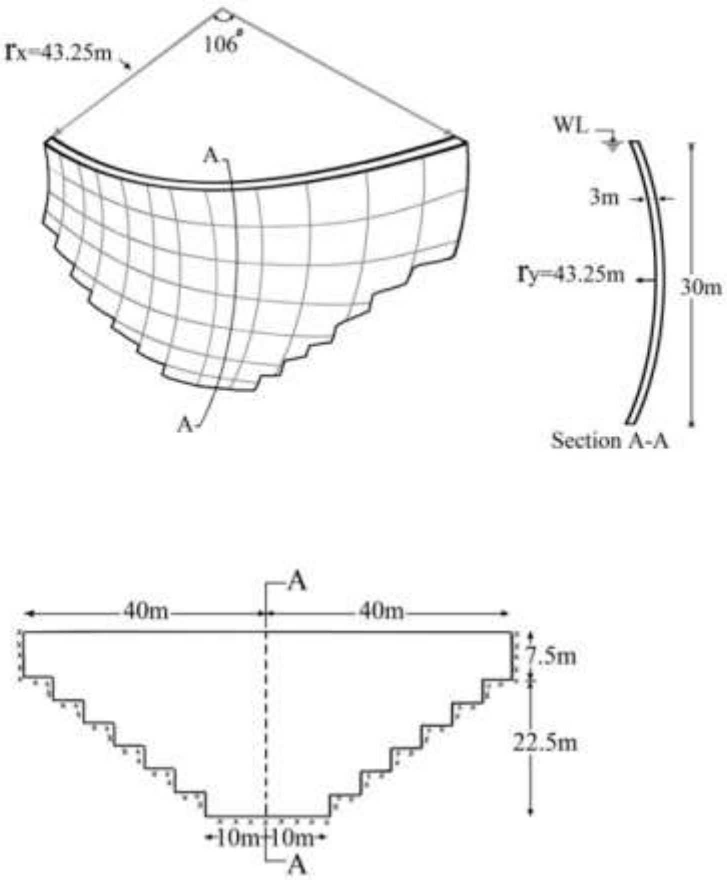


Fig.4 General dimensions of hyperbolic paraboloid dam

This problem was analyzed previously in Ref.17 using hyper rectangular element with five degrees of freedom at each corner node. Now the new strain based triangular hyperbolic paraboloid element developed in the present paper is applied to analyze the same problem, the mesh considered in the analysis is shown in Figure (5). Figure 5 also shows the mesh used previously in Ref. 17. Due to symmetry only one half of the dam is analyzed.

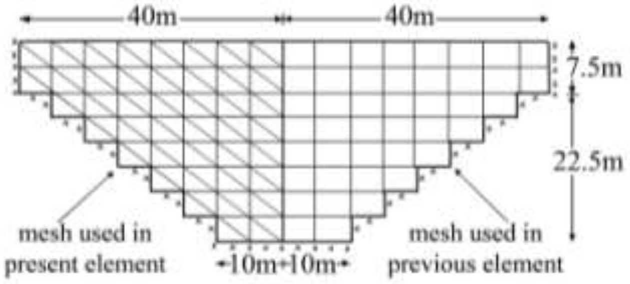


Fig.5 Finite element mesh in the H.P. dam

The results for the normal deflection along the vertical central line (crown line) are given in Figure (6). Figures (7) to (10) give the results for the vertical stresses and hoop stresses on both the upstream and down-stream faces, respectively

The figures show that the present element gives a very good agreement results.

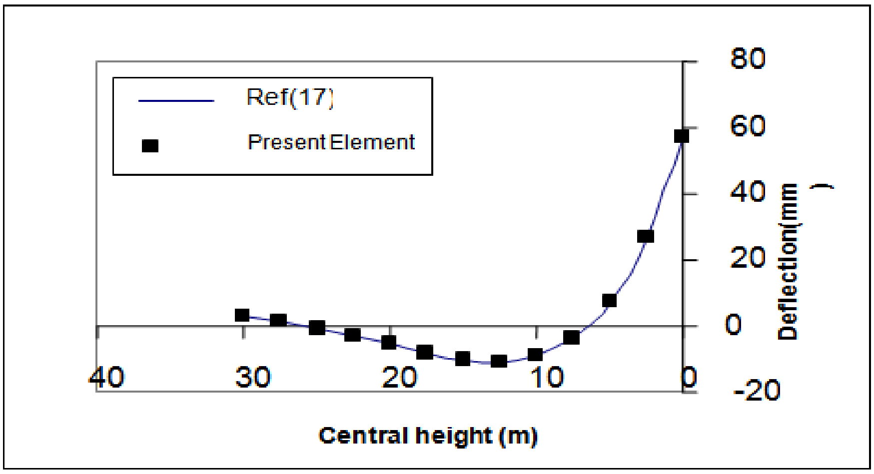


Fig.6 Radial deflection on central line of dam.

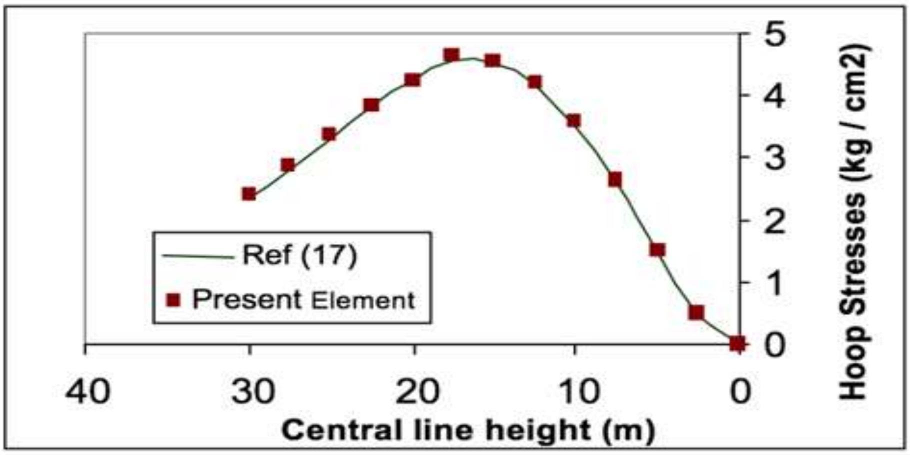


Fig.7 Vertical stresses on central cantilever for dam (upstream).

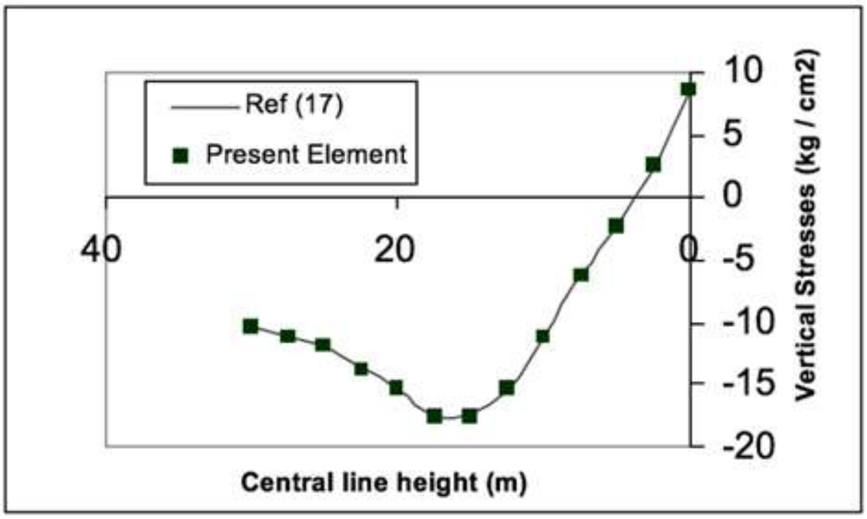


Fig.8 Vertical stresses on central of dam (downstream).

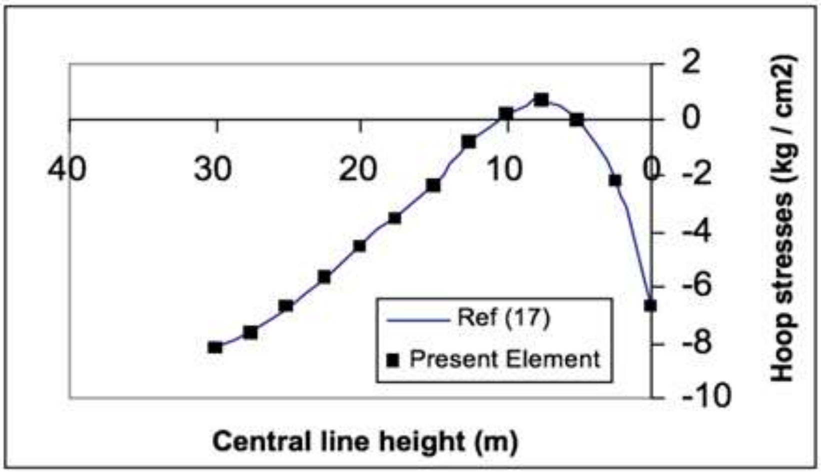


Fig.9 Hoop stresses on central cantilever of dam (upstream).

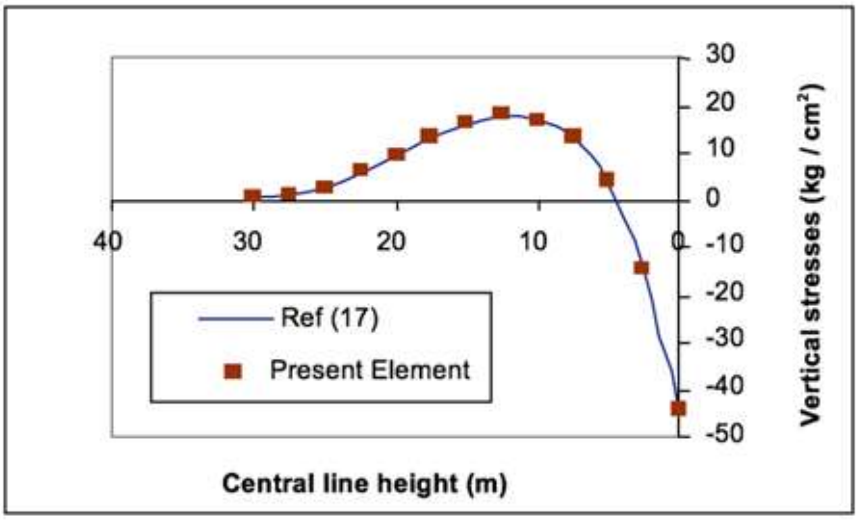


Fig.10 Hoop stresses on central cantilever of dam (downtream).

II.I. *Cylindrical shell dam structure*

To check the efficiency of the developed hyper element to analyze the singly curvature such as cylindrical shell problem, we used the same shape function of the doubly curvature element developed in this paper using ry = infinity (i.e. 1/ry = 0) and then applied the revised shape function on analysis of cylindrical shell problem. The problem considered is that of a cylindrical shape shell dam where he boundary nodes are supported by a rigid valley, the dam subtending a central angle of 106 degrees, 3m thick , 30m high and has a radius of 43.25m, young's modulus and Poisson's ratio are 20 x 108 kg/m2 and 0.15 respectively. (Fig. 11)

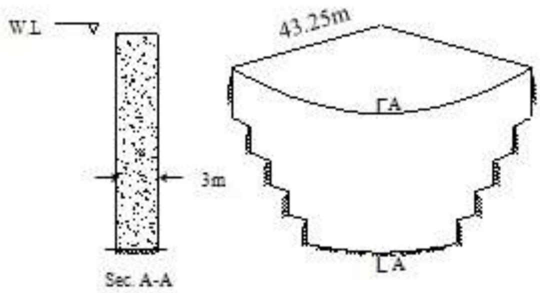


Fig.11 General dimensions of the cylindrical dam.

Due to symmetry only one half of the dam is analyzed. Two meshes as shown in Fig. 12 were considered.

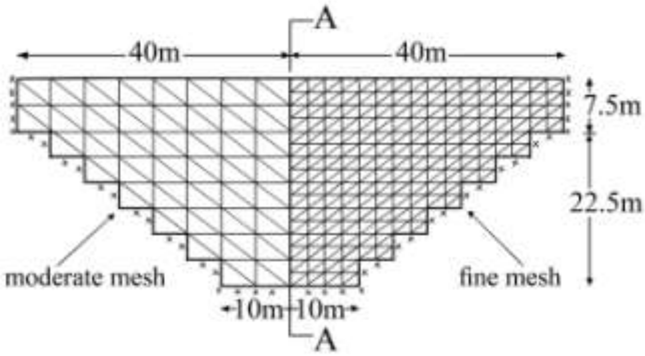


Fig.12 Finite element meshes.

The results for the radial deflection at the central line of the dam (crown line) for both meshes are shown in Fig.13. This figure also contains the results of the finite element solution obtained in reference [17] where a rectangular element with five degrees of freedom at each corner node is considered.

These results show that the two meshes give similar results and they are in very good agreement with these of finite element solution obtained in reference [17].

The direct stress resultants Nx&Ny in the x and y directions respectively and the bending moments stress resultants Mx&My about y and x axes respectively are compared along the crown line. The results are presented in Figs. 14-17 respectively.

It is clearly seen that the two meshes gave similar results and they are in very good agreement with those obtained in reference [17].

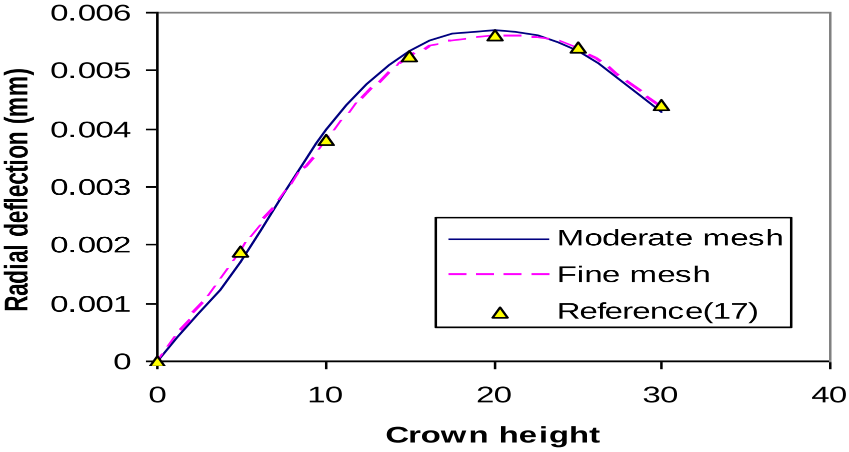


Fig.13 Redial deflection at the crown line.

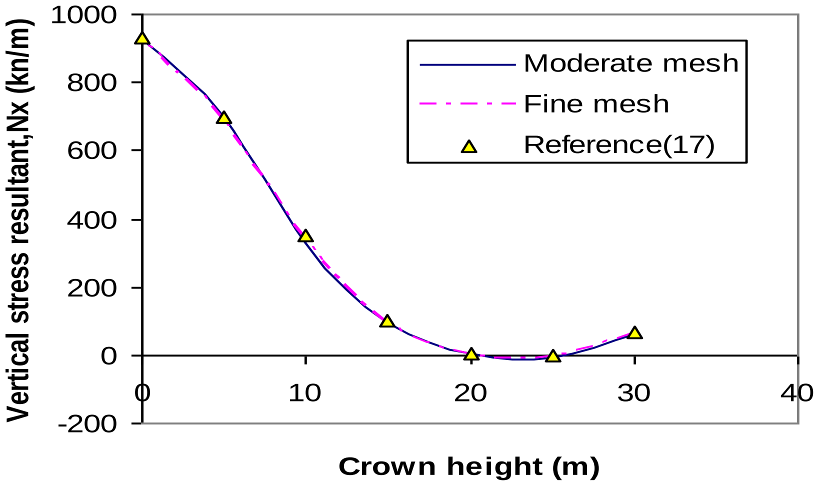


Fig.14 Direct stress resultant (Nx) at the crown line.

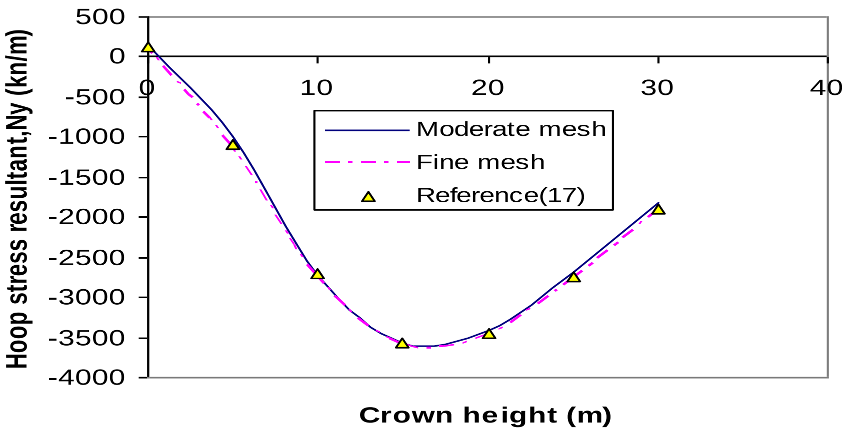


Fig.15 Direct stress resultant (Nx) at the crown line.

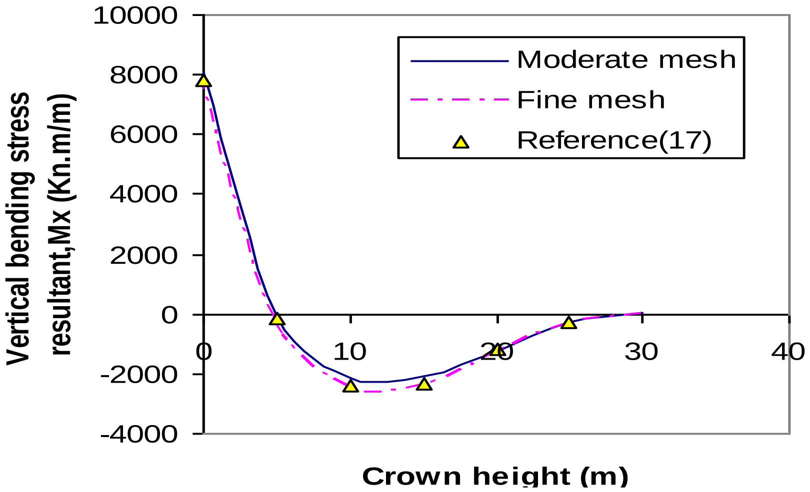


Fig.16 Bending stress resultant (Mx) of the crown line

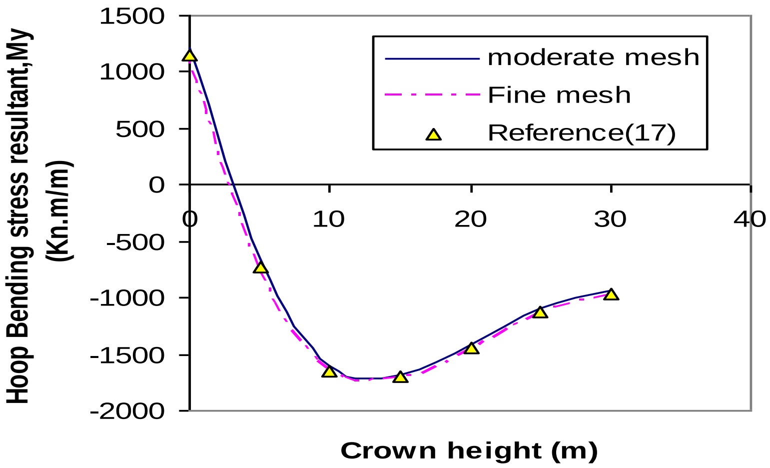


Fig.17 Bending stress resultant (My) at the crown line

V. CONCLUSION

A new triangular strain-based finite hyperbolic paraboloid shell element has five degrees of freedom at each corner node is developed in the present paper. The element has been applied for patch hyperbolic paraboloid shell problem. The results obtained are compared with other available in the literatures and very good rate of agreements with other solutions were obtained. . Then the new element is used to analyze a hyper shell dam The distribution of various components of stresses is obtained. The developed element is also applied for analysis of cylindrical dam shell and results approved the efficiency of the element to analyze of the cylindrical problems. the distribution of various components of stresses as well obtained. These distribution of stresses for both hyper dam and cylindrical dam are to give the designer an insight for the behavior of such structures.

REFERENCES

[1]. Grafton, P. E., and Strome, D.R. (1963) "Analysis of ax symmetric shells by the direct stiffness method.", AIAA. J., Vol. 1, No. 10, pp2337-2342.

[2]. Jones, R.E. and Strome, D.R. (1966) "Direct stiffness method analysis of shells of revolution utilizing curved elements", AIAA. J. Vol. 4, pp1525-1551.

[3]. Grafton P.E. and Strome. D.R., Analysis of axisymmetric shells by the direct stiffness method. AIAA. J.1.1963, 2342-2347.

[4]. Jones. RE and Strome. D.R., Direct stiffness method analysis of shells of revolution utilizing curved dements, AIAA. J.(4).1966, 15159-1525.

[5]. Conner. J.J. and Brcbbia, C., Stiffness matrix for shallow rectangular shell element,J. Eng. Mech. Div. ASCE.93. No EMS. 1967, 41-65.

[6]. Bogner. F K. Fox. R L. and Schmit. I. A., A cylindrical shell discrete element, J. AIAA. 5(4).1967, 745-750.

[7]. Cantin. G and Clough. R. W., A curved cylindrical shell finite element,AIAA J.16.1968, 1057-1062.

[8]. Sabir. A.B. and Lock A.C., A curved cylindrical shell finite element,Int. J. Mech. Sci.14. 1972, 125-135.,
[9]. Lindbergy. G.M. Cowper. G. R. and Olson. M.D., A shallow shell finite element of triangular shape,Int. J. Solids Struc 14.1970, 1133-1156.
[10]. Dawe. D.J., High order triangular finite element for shell and analysis,Int. J. Solids Struct.11.1975, 1097-1110.
[11]. Ashwell. D. G. Sabir. A. B. and Roberts T. M., Further studies in the application of curved finite element to circular studies,Int. J. Mech. Sci.14.1971, 507-517.
[12]. Ashwell. D. G. and Sabir. A. B, A new cylindrical shell finite element based on simple independent strain functions, Int. J. Mech. Sci.14.1972, 171-183.
[13]. Sabir. A. B., Stiffness matrices for general deformation (out of plane and in-plane) of curved beam members based on independent strain functions, The Maths of Finite Elements and Applications II. Academic Press.34.1975, 411-421.
[14]. Sabir. A. B. and Charchafchi. T. A., Curved rectangular and general quadrilateral shell element for cylindrical shells,TheMaths of Finite Elements and Applications IV. Academic Press.1982, 231-238.
[15]. Sabir. A. B., Strain based finite for the analysis of cylinders with holes and normally intersecting cylinders,Nuclear Eng. and Design 76.1983, 111-120.
[16]. Sabir. A B. Strain based shallow spherical shell element,Proceedings Int. Conf. on the Mathematics of Finite Element and Applications, Bronel University, 1987.
[17]. El-Erris, H. F., Behavior of hipped roof structures,Proc. 2nd. MCE Eng. Con., CEI, Baghdad, 1994.
[18]. El-Erris, H. F., Effect of eccentricity of crown and edge beams on the behavior of hipped roof structures, accepted for publishing in the journal of military college of engineering, Baghdad, 1995.
[19]. Mousa A. I. and Sabir. A. B., Finite element analysis of doubly curved shell structure , Internal Report , University of Wales , Cardiff ,U.K ,1991
[20]. Mousa. A. I. and Sabir. A. B., Finite element analysis of fluted conical shell roof structures,Computational structural engineering in practice. Civil Comp. Press, ISBN 0-948748-30-X . 1994, 173-181.
[21]. Mousa. A. I., Finite element analysis of a gable shell roof,Advances in Civil and Structural of Engineering Computing for Paretic civil-comp press, 1998. 26-268.
[22]. Mousa A.I, Aljuadi A.E, Kameshki E. Dahman, N, New strain cylindrical rectangular finite element for the analysis of arch dam, Canadians Journal on Environment, Construction and Civil Engineering, Canada. 2012.
[23]. Bathe K. J , finite element procedures ,prentice –Hall . Massachusetts (MIT0. Englewwod Cliffs , Nj ,1996
[24]. Dvorkin , E . Bathe , K. J A continuum mechanics based four –node shell element for general nonlinear analysis , Engrg. Comput. ,1985
[25]. Nguyen-Thanh , N., TimonRabczuk , H. Nguyen –Xuan ,Stephane P.A , A smoothed finite element method for shell analysis , Comput. Methods appl. Mech. Engrg. ,198 ,2008 ,pp 165-177
[26]. Bathe , K. J , Iosilevich , A. &Chapelee , D. , An evaluation of the MITC shell elements . Computers and Structures . 75. ,2000 . pp 1-30

Attia Mousa"A Shallow Shell Triangular Finite Element For The Analysis Of Hyper Shell Dam Structure."American Journal of Engineering Research (AJER), vol. 7, no. 07, 2018, pp. 194-204.