

## An EPQ Model for Deteriorating Items with Probabilistic Demand and Variable Production Rate

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**ABSTRACT:** In this paper, we have developed a production inventory model for deteriorating items with variable production rate. In reality it is observed that at the beginning of any production process, the rate of production in any manufacturing firm remain almost static up to certain time, but after that the production rate follows a decreasing trend due to some inherent problems, like machinery fault, lethargy of the personnel for continuous work, delaying in the supply of raw materials etc. So, in this model, we have considered variable production rate. Moreover, the demand for the items is assumed as probabilistic. Under these circumstances, a cost function of the model has been formulated. Finally, the proposed model has been demonstrated taking numerical examples and the sensitivity analysis of the optimal solution is provided with respect to key parameters of the system.

**KEYWORDS:** EPQ model, inventory, deterioration, probabilistic demand, variable production rate.

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### I. INTRODUCTION

In any manufacturing or business operation it is very crucial to maintain a good inventory for the smooth and efficient functioning of the system. If the firm does not have the required quantity of items in stock when the customer arrives, he/she may look elsewhere to fulfill his requirement, which arises lost sales and loss of goodwill. Some recent reviews on inventory management system are presented by Chen et al. (2006), Patra (2010), Prasad and Mukherjee (2016), Palanivel and Uthayakumar (2017), Saha and Chakrabarti (2017b) and Saha and Chakrabarti (2017c)[1–6].

Again, there is also an important issue associated with the stock of physical goods, which is deterioration. Most physical goods deteriorate over time, so the control and maintenance of production inventories of deteriorating items have received much attention of several researchers in the recent years. It is well known that certain products such as vegetables, medicine, gasoline, and radioactive chemicals get deteriorated or spoiled during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored. Researchers considered the deterioration of the products while developing their model, among them the work done by Hou (2006), He et al. (2010), Min et al., (2012), Sicilia et al. (2014), Ghiami and Williams (2015) [7–11] are worth mentioning.

In any production inventory system, it is very important to determine an optimum production rate, because if the manufacturer produces a huge quantum of goods, it may result in a loss due to deterioration of the products, holding cost of the excess items and huge investment in the production. Also, the products may get obsolete and hence remained unsold. On the other hand, an insufficient amount of stock may result in a shortage. Researchers have developed inventory models taking various types of production rate. Su and Lin (2001)[12] developed a production inventory model considering production rate as demand and inventory level dependent. Samanta & Ajanta (2004) [13] discussed deterministic inventory model of deteriorating items with two rates of production and shortages. Bhowmick et al. (2011) [14] worked with deterministic inventory model of deteriorating items with two rates of productions, shortages and variable production cycle. Manna et al. (2016) [15] derived an economic order quantity model with ramp type demand rate, constant deterioration rate and unit production cost, where they considered the production rate as demand dependent.

Besides deterioration and production rate another important factor in any inventory system is demand. Authors have studied inventory models with different types of demand, such as price-dependent demand, time-dependent demand, inventory level-dependent demand, fuzzy demand etc. Inventory models with price-dependent demand analyzed by Saha and Chakrabarti (2017), Tripathy and Mishra (2010) and Maihami and Nakhai Kamalabadi (2012)[16–18]. Inventory models with time dependent demand studied by Giri and Maiti (2012), Maiti (1998), Prasad and Mukherjee (2016) and Zhao, Wu and Yuan (2016)[2, 19–21] etc. Inventory models with stock level dependent presented by Alfares (2007), Hou (2006), Sarkar and Sarkar (2013) and Zhou, Min and Goyal (2008) [9, 22–24] etc. and models with fuzzy demand is studied by Mahata and Mahata (2011), Pan and Yang (2008), Shabani, Mirzazadeh and Sharifi (2016) and (Singh and Singh, 2011) [25–28] etc.

In this paper, we have developed a pricing model for deteriorating items with variable production rate. The production rate is constant at the beginning of the production process, but after some time the rate of production decreases due to various problems associated with the production system. We have considered development cost to reduce the interruptions in the production process. Moreover, it is assumed that the demand rate follows uniform distribution. Finally, we have derived cost function of the proposed model. To estimate the total cost and total production time a numerical example has been illustrated.

The rest of this paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we have developed the mathematical model. In section 4, we have provided numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 5. Finally, we draw the conclusions and future research in section 6.

## II. NOTATIONS AND ASSUMPTIONS

### 2.1 Notations:

The following notations are used for developing the model:

- i)  $D$ : customers demand rate, which is random in nature,
- ii)  $K$ : production rate per unit time,
- iii)  $\theta$ : constant deterioration rate,
- iv)  $t_1$ : duration of constant production,
- v)  $t_2$ : total production period,
- vi)  $T$ : total business period,
- vii)  $Q_1$ : inventory level at time  $t_1$ ,
- viii)  $Q_2$ : inventory level at time  $t_2$ ,
- ix)  $A_m$ : manufacturer setup cost,
- x)  $C_p$ : production cost per unit time,
- xi)  $h_m$ : holding cost per unit item per unit time,
- xii)  $d_m$ : deteriorating cost per unit,
- xiii)  $d_c$ : development cost of the manufacturer,
- xiv)  $TC$ : total cost of the manufacturer,
- xv)  $I(t)$ : inventory level at time  $t$ .

### 2.2 Assumptions:

The proposed production inventory model has been developed under the following assumptions.

- i) A constant fraction  $\theta$  ( $0 < \theta < 1$ ) of the on-hand inventory deteriorates per unit time.
- ii) The production system produces a single item.
- iii) Shortages are not allowed in the inventory.
- iv) The business period  $T$  is constant.
- v) The variable production rate  $K$  is taken as –

$$K = \begin{cases} k_0, & \text{for } 0 \leq t \leq t_1 \\ k_0 e^{-\mu(t-t_1)}, & \text{for } t_1 \leq t \leq t_2 \end{cases}$$

where  $\mu$  is a constant ( $0 < \mu < 1$ ) (according to Patra and Mondal (2015).[29]).

- vi) In reality we observe that, when production is going on in the factory then initially up to certain time period the production process produces the products at a constant rate but after some time the production rate decreases due to some inherent problems associated with the production system like machinery fault, lethargy of the labors, delay in receiving raw materials etc. These unwanted situations may impact adversely on the profitability of the system. Today's business market is so competitive that it is very challenging to sustain

company's existence in the business world and due to this reason manufacturers do not tolerate such unwanted situations. So, to avoid such unexpected situations, it is wise for them to adopt a maintenance strategy and to do so the manufacturer pay an extra cost, known as development cost so that it is possible to reduce the interruptions in production. Here we have considered the development cost as a function of initial production rate, i.e.,

$$d_c = \beta k_0,$$

where  $\beta$  is a constant.

### III. MATHEMATICAL MODEL

The production rate is constant ( $k_0$ ) in the interval  $[0, t_1]$ , then the production rate starts decreasing with time and continues up to time  $t_2$ . After that the production process remains idle in the time interval  $[t_2, T]$  and the inventory level reaches zero at  $t = T$  due to both demand and deterioration. The differential equations describing the state of  $I(t)$  in the interval  $[0, T]$  are given by –

$$\frac{dI(t)}{dt} + \theta I(t) = \begin{cases} k_0 - D, & \text{when } 0 \leq t \leq t_1 \\ k_0 e^{-\mu(t-t_1)} - D, & \text{when } t_1 \leq t \leq t_2 \dots \dots \dots (1) \\ -D, & \text{when } t_2 \leq t \leq T \end{cases}$$

With the boundary conditions

$$I(t) = 0, I(t_1) = Q_1, I(t_2) = Q_2, I(T) = 0 \dots \dots \dots (2)$$

Solving above differential equations using boundary conditions we get,

$$I(t) = \begin{cases} \frac{(k_0 - D)}{\theta} (1 - e^{-\theta t}), & 0 \leq t \leq t_1 \\ \frac{k_0 e^{-\mu(t-t_1)}}{(\theta - \mu)} - \frac{D}{\theta} + \left(Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta}\right) e^{\theta(t_1-t)}, & t_1 \leq t \leq t_2 \dots \dots \dots (3) \\ -\frac{D}{\theta} + \left(Q_2 + \frac{D}{\theta}\right) e^{\theta(t_2-t)}, & t_2 \leq t \leq T \end{cases}$$

Now, using boundary conditions from (3) we have –

$$Q_1 = \frac{(k_0 - D)}{\theta} (1 - e^{-\theta t_1}) \dots \dots \dots (4)$$

And,

$$Q_2 = \frac{D}{\theta} (e^{\theta(T-t_2)} - 1) \dots \dots \dots (5)$$

Now, set up cost of the manufacturer =  $A_m$ .

Production cost =  $C_p DT$ .

Holding cost of the manufacturer

$$= h_m \int_0^T I(T) dt$$

Now,

$$\begin{aligned} & \int_0^T I(T) dt \\ &= \int_0^{t_1} I(T) dt + \int_{t_1}^{t_2} I(T) dt + \int_{t_2}^T I(T) dt \\ &= \int_0^{t_1} \frac{(k_0 - D)}{\theta} (1 - e^{-\theta t}) dt + \int_{t_1}^{t_2} \left[ \frac{k_0 e^{-\mu(t-t_1)}}{(\theta - \mu)} - \frac{D}{\theta} + \left(Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta}\right) e^{\theta(t_1-t)} \right] dt \\ & \quad + \int_{t_2}^T \left\{ -\frac{D}{\theta} + \left(Q_2 + \frac{D}{\theta}\right) e^{\theta(t_2-t)} \right\} dt \\ &= \frac{(k_0 - D)}{\theta} \left\{ t_1 + \frac{1}{\theta} (e^{-\theta t_1} - 1) \right\} - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(t_2-t_1)} - 1) - \frac{D}{\theta} (t_2 - t_1) \\ & \quad - \frac{1}{\theta} \left( Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta} \right) (e^{-\theta(t_2-t_1)} - 1) - \frac{D}{\theta} (T - t_2) - \frac{1}{\theta} \left( Q_2 + \frac{D}{\theta} \right) (e^{-\theta(T-t_2)} - 1) \end{aligned}$$

Therefore, holding cost of the manufacturer –

$$= h_m \left[ \frac{(k_0 - D)}{\theta} \left\{ t_1 + \frac{1}{\theta} (e^{-\theta t_1} - 1) \right\} - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(t_2 - t_1)} - 1) - \frac{D}{\theta} (t_2 - t_1) \right. \\ \left. - \frac{1}{\theta} \left( Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta} \right) (e^{-\theta(t_2 - t_1)} - 1) - \frac{D}{\theta} (T - t_2) - \frac{1}{\theta} \left( Q_2 + \frac{D}{\theta} \right) (e^{-\theta(T - t_2)} - 1) \right]$$

Deterioration cost –

$$= d_m \int_0^T \theta I(T) dt \\ = \theta d_m \left[ \frac{(k_0 - D)}{\theta} \left\{ t_1 + \frac{1}{\theta} (e^{-\theta t_1} - 1) \right\} - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(t_2 - t_1)} - 1) - \frac{D}{\theta} (t_2 - t_1) \right. \\ \left. - \frac{1}{\theta} \left( Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta} \right) (e^{-\theta(t_2 - t_1)} - 1) - \frac{D}{\theta} (T - t_2) - \frac{1}{\theta} \left( Q_2 + \frac{D}{\theta} \right) (e^{-\theta(T - t_2)} - 1) \right]$$

Therefore, the total cost of the manufacturer,

$$TC = \frac{1}{T} \left[ A_m + C_p DT + \beta k_0 \right. \\ \left. + (h_m + \theta d_m) \left\{ \frac{(k_0 - D)}{\theta} \left( t_1 + \frac{1}{\theta} (e^{-\theta t_1} - 1) \right) - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(t_2 - t_1)} - 1) - \frac{D}{\theta} (t_2 - t_1) \right. \right. \\ \left. \left. - \frac{1}{\theta} \left( Q_1 - \frac{k_0}{(\theta - \mu)} + \frac{D}{\theta} \right) (e^{-\theta(t_2 - t_1)} - 1) - \frac{D}{\theta} (T - t_2) - \frac{1}{\theta} \left( Q_2 + \frac{D}{\theta} \right) (e^{-\theta(T - t_2)} - 1) \right\} \right] \dots \dots \dots (6)$$

Putting the values of  $Q_1$  and  $Q_2$  from the equations (4) and (5) we have –

$$TC = \frac{1}{T} \left[ A_m + C_p DT + \beta k_0 \right. \\ \left. + (h_m + \theta d_m) \left\{ \frac{(k_0 - D)}{\theta} \left( t_1 + \frac{1}{\theta} (e^{-\theta t_1} - 1) \right) - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(t_2 - t_1)} - 1) - \frac{D}{\theta} (t_2 - t_1) \right. \right. \\ \left. \left. + \frac{1}{\theta^2} \left( \frac{k_0 \mu}{(\theta - \mu)} + (k_0 - D) e^{-\theta t_1} \right) (e^{-\theta(t_2 - t_1)} - 1) - \frac{D}{\theta} (T - t_2) \right. \right. \\ \left. \left. - \frac{D}{\theta^2} (1 - e^{\theta(T - t_2)}) \right\} \right] \dots \dots \dots (7)$$

Now, let  $t_1 = \gamma t_2$ , then the equation (7) can be rewritten as –

$$TC = \frac{1}{T} \left[ A_m + C_p DT + \beta k_0 \right. \\ \left. + (h_m + \theta d_m) \left\{ \frac{(k_0 - D)}{\theta} \left( \gamma t_2 + \frac{1}{\theta} (e^{-\theta \gamma t_2} - 1) \right) - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(1 - \gamma)t_2} - 1) - \frac{D}{\theta} (1 - \gamma)t_2 \right. \right. \\ \left. \left. + \frac{1}{\theta^2} \left( \frac{k_0 \mu}{(\theta - \mu)} + (k_0 - D) e^{-\theta \gamma t_2} \right) (e^{-\theta(1 - \gamma)t_2} - 1) - \frac{D}{\theta} (T - t_2) \right. \right. \\ \left. \left. - \frac{D}{\theta^2} (1 - e^{\theta(T - t_2)}) \right\} \right] \dots \dots \dots (7)$$

Now, we have considered that the demand rate  $D$  follows uniform distribution as  $D = E[f(x)] = \frac{(l+m)}{2}, l > 0, m > 0$  and  $l < m$ . Then the equation (6) can be rewritten as –

$$TC = \frac{1}{T} \left[ A_m + C_p \frac{(l + m)}{2} T + \beta k_0 \right. \\ \left. + (h_m + \theta d_m) \left\{ \frac{(2k_0 - (l + m))}{2\theta} \left( \gamma t_2 + \frac{1}{\theta} (e^{-\theta \gamma t_2} - 1) \right) - \frac{k_0}{\mu(\theta - \mu)} (e^{-\mu(1 - \gamma)t_2} - 1) \right. \right. \\ \left. \left. - \frac{1}{\theta} \frac{(l + m)}{2} (1 - \gamma)t_2 + \frac{1}{\theta^2} \left( \frac{k_0 \mu}{(\theta - \mu)} + \left( k_0 - \frac{(l + m)}{2} \right) e^{-\theta \gamma t_2} \right) (e^{-\theta(1 - \gamma)t_2} - 1) \right. \right. \\ \left. \left. - \frac{1}{\theta} \frac{(l + m)}{2} (T - t_2) - \frac{(l + m)}{2\theta^2} (1 - e^{\theta(T - t_2)}) \right\} \right] \dots \dots \dots (8)$$

Now, the optimum value of  $t_2, T, i.e. t_2^*, T^*$  which minimizes  $TC$  can be found from the necessary conditions  $\frac{\partial(TC)}{\partial t_2} = 0$  and  $\frac{\partial(TC)}{\partial T} = 0$ .

$$\frac{\partial(TC)}{\partial t_2} = 0 \text{ gives,}$$

$$\frac{k_0\gamma}{\theta} + \frac{k_0(1-\gamma)}{(\theta-\mu)}e^{-\mu(1-\gamma)t_2} - \frac{\gamma(2k_0-l-m)}{2\theta}e^{-\theta t_2} - \frac{(1-\gamma)}{\theta} \left\{ \frac{k_0\mu}{(\theta-\mu)} + \left( k_0 - \frac{(l+m)}{2} \right) e^{-\theta\gamma t_2} \right\} e^{-\theta(1-\gamma)t_2} - \frac{(l+m)}{2\theta}e^{\theta(T-t_2)} = 0 \dots \dots \dots (9)$$

$$\frac{\partial(TC)}{\partial T} = 0 \text{ gives,}$$

$$-\frac{1}{T^2} \left[ A_m + C_p \frac{(l+m)}{2} T + \beta k_0 + (h_m + \theta d_m) \left\{ \frac{(2k_0-(l+m))}{2\theta} \left( \gamma t_2 + \frac{1}{\theta} (e^{-\theta\gamma t_2} - 1) \right) - \frac{k_0}{\mu(\theta-\mu)} (e^{-\mu(1-\gamma)t_2} - 1) - \frac{1}{\theta} \frac{(l+m)}{2} (1-\gamma)t_2 + \frac{1}{\theta^2} \left( \frac{k_0\mu}{(\theta-\mu)} + \left( k_0 - \frac{(l+m)}{2} \right) e^{-\theta\gamma t_2} \right) (e^{-\theta(1-\gamma)t_2} - 1) - \frac{1}{\theta} \frac{(l+m)}{2} (T-t_2) - \frac{(l+m)}{2\theta^2} (1 - e^{\theta(T-t_2)}) \right\} \right] + \frac{1}{T} \left\{ C_p \frac{(l+m)}{2} + (h_m + \theta d_m) \frac{(l+m)}{2\theta} (e^{\theta(T-t_2)} - 1) \right\} = 0 \dots \dots \dots (10)$$

From equation (8) we can determine the optimum value of  $TC$ , say  $TC^*$ , provided that  $t_2^*, T^*$  satisfy the sufficient conditions  $\frac{\partial^2(TC)}{\partial t_2^2} > 0, \frac{\partial^2(TC)}{\partial T^2} > 0$  and  $\left( \frac{\partial^2(TC)}{\partial t_2^2} \cdot \frac{\partial^2(TC)}{\partial T^2} \right) - \left( \frac{\partial^2(TC)}{\partial t_2 \partial T} \right)^2 > 0$  at  $t_2 = t_2^*, T = T^*$ .

If the solutions obtained from (9) and (10) do not satisfy the sufficiency conditions, then we conclude that no feasible solution will be optimal for the set of parameters taken to solve (9) and (10). Such a situation will imply that the parameter values are inconsistent and there are some errors in their estimation.

IV. NUMERICAL EXAMPLES

**Example-1:** We have considered the following numerical values of the variables to illustrate the proposed model:

$k_0 = 140, A_m = 363, C_p = 15, \theta = 0.07, h_m = 3, \beta = 0.03, \mu = 0.007, \gamma = 0.6, d_m = 5, l = 55, m = 85.$

We have the optimum solution as  $t_2^* = 1.31, T^* = 2.50$  and  $TC^* = 1343.22$ . Figure-1 depicts the convexity of the cost function.

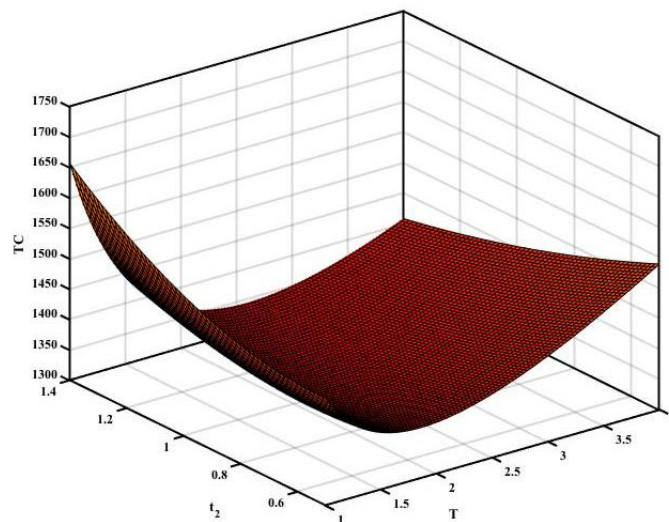


Figure-1: Total cost (TC) vs production time ( $t_2$ ) and cycle time (T).

**Example-2:** We have considered the following numerical values of the variables to illustrate the proposed model:

$$k_0 = 125, A_m = 355, C_p = 12, \theta = 0.06, h_m = 2, \beta = 0.02, \mu = 0.01, \gamma = 0.7, d_m = 4, l = 50, m = 80.$$

We have the optimum solution as  $t_2^* = 1.75, T^* = 3.21$  and  $TC^* = 1003.06$ .

### V. SENSITIVITY ANALYSIS

We have studied the effect of the changes in the values of the inventory parameters  $k_0, \theta, h_m$  and  $d_m$  on the optimum values. We have changed one parameter at a time keeping another parameter unchanged and summarized the optimum results in table-1 and the corresponding curves of the total cost are presented in figure-2, figure-3, figure-4 and figure-5 respectively.

Table-1: Sensitivity analysis for different parameters associated with the model (for example-1)

Changing parameters	Change in parameters	$Q_1$	$Q_2$	$t_2$	$T$	$TC$
$k_0$	140	53.51	87.53	1.31	2.50	1343.22
	150	55.29	90.61	1.18	2.42	1353.56
	160	56.81	93.24	1.08	2.35	1362.37
	170	58.13	95.52	0.99	2.29	1370.00
$\theta$	0.05	54.07	88.89	1.31	2.54	1338.90
	0.07	53.51	87.53	1.31	2.50	1343.22
	0.09	52.97	86.22	1.31	2.47	1347.45
	0.1	52.71	85.58	1.30	2.46	1349.53
$h_m$	2	64.11	104.48	1.58	3.00	1295.52
	2.5	58.10	94.88	1.42	2.72	1320.42
	3	53.51	87.53	1.31	2.50	1343.22
	3.5	49.85	81.66	1.22	2.34	1364.37
$d_m$	3	54.68	89.42	1.34	2.56	1337.02
	4	54.09	88.46	1.32	2.53	1340.14
	5	53.51	87.53	1.31	2.51	1343.22
	6	52.95	86.63	1.30	2.48	1346.28

It is observed from the table that –

- i) With the increase of the initial of the initial production rate ( $k_0$ ) the production runtime ( $t_2$ ) and the cycle time ( $T$ ) both decrease, but the total cost of the system ( $TC$ ) increases.

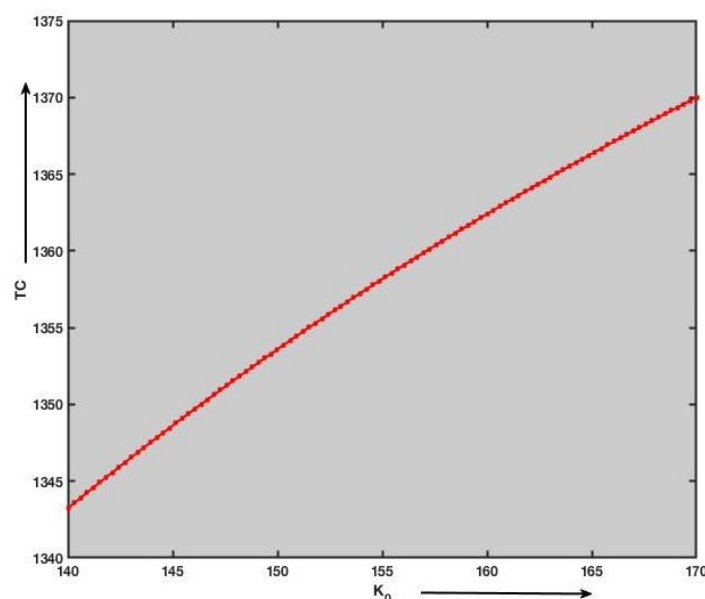


Figure-2: Initial production rate ( $k_0$ ) versus total cost( $TC$ ).

ii) As the rate of deterioration increases, the on-hand inventory level at time  $t_1$  and  $t_2$  both decrease, but the total cost of the system increases.

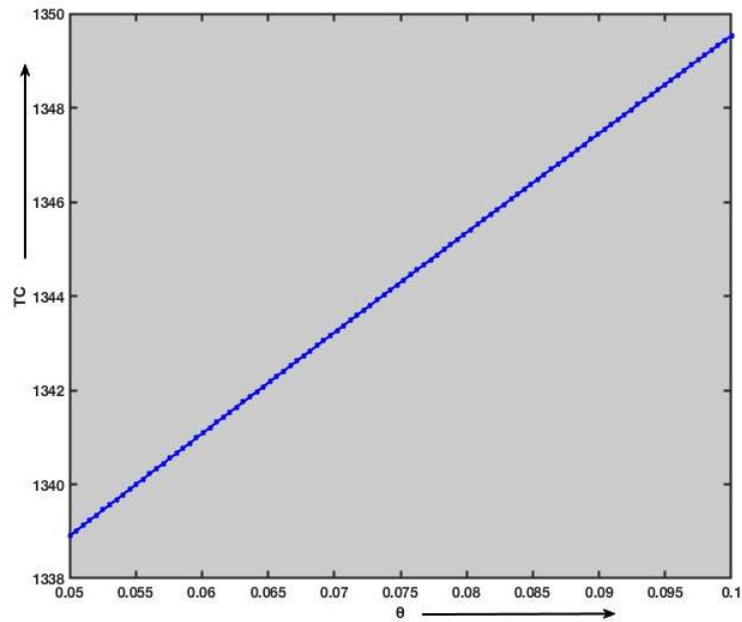


Figure-3: Rate of deterioration ( $\theta$ ) versus total cost (TC).

iii) The production runtime ( $t_2$ ) and the cycle time ( $T$ ) both decrease with the increase in the holding cost and deterioration cost, but the total cost of the system increases. Moreover,  $t_2$ ,  $T$  and  $TC$  are more sensitive to  $h_m$  than  $d_m$ .

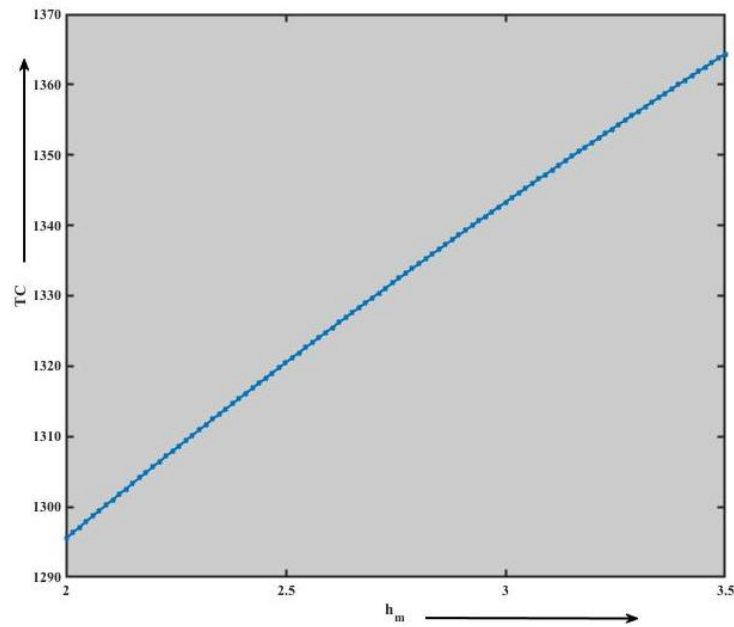


Figure-4: Holding cost ( $h_m$ ) versus total cost (TC).

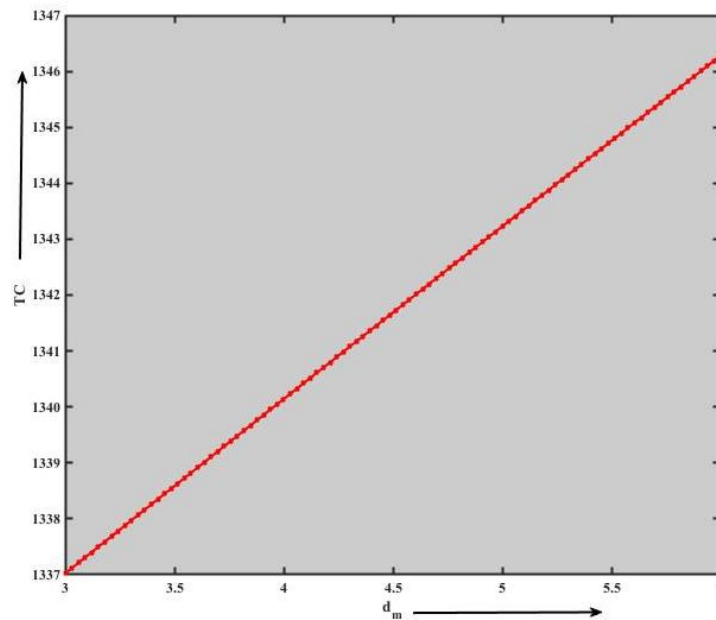


Figure-5: Deterioration cost ( $d_m$ ) versus total cost (TC).

## VI. CONCLUSIONS

This paper presents a generalized EPQ model for deteriorating items. We have considered variable production rate which decreases gradually with respect to time and this assumption makes our study close to reality as almost every manufacturing firms face such situation due to various causes like machinery fault, lethargy of the workers, delay in supply of raw materials etc. Furthermore, in this model we have assumed the demand rate as probabilistic which follows the uniform distribution. The sensitivity analysis is performed to study the effect of the parameters associated with the model.

The proposed model can be extended in several ways. Researchers can do more work on several types of deterioration rate and demand. Moreover, we can consider trade credit period offered by the supplier to the manufacturer or offered by the manufacturer to the retailer to extend the following model.

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