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Modified Radial Diffusivity Equation with Hydrostatic Gravity Correction: Laplace Transform Solution and Field Validation for Single-Phase Oil Reservoirs

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Abstract
In this study the effects of gravity on calculating the pressure drop in single-phase oil reservoirs are analyzed by revising the radial diffusivity equation (RDE). Gravitational effects are not considered in the traditional formulations for RDE, which would cause inaccuracies in analysis of pressure transients and prediction of reservoir performance.
The hydrostatic pressure term was included in Darcy's law and the RDE was modified to include it in order to obtain better accuracy of the pressure drop estimation especially in the deep wells where there is a strong influence of fluid segregation on the pressure drop distribution. The research method called for the developers to develop a nonlinear model of the RDE that would be able to capture variations in the density of the fluid as well as the effect of the hydrostatic pressure. The modified equation was solved using Laplace transformation. A suite of four case studies were used for validation of the model, covering tight shale reservoirs from the Eagle Ford and Bakken plays (USA) in addition to conventional sandstone reservoirs (Pudonghe bay, Alaska, and a field in the Niger Delta Basin, Nigeria) with depths ranging from about 2000m to 3500m.
The analysis is done by comparing classical RDE solution to modified RDE solution. The analysis indicates that the modified model has better prediction ability for pressure distribution, because of the gravitational effects. The ANOVA results show that the predictions of the modified model agree with the measured data at a 5% significance level, while the predictions of the classical model deviate significantly from the measured data. But the prediction accuracy cannot be quantified with ANOVA, so the RMSE analysis is used to assess model performance; and the modified model shows lower error value each time.
In the case of tight shale reservoirs (Eagle Ford and Bakken formations), the flow behavior is non-Darcy, but the single-phase RDE is used here as a first-order approximation in the presence of liquid-phase flow. Thus, the results obtained in tight formations should be considered as approximate.
Keywords
Radial Diffusivity Equation, Laplace Transformation, Analysis of Variance, Partial Differential Equation

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I. Introduction

Underground reservoir fluids including petroleum exist due to the important role gravity performs in shaping their distribution patterns and behavioural characteristics. The force of gravity causes the segregation of fluids which exist between rocks in the Earth's underground structure, producing petroleum reservoirs and determining the spatial distribution of fluids both horizontally and vertically. One of the most important equations in reservoir engineering derived by Muskat (1936) as an equation for fluid flow in porous media and adopted in petroleum engineering was developed under the assumption of negligible gravity effect. This assumption is also inherent in classical pressure transient analysis formulations based on the radial diffusivity equation, which are commonly solved using analytical techniques such as Laplace transformation (e.g., Weijermars and Afagwu, 2024). However, researches have indicated that the effect of gravity on reservoir fluids is not in fact negligible. According to Dake (1978), the flow direction together with rate from gravity-driven pressure gradients directly influences hydrocarbon recovery. Perm Inc (2016) studied the effect of gravity on macroscopic displacement efficiency during miscible displacement, explaining how gravity segregation affects fluid movement in inclined reservoirs.

Previous studies on gravity effects in reservoirs have largely focused on multiphase processes such as gravity drainage (Tang et al., 2019; Alipour & Madani, 2023), where density differences drive fluid segregation. In contrast, the present study addresses single-phase flow, where gravity influences pressure distribution solely through a hydrostatic term. This distinction limits the direct applicability of gravity drainage models to the current formulation. Mancinelli (2021) carried out an extensive investigation of fluid storage and withdrawal gravity effects through research on the Volve field located in the North Sea. The research used 3D forward gravity modeling to analyze how gravity indications relate to post-hydrocarbon operations like water flooding. It is however important to note that Gravity drainage involves multiphase flow and fluid segregation driven by density differences, whereas the present work considers single-phase flow, in which gravity influences pressure distribution solely through a hydrostatic term without inducing phase movement. The radial diffusivity equation is widely used to model pressure behaviour in slightly compressible reservoirs under radial flow conditions. In well test analysis, pressure measurements are commonly adjusted to a reference depth through hydrostatic (datum) correction to account for the weight of the fluid column. This practice is particularly important in deep wells, where vertical pressure variation can be significant (Raghavan, 1993; Bourdet, 2002). These corrections are routinely applied during pressure buildup and drawdown interpretation, reflecting the recognized influence of gravity on measured pressure data. Studies have shown that failure to account for such effects can introduce substantial errors in well test interpretation (McLean and Zarrouk, 2015). However, such adjustments are typically applied externally to the measured data rather than being incorporated directly into the governing radial diffusivity equation. Recent studies have also incorporated gravity effects within porous media flow formulations, either explicitly or implicitly through body force terms. For example, Ababou et al. (2025) and Berre et al. (2020) include gravity in single-phase flow equations, while more recent computational approaches (Ballini et al., 2024) retain gravity within reduced-order modeling frameworks. Additionally, gravity-driven flow behavior has been extensively analyzed in porous systems (Kmec and Šír, 2024), although often in multiphase or unsaturated contexts. However, limited attention has been given to the explicit incorporation of hydrostatic gravity correction in radial diffusivity equation (RDE)-based pressure models, which is the focus of the present study. Traditional formulations typically neglect gravitational effects. Although various modifications of the equation have been proposed, limited analytical work exists on explicitly incorporating gravity into the radial diffusivity equation. This study therefore addresses this gap by deriving a modified radial diffusivity equation that incorporates gravity and obtaining a closed-form analytical solution using Laplace transformation. The resulting formulation is then applied to estimate pressure drop between the reservoir and wellbore and validated using reservoir data from selected oil fields.

The radial diffusivity equation is a primary equation in reservoir engineering used to describe the flow of fluids through porous media, derived from Darcy's law, the equation of state and the principle of conservation of mass. This equation is critical in determining key reservoir parameters such as reservoir initial pressure, permeability, productivity index, and flow efficiency, aiding the understanding of reservoir behaviour. However, the equation assumes the following:

- Darcy's law application
- Rock qualities that are independent of pressure
- Negligible gravity impact
- Assumption of radial flow
- Small pressure gradient
- Single phase reservoir

Combining the conservation of mass with Darcy's law for the isothermal flow of fluids of small and constant compressibility, a partial differential equation is found that in field units reduces to

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\emptyset \mu c}{0.000264k}\frac{\partial p}{\partial t} \tag{1}$$

II. Method

To address the theoretical gap identified in literature, a set of objectives were identified with appropriate method applied in the achievement of those set objectives leading to the development of a new radial diffusivity equation (RDE.

1. To derive a modified form of the radial diffusivity equation that incorporates gravitational (hydrostatic) effects on subsurface fluid flow.
2. To obtain a closed-form analytical solution to the modified RDE using Laplace transformation.
3. To validate the modified RDE solution against measured pressure drop data from four field case studies.
4. To statistically compare the performance of the modified model with the classical RDE solution using ANOVA and RMSE metrics.

2.1 Derivation of the Modified Radial Diffusivity Equation

The radial diffusivity equation is derived from Darcy's law, equation of state for slightly compressible fluids and the principle of conservation of mass. However, to account for the effect of gravity on subsurface fluid, we modify Darcy's law. To modify the existing Darcy's law to account for the effect of gravity on subsurface fluid, we introduce hydrostatic pressure into the pressure term in Darcy's law to account for the resultant pressure on the fluid. Hydrostatic pressure is the pressure acting on a fluid under gravity in a downward negative direction. It is given by:

$$\text{Hydrostatic Pressure} = \rho g h \tag{2}$$

Where ρ the density of the fluid, g is is the gravity term and h is the height of the fluid column from the surface to the point of interest. The pressure term p is the force exerted on the fluid due to stresses present in subsurface reservoir rocks and other subsurface forces, causing the fluid to move in an upward direction towards the surface in a positive direction. The resultant pressure P acting on the fluid is given by:

$$P = p - \rho g h \tag{3}$$

The existing Darcy's law is given by:

$$Q = \frac{kA}{\mu}\frac{dp}{dl} \tag{4}$$

Modifying the existing Darcy's law to account for the effect of gravity by replacing the pressure term with the resultant pressure term, we have:

$$Q = \frac{kA}{\mu}\frac{d(p - \rho g h)}{dl} \tag{5}$$

The equation of state for slightly compressible fluids is given by:

$$\rho = \rho_i e^{c(p - p_i)} \tag{6}$$

The term c is the isothermal compressibility and it is given by:

$$c = -\frac{1}{v}\frac{dv}{dp} = \frac{1}{\rho}\frac{d\rho}{dp} \tag{7}$$

The law of conservation of mass states that in a closed system, mass cannot be created nor destroyed but can change from one form to another.

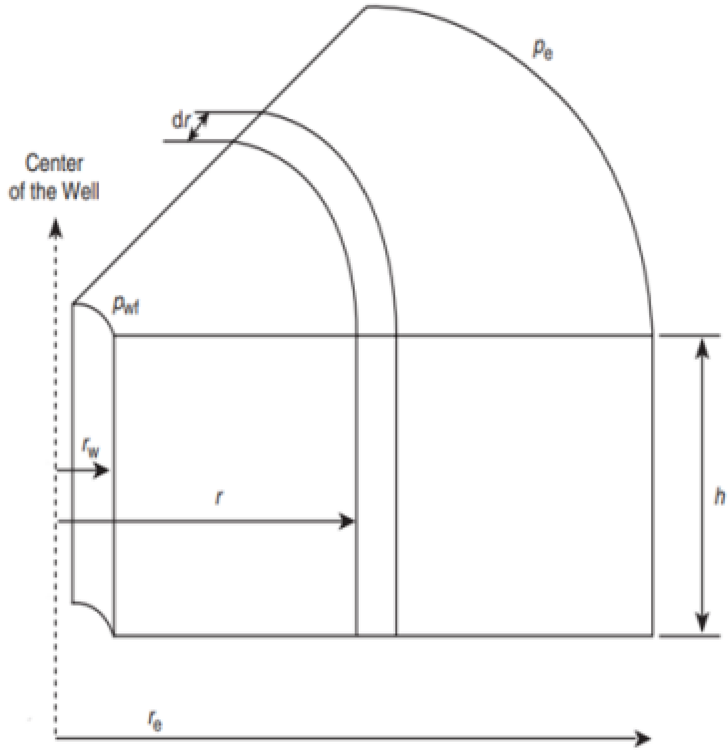

Figure 1: Cross section of radial flow in a reservoir (Ahmed, 2010)

From Figure 1, applying the law of conservation of mass on fluid flow in radial direction, we have:

$$(\rho Q)_{r+dr} - (\rho Q)_r = \emptyset \frac{d\rho}{dt} dv \quad (8)$$

Subtracting, we have the continuity equation given by:

$$d(\rho Q) = \emptyset \frac{d\rho}{dt} dv \quad (9)$$

Substituting q from equation (5) above, we have:

$$d\left(\rho \frac{kA}{\mu} \frac{d(p-\rho gh)}{dr}\right) = \emptyset \frac{d\rho}{dt} dv \quad (10)$$

But *dv* can be written as *A.dr* and *A* = $2\pi rh$. Substituting these into the equation, we have:

$$d\left(\rho 2\pi rh \frac{kd(p-\rho gh)}{\mu dr}\right) = \emptyset \frac{d\rho}{dt} 2\pi rhdr \quad (11)$$

Dividing both side by dr and $\frac{k}{\mu}$, we have:

$$\frac{d}{dr}\left(\rho 2\pi rh \left(\frac{d(p-\rho gh)}{dr}\right)\right) = \frac{\emptyset\mu}{k}\frac{d\rho}{dt} 2\pi rh \quad (12)$$

Dividing by $2\pi rh$, we have:

$$\frac{1}{r}\frac{d}{dr}\left(\rho r \left(\frac{d(p-\rho gh)}{dr}\right)\right) = \frac{\emptyset\mu}{k}\frac{d\rho}{dt} \quad (13)$$

Introducing the term $\frac{dp}{dp}$ to the RHS, we have:

$$\frac{1}{r}\frac{d}{dr}\left(\rho r \left(\frac{d(p-\rho gh)}{dr}\right)\right) = \frac{\emptyset\mu}{k}\frac{d\rho}{dt}\frac{dp}{dp} \quad (14)$$

Rearranging the RHS, we have:

$$\frac{1}{r}\frac{d}{dr}\left(\rho r \left(\frac{d(p-\rho gh)}{dr}\right)\right) = \frac{\emptyset\mu}{k}\frac{d\rho}{dp}\frac{dp}{dt} \quad (15)$$

Dividing both sides by ρ, we have:

$$\frac{1}{r}\frac{d}{dr}\left(r \frac{d(p-\rho gh)}{dr}\right) = \frac{\emptyset\mu}{k}\frac{1}{\rho}\frac{d\rho}{dp}\frac{dp}{dt} \quad (16)$$

But from equation (7), $\frac{1}{\rho}\frac{d\rho}{dp}$ = c. Therefore, the modified radial diffusivity equation is given by:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d(p-\rho gh)}{dr}\right) = \frac{\emptyset\mu c}{k}\frac{dp}{dt} \tag{17}$$

Applying chain rule to the LHS and expanding, we have:

$$\frac{1}{r}\left(r\frac{d^2(p-\rho gh)}{dr^2} + \frac{d(p-\rho gh)}{dr}\right) = \frac{\emptyset\mu c}{k}\frac{dp}{dt} \tag{18}$$

Multiplying by $\frac{1}{r}$, we have:

$$\frac{d^2(p-\rho gh)}{dr^2} + \frac{1}{r}\frac{d(p-\rho gh)}{dr} = \frac{\emptyset\mu c}{k}\frac{dp}{dt} \tag{19}$$

2.2 Steps in the solution of a PDE by means of an integral transform

STEP 1: Let u(x, t) depend on two variables x, t, assuming it is a solution to a partial differential equation. Then, by applying an appropriate integral transform concerning one of its independent variables, we can transform u(x, t); for example, if the transformation is applied with respect to x, the new variable U(α, t) is formed, where α denotes the transformation variable. In cases where a Laplace transform is suitable, the transformation variable α will be represented by s. Subsequently, rearrange the result to derive an ordinary differential equation for the transformed variable U(α, t), where t serves as the sole independent variable and α acts as a parameter.

STEP 2: Determine the general solution of the ordinary differential equation for U(α, t) as a function of t, while still keeping the transform variable α as a parameter in the solution. Additionally, utilize the boundary and/or initial conditions from the original problem to find the specific form of the transform U(α, t).

STEP 3: To find the required solution u(x,t), you need to invert the transform U(α,t). Typically, the appropriate inversion integral is used to reverse U(α,t), although in more straightforward cases, the inversion can be performed using a table of transform pairs.

The region where a solution is required, along with the initial and boundary conditions of the original problem, influences the choice of transform to apply and the independent variable in u(x, t) that guides the essential steps to be taken when addressing a PDE through an integral transform.

2.3 Assumptions

1. The reservoir contains a single-phase, slightly compressible fluid, and multiphase effects are neglected.
2. The reservoir is homogeneous and isotropic.
3. Flow is radial toward the wellbore.
4. Fluid and rock compressibility are small and constant, consistent with slightly compressible flow conditions.
5. The reservoir initially has a uniform pressure distribution, such that $P(r, 0) = P_i$
6. The reservoir is infinite-acting, and pressure remains finite as radial distance approaches infinity.
7. The density of the reservoir fluid is constant.

2.4 Application of Laplace Transform to Solving the Modified Radial Diffusivity Equation for Slightly Compressible Fluids

$$\frac{\partial^2(p-\rho gh)}{\partial r^2} + \frac{1}{r}\frac{\partial(p-\rho gh)}{\partial r} = \frac{\emptyset\mu c}{k}\frac{\partial p}{\partial t} \tag{20}$$

Let us denote $F(r, t) = p(r, t) - \rho gh$. Assuming a constant density and differentiating F(r, t), we have:

$$\frac{\partial F}{\partial t} = \frac{\partial p}{\partial t}$$

Substituting *F(r, t),* the equation becomes:

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} = \frac{\emptyset\mu c}{k}\frac{\partial F}{\partial t} \tag{21}$$

Solving this using Laplace transform, let $L\{F(r, t)\} = F(r, s)$, where s is the Laplace parameter. The Laplace transform of the time derivative is:

$$L\left\{\frac{\partial F}{\partial t}\right\} = sF(r,s) - F(r,0) \tag{22}$$

Where F(r, 0) is the initial condition. Applying Laplace transform to the equation, we have:

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} = \frac{\emptyset\mu c}{k}\left(sF - F(r,0)\right) \tag{23}$$

This is now a second order partial differential equation in r. To solve this equation, we introduce boundary and initial conditions, solve the transformed equation for F(r, s) and finally apply the inverse Laplace transform to obtain F(r, t). Let $F(r,0) = F_0(r)$ be the initial condition and let$\alpha^2 = \frac{\emptyset\mu c}{k}$, the equation becomes:

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} - \alpha^2 sF = -\alpha^2 F_0(r) \quad (24)$$

This is a modified Bessel Equation of the form:

$$r^2\frac{\partial^2 F_h}{\partial r^2} + r\frac{\partial F_h}{\partial r} - (v^2 + r^2k^2)F_h = 0 \quad (25)$$

Where v = 0 and k = $\sqrt{\alpha^2 s}$. The solution to this form of the equation is a modified Bessel function where $I_0(x)$ is a modified Bessel function of the first kind (finite as r → 0) and $K_0(x)$ is a modified Bessel function of the second kind (diverges logarithmically as r → 0). Thus, the solution to the homogenous equation is given by:

$$F_h(r,s) = AI_0\left(\sqrt{\alpha^2 s}\, r\right) + CK_0\left(\sqrt{\alpha^2 s}\, r\right) \quad (26)$$

Where A and C are constants determined by boundary conditions. The inhomogeneous term $-\propto^2 F(r,0)$ assume a constant particular solution:

$$F_p(r,s) = \frac{F(r,0)}{s} \quad (27)$$

The particular solution Fp is obtained under the assumption of a spatially uniform initial condition. This implies that the reservoir pressure is constant throughout the domain at t=0, which is a standard assumption in pressure transient analysis, as reservoirs are typically in equilibrium prior to production.

The general solution is given as the sum of the homogenous solution and the particular solution expressed as:

$$F(r,s) = AI_0\left(\sqrt{\alpha^2 s}\, r\right) + CK_0\left(\sqrt{\alpha^2 s}\, r\right) + \frac{F(r,0)}{s} \quad (28)$$

Applying the following boundary condition to determine A and C:

1. At $r = r_w$, which is the well bore condition, the pressure gives only one equation involving A and C.
2. At $r \to \infty$, representing an infinite reservoir, to ensure that the pressure remains finite, A = 0, i.e $I_0\left(\sqrt{\alpha^2 s}\, r\right)$ is 0.

Therefore, the equation reduces to:

$$F(r,s) = CK_0\left(\sqrt{\alpha^2 s}\, r\right) + \frac{F(r,0)}{s} \quad (29)$$

Taking the inverse Laplace transform of the equation, we have:

$$F(r,t) = L^{-1}\left\{CK_0\left(\sqrt{\alpha^2 s}\, r\right) + \frac{F(r,0)}{s}\right\} \quad (30)$$

The term $\frac{F(r,0)}{s}$is correspondent to the initial condition and the inverse Laplace transform of $K_0\left(\sqrt{\alpha^2 s}\, r\right)$ gives the transient response. The full solution in time domain is given by:

$$F(r,t) = F(r,0) + \int_0^t G(r,t)dt \quad (31)$$

Where *G(r, t)* is referred to as Green's function and is derived from the transient term involving K_0. The function *G(r, t)* is the inverse Laplace transform of $K_0\left(\sqrt{\alpha^2 s}\, r\right)$, scaled by C. To derive *G(r, t)*, we need to solve the inverse Laplace transform of the homogenous equation explicitly:

$$F_h(r,s) = CK_0\left(\sqrt{\alpha^2 s}\, r\right) \quad (32)$$

$$G(r,t) = L^{-1}\left\{K_0\left(\sqrt{\alpha^2 s}\, r\right)\right\} \quad (33)$$

The inverse Laplace transform of the function can be found in the table of inverse of Laplace transform for special functions and its given by:

$$L^{-1}\left\{K_0\left(\sqrt{\alpha^2 s}\, r\right)\right\} = \frac{1}{2\alpha^2 t}\exp\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (34)$$

Where r is the radial distance and $\alpha^2 = \frac{\emptyset\mu c}{k}$. Substituting back into *G(r, t)*, we have:

$$G(r,t) = \frac{C}{2\alpha^2 t}\exp\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (35)$$

Determining C from boundary condition of a point source well producing at a constant rate qB, the total mass flow is given by:

$$-2\pi r\frac{\partial F}{\partial r}_{r=0} = qB \quad (36)$$

Substituting the homogenous equation, we have:

$$\frac{\partial F}{\partial r} = -C\sqrt{\alpha^2 s}K_1\left(\sqrt{\alpha^2 s}\, r\right) \quad (37)$$

At r = 0, we will use the asymptotic property of the Bessel function K_1 where $K_1(x) \sim \frac{1}{x}$ as x → 0. Therefore, we have:

$$\frac{\partial F}{\partial r}_{r=0} = -\frac{C\sqrt{\alpha^2 s}}{\sqrt{\alpha^2 s}r} = -\frac{C}{r} \quad (38)$$

Considering flow rate condition, we have:

$$qB = -2\pi rh \cdot \frac{k}{\mu} \cdot \frac{\partial F}{\partial r}_{r=0} \quad (39)$$

Substituting $-\frac{C}{r}$ for$\frac{\partial F}{\partial r}$, we have:

$$qB = 2\pi rh \frac{k}{\mu}\frac{C}{r} (40)$$

$$C = \frac{qB\mu}{2\pi kh} \quad (41)$$

Substituting C into *G(r, t)*, we have:

$$G(r,t) = \frac{qB\mu}{4\pi kh\alpha^2 t} \exp\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (42)$$

Substituting the value of G(r, t) into F(r, t) and assuming that F(r, 0) is F_i, we have:

$$F(r,t) = F_i + \int_0^t \frac{qB\mu}{4\pi kh\alpha^2 t} \exp\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (43)$$

Simplifying the integral, we have:

$$F(r,t) = F_i + \frac{qB\mu}{4\pi kh\alpha^2} \int_0^t \frac{1}{t} \exp\left(-\frac{r^2}{4\alpha^2 t}\right) dt \quad (44)$$

Let u = $\frac{r^2}{4\alpha^2 t}$and t = $\frac{r^2}{4\alpha^2 u}$and dt = t = $\frac{r^2}{4\alpha^2 u^2} du$. Substituting into the integral, we have:

$$\int_0^t \frac{1}{t} \exp\left(-\frac{r^2}{4\alpha^2 t}\right) dt = \int_\infty^{\frac{r^2}{4\alpha^2 t}} \frac{1}{\frac{r^2}{4\alpha^2 u}} \exp(-u)\left(-\frac{r^2}{4\alpha^2 u^2}\right) du \quad (45)$$

Simplifying, we have:

$$\int_\infty^{\frac{r^2}{4\alpha^2 t}} \frac{4\alpha^2 u}{r^2} \cdot \frac{r^2}{4\alpha^2 u^2} \exp(-u) du = \int_\infty^{\frac{r^2}{4\alpha^2 t}} \frac{\exp(-u)}{u} du \quad (46)$$

Therefore, the exponential integral function is given by:

$$\int_0^t \frac{1}{t} \exp\left(-\frac{r^2}{4\alpha^2 t}\right) dt = E_i\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (47)$$

Substituting back into *F(r, t)*, we have:

$$F(r,t) = F_i + \frac{qB\mu}{4\pi kh} E_i\left(-\frac{r^2}{4\alpha^2 t}\right) \quad (48)$$

Substituting $\alpha^2 = \frac{\emptyset\mu c}{k}$ and converting into practical field unit, we have:

$$F(r,t) = F_i - 70.6\frac{qB\mu}{kh} E_i\left(-\frac{948\emptyset\mu c r^2}{kt}\right) + \rho gh \quad (49)$$

III. Results and Discussion

After obtaining the modified radial diffusivity Eq. (49), Laplace transformation was used to obtain a solution to the radial diffusivity equation. The modified radial diffusivity equation solution was thus used to estimate the pressure drop from the reservoir to the wellbore. The pressure drop from the estimate of the modified radial diffusivity equation solution is compared with the pressure drop estimate of the classical solution of the radial diffusivity equation, and a statistical test, the analysis of variance (ANOVA), is used to determine if there is a statistical significant difference between the developed and classical model pressure estimate and the actual measured pressure drop as obtained from selected oil field production and characterization data. The analysis of variance tests two hypotheses: Null hypothesis (There is no significant difference between the measured pressure drop and the classical model) and Alternative hypothesis (There is significant difference between the measured pressure drop and the classical model) at 5% level of significance.

3.1 Classical and New RDE Model Application

The classical radial diffusivity equation (RDE) is derived under the following conditions: Darcy flow, single-phase, slightly compressible, and a homogeneous medium. Permeabilities of tight shale reservoirs can range from nano- to microdarcies, and a tight shale reservoir may have non-Darcy effects (such as gas slippage/Klinkenberg effect and threshold pressure gradients) or multiphase flow. The model of the RDE is adopted as a first-order model, so that the effect of the proposed gravity (hydrostatic) correction on the pressure behavior is isolated. The analysis, therefore, assumes single-phase flow for the time period analyzed, and conditions of Darcy domination.

3.2 Case Study 1

The reservoir under investigation is the Eagleville Ford reservoir, which is classed as an ultra-tight shale reservoir with a permeability of microdarcy to nanodarcy, reservoir depths of >3000 m and high initial pressure. Data obtained from production and reservoir characterization in Table 1 was used in estimating pressure drop across the reservoir using the solution to the modified radial diffusivity equation and the classical solution to the radial diffusivity equation and pressure distribution plot results is obtained Figure 2.

Table 1: Eagleville Ford reservoir properties

Parameter	**Eagleville Bay**
Reservoir type	Tight shale formation
Permeability	10 – 500 nD
Porosity	6 – 12%
Depth	2500 – 3000 m
Initial pressure	4000 – 4500 psi
Fluid type	Oil /gas condensate
Flow assumption	Single-phase, radial

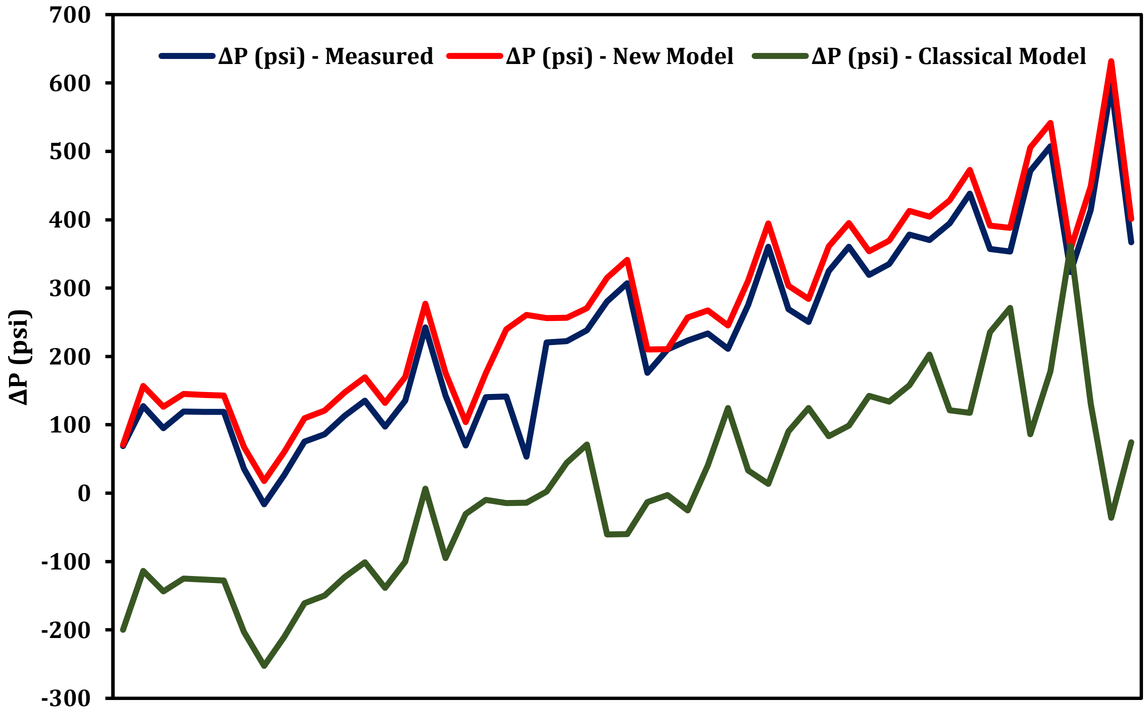

Figure 2: Case 1 - Eagleville Ford reservoir pressure distribution curves from measured, and estimation from new and classical models

The ANOVA results reveal an F-value of 1.828, which is below the critical F value of 3.936, and with a p-value of 0.179.At a 5% level of significance the F-value is below the F-critical and the p-value is above 5%, and so, at this level of significance, the null hypothesis, H_0, is accepted. This means that the difference between the pressure drop measured and the developed model is not statistically significant. The F value obtained in the ANOVA is 70.653, which is greater than the F critical value 3.936, with a corresponding p value of 3.03x 10-13.Since the F value is higher than the F value for 5%, and the p value is lower than 5%, we reject the null hypothesis (H_0). This means that the difference between the measured pressure drop and the classical model are statistically different.
The deviation of the developed model from the classical model is calculated using the RMSE method as in Eq. (50) and compared with the RMSE of the classical model and the actual measured pressure drop.

$$Root\ mean\ squared\ error\ (RMSE)\ =\ \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y_i})^2} \tag{50}$$

The developed model gives a root mean squared error of 45.35 compared to the actual measured pressure drop, whereas the classical model yields a root mean squared error of 733.92 compared to the actual measured pressure drop. Therefore, the developed model is closer to the actual measured pressure drop than the classical model, as the RMSE for the developed model's comparison is lesser than the RMSE for the classical model's comparison.

3.3 Case Study 2

Pressure data for Bakken oil field used in this study presented in Table 2 correspond to drawdown behavior over a limited production period, with radial flow conditions assumed for modeling purposes. The reservoir fluid which is predominantly light oil, with a single-phase under the conditions considered. Data obtained from production and reservoir characterization was used in estimating pressure drop across the reservoir using the solution to the modified radial diffusivity equation and the classical solution to the radial diffusivity equation and the pressure distribution result is shown in Figure 3.

Table 2: Bakken oil field reservoir properties

Parameter	**Bakken Formation**
Reservoir type	Tight shale (unconventional)
Permeability	0.1 – 1.0 µD
Porosity	8 – 12%
Depth	2500 – 3200 m
Initial pressure	3500 – 4500 psi
Fluid type	Light oil
Flow assumption	Single-phase (model approximation), radial

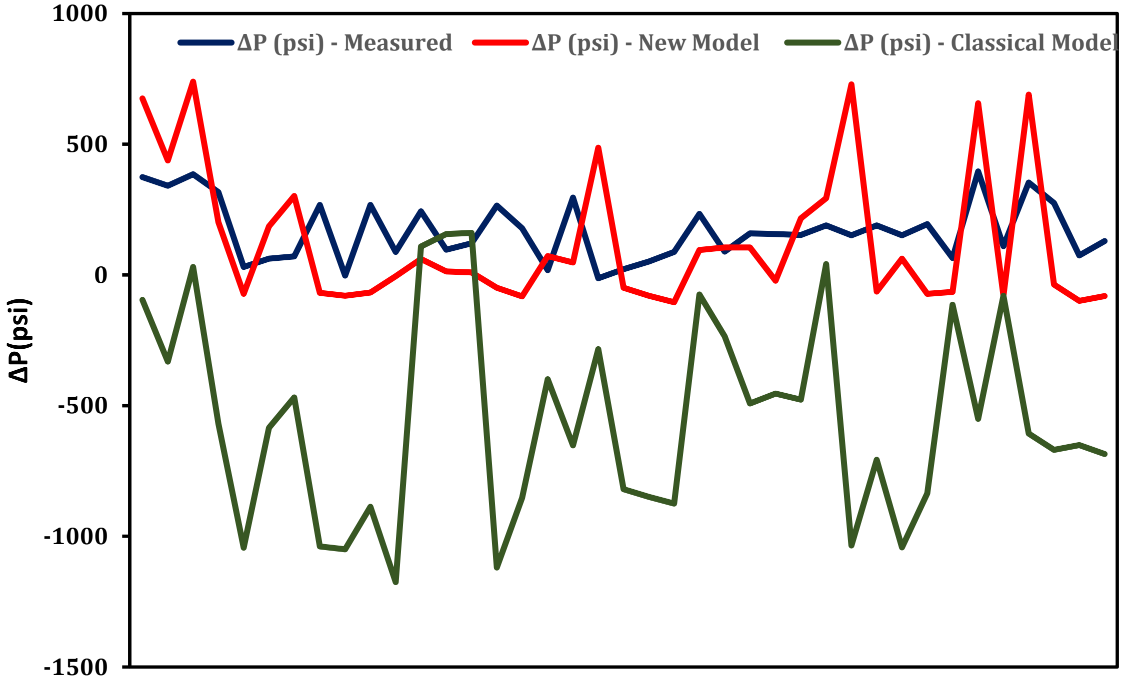

Figure 3: Bakken oil field reservoir pressure distribution curves from measured, and estimation from new and classical models

The ANOVA results indicate an F-value of 167.989, which is higher than the critical F-value of 3.934, with a corresponding p-value of 2.74 x 10^{-23}. Using a significance level of 5%, since the F-value is greater than F-critical and the p-value is less than 5%, we reject the null hypothesis, H_0. This indicates that there is statistically significant difference between the measured pressure drop and the classical model. The ANOVA results indicate an F-value of 2.435, which is lower than the critical F-value of 3.934, with a corresponding p-value of 0.122. Using a significance level of 5%, since the F-value is less than F-critical and the p-value is greater than 5%, we fail to reject the null hypothesis, H_0. This indicates that there is no statistically significant difference between the measured pressure drop and the developed model.

The root mean squared error of the classical model compared to the actual measured pressure drop is 780.66 while the root mean squared error of the developed model compared to the actual measured pressure drop is 247.76. Since the RMSE for the developed model comparison is less than the RMSE for the classical model comparison, therefore, the developed model is closer to the actual measured pressure drop and more accurate than the classical model.

3.4 Case study 3

The Prudhoe Bay oil field reservoir property is presented in Table 3. The reservoir fluid consists of undersaturated crude oil with associated gas, and for the purpose of this study, flow is approximated as single-phase slightly compressible liquid. The table below contains a summary of the reservoir properties. Data obtained from production and reservoir characterization was used in estimating pressure drop across the reservoir using the solution to the modified radial diffusivity equation and the classical solution to the radial diffusivity equation and the pressure distribution plot is presented in Figure 4.

Table 3: Prudhoe Bay Oil Field reservoir properties

Parameter	Prudhoe Bay
Reservoir type	Conventional oil
Permeability	100 – 1000 mD
Porosity	20 – 30%
Depth	2500 – 3000 m
Initial pressure	4000 – 4500 psi

Parameter	Prudhoe Bay
Fluid type	Oil (slightly compressible)
Flow assumption	Single-phase, radial

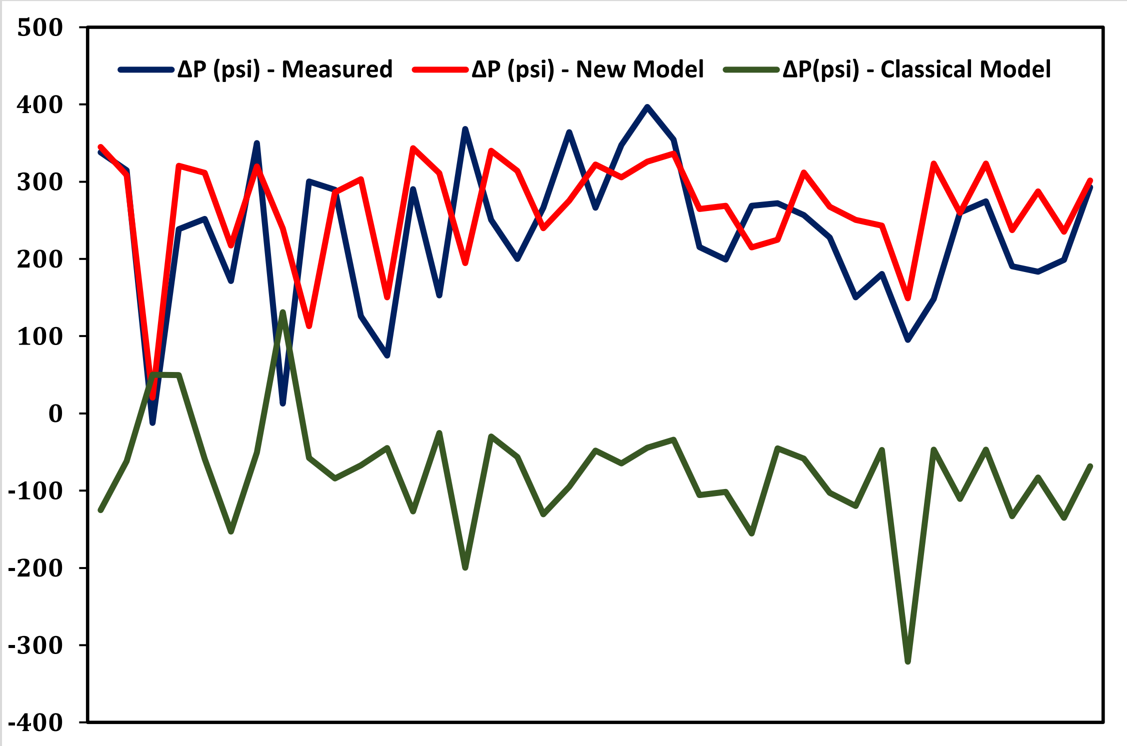

Figure 4: Prudhoe Bay Oil Field reservoir pressure distribution curves from measured, and estimation from new and classical models

The ANOVA results indicate an F-value of 393.612, which is higher than the critical F-value of 3.934, with a corresponding p-value of 8.55 x 10^{-37}. Using a significance level of 5%, since the F-value is greater than F-critical and the p-value is less than 5%, we reject the null hypothesis, H_0. This indicates that there is statistically significant difference between the measured pressure drop and the classical model.

In determining the accuracy of the developed models, we estimate the root mean squared error of the two models in comparison to the actual measured pressure drop. The root mean squared error of the classical model compared to the actual measured pressure drop is 337.56 while the root mean squared error of the developed model compared to the actual measured pressure drop is 86.31.

3.5 Case study 4

The reservoir considered is representative of a Niger Delta oil field, characterized by moderate to high permeability in the range of 100–300 mD and a net pay thickness of approximately 30–40 ft as summarized in Table 4. The formation exhibits porosity values between 20% and 30%, indicating good storage capacity. The table below contains a summary of the reservoir properties.

Table 4: Niger Delta reservoir properties

Permeability (mD)	Height (ft)	Oil FVF (rb/stb)	Viscosity (cp)	Porosity (%)	Density (lbm/ft^3)
100-300	30-40	1.2-1.6	0.9-1	20-30	52-57

Fluid properties show an oil formation volume factor ranging from 1.2 to 1.6 rb/stb and a viscosity between 0.9 and 1.0 cp, suggesting relatively mobile crude. The fluid density lies within the range of 52–57 lbm/ft³, consistent with light to moderately dense oil systems typical of the region. Data obtained from production and reservoir characterization was used in estimating pressure drop across the reservoir using the solution to the modified radial diffusivity equation and the classical solution to the radial diffusivity equation and the result of pressure drop distribution is presented in Figure 5.

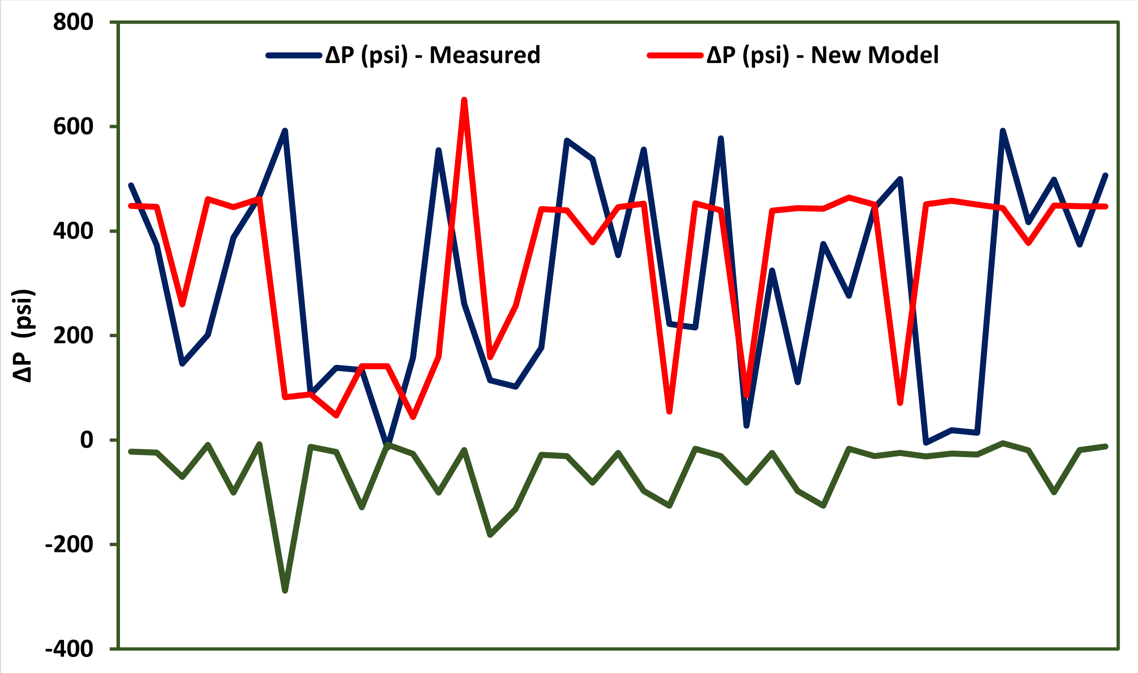

Figure 5: Niger Delta oil field reservoir pressure distribution curves from measured, and estimation from new and classical models

The ANOVA results indicate an F-value of 120.791, which is higher than the critical F-value of 3.967, with a corresponding p-value of 2.3×10^{-17}. Using a significance level of 5%, since the F-value is greater than F-critical and the p-value is less than 5%, we reject the null hypothesis, H_0. This indicates that there is statistically significant difference between the measured pressure drop and the classical model.

Determining the accuracy of the developed models, we estimate the root mean squared error of the two models in comparison to the actual measured pressure drop. The root mean squared error of the classical model compared to the actual measured pressure drop is 417.41 while the root mean squared error of the developed model compared to the actual measured pressure drop is 223.33. Since the RMSE for the developed model comparison is less than the RMSE for the classical model comparison, therefore, the developed model is closer to the actual measured pressure drop and more accurate than the classical model.

It is observed that the classical radial diffusivity equation yields negative pressure drop values in some cases, which is physically inconsistent for a production scenario where pressure is expected to decrease from the reservoir toward the wellbore. This behavior arises from the simplifying assumptions of the classical formulation, which neglects gravity and considers only pressure-driven flow. Furthermore, the mathematical structure of the solution and the sign convention adopted for pressure drop may contribute to this anomaly when applied to real reservoir conditions. In deep reservoirs, where hydrostatic pressure contributions are significant, neglecting gravity can lead to an incomplete representation of the pressure field.

IV. Conclusion and Recommendation

4.1 Conclusion

In this work, the effect of gravity on reservoir fluids and in reservoir pressure determination was studied by modifying the classical radial diffusivity equation, an important equation for estimating reservoir, well bore flowing pressure and pressure drop across the reservoir to the well bore. The radial diffusivity was modified by incorporating gravity into Darcy's law, an important equation governing the flow of fluids.

The modified radial diffusivity equation was solved using Laplace transformation while making some assumptions and the solution to the modified radial diffusivity equation was used in estimating pressure drop across the reservoir to the well bore for some selected oil fields. Furthermore, the solution of the modified radial diffusivity equation was compared to the solution obtained by using the classical radial diffusivity equation to test for the accuracy of the developed model. The results demonstrate that the inclusion of gravity effects improves pressure drop estimation across the reservoir and leads to more accurate reservoir pressure predictions.
In summary, the significance of gravity correction depends primarily on reservoir depth and fluid density. The hydrostatic term (ρgh) increases linearly with depth and becomes comparable to pressure drawdown in deep reservoirs. The impact of gravity correction varies across the case studies due to differences in reservoir depth, fluid density, and permeability. Deeper reservoirs exhibit larger hydrostatic contributions, leading to more pronounced corrections. In contrast, shallower reservoirs show relatively smaller gravity effects. Additionally, lower permeability systems tend to exhibit smaller pressure gradients from flow, making the relative contribution of the gravity term more significant.

4.2 Recommendation

1. The model assumes single-phase flow, whereas real reservoirs particularly unconventional systems such as the Eagle Ford Formation and Bakken Formation often involve multiphase behavior, which may affect pressure predictions. Future works should examine a multiphase flow regime while considering gravity effect.
2. The formulation is based on Darcy's law and does not account for gas slippage (Klinkenberg effect), threshold pressure gradients, or inertial effects, which can be significant in low-permeability formations. This effect can be incorporated in future studies.
3. The reservoir is assumed to have uniform properties, whereas actual reservoirs are heterogeneous and anisotropic, potentially influencing flow behavior and pressure distribution. Heterogeneous and anisotropic conditions should be incorporated when considering the development of more robust RDE model.
4. Gravity is incorporated only as a hydrostatic term (ρgh), without accounting for vertical flow, fluid segregation, or multiphase gravity effects. These factors and effects may be integrated in future RDE model development.
5. The use of ANOVA provides only a simplified comparison of model performance. Other more robust statistical methods can be deployed to understand relationships and interaction across models (classical and traditional).
6. Although trends are discussed, direct quantitative comparison with previous studies is limited, which may affect the broader generalization of results.

Data Availability Statement: Data will be made available from the authors upon reasonable request.
Conflicts of Interest: The authors declare no conflicts of interest.

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