

Assessment of Hue Transformation Algorithms For SSLCS Studies

Syed Qasim Zaheer¹, Peter J. Disimile¹ and Norman Toy²

¹Department of Aerospace Engineering and Engineering Mechanics, College of Engineering and Applied Sciences, University of Cincinnati, OH 45221, United States

²Engineering & Scientific Innovations Inc, Fairfield, OH, 45014, United States

ABSTRACT: Surface flow characteristics in aerodynamic testing and analysis such as flow transition, separation and reattachment, recirculation can be evaluated using distribution of magnitude and direction of wall shear stress. In this regard, shear stress sensitive liquid crystals (SSLCSs), owing to their color changing characteristics under the influence of applied shear stress, provide a non-intrusive method for shear stress measurement. However, the degree of accuracy of quantitative analysis of wall shear stress using this technique highly depends on accurate calculation of Hue of the color play obtained from SSLC coated surface. In this paper, different mathematical expressions for Hue calculation used in shear sensitive liquid crystal studies are compared for their accuracy using color values from standard color chart and an actual image of model from liquid crystal study. The limitations of certain algorithms are also discussed. Moreover, the influence of spatial resolution for image processing to get localized hue values is also explored. The results are helpful for measuring systems where correct color identification is of prime importance.

KEYWORDS: Liquid Crystals, Hue Transformation Algorithms, Shear Stress

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I. INTRODUCTION

Liquid Crystals (LCs) have intermediate properties of a crystalline solid and a liquid substance. The physical properties of LCs resemble that of a liquid, but they possess optical properties similar to crystalline solids. Some LCs are highly sensitive to external applied stimuli like temperature changes or applied shear stress and they react to these variations by changing color. This peculiar optical behavior, known as "optical activity", is due to inherent molecular structure of these materials which is very sensitive to external stimuli and is very advantageous in aerodynamic, aerothermal and heat transfer measurements as compared to other point measurement techniques. Depending upon molecular arrangements, as shown in Figure 1, smectic phase is optically inactive whereas the chiral nematic (cholesteric) phase is optically active [1,2]. The use of LCs in qualitative analysis and quantitative assessment of surface shear stress vectors in aerodynamic testing provide a non-intrusive technique by which a global distribution of investigated parameter can be achieved. However, for the quantitative analysis of surface shear stress using LCs, the color play obtained must be calibrated against known values of applied shear stress using a calibration technique. Hence, the accurate calculation of hue of color play spectrum i.e. the color carrying information is very important for accurate quantitative analysis of surface shear stress. This paper deals with comparative study of the mathematical definitions of hue obtained from an RGB signal, mentioned in literature, to assess their limitations and variation in the resultant hue value (if any). This research effort will help in providing an insight into color (hue) definition/analysis in the engineering community and also provide the importance of correct color when it is to be defined for a measurement system such as shear stress measurement using LCs.

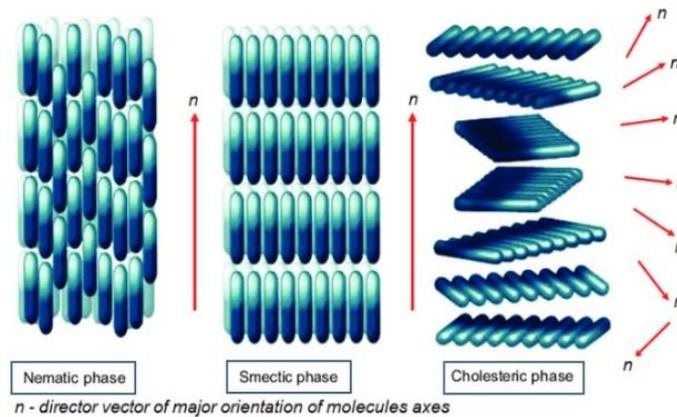


Fig 1: A typical representation of molecular structure of Nematic, Smectic and Cholesteric phases of Liquid Crystals

All LCs are optically sensitive to temperature and shear stress and the one used in shear stress measurements generally referred to as Shear Sensitive Liquid Crystals (SSLCs). Wall shear stress is an important parameter to be measured as its accurate mapping or distribution on surface can reveal information in better understanding of flow characteristics such as the state of a boundary layer, recirculation regions, flow separation and reattachment, turbulent spots and shock cells [3,4]. Liquid crystals have been extensively used for visualization of such flow fields but these studies only provide qualitative distribution of shear stresses. In order to extract magnitude and direction of surface shear stress using LCs, calibration of reflected color play spectrum must be carried out for different physical and optical properties of LCs, as well as experimental setup including lighting and viewing angles, intensity of white light, thickness of LC coating and shear stress vector. For the calibration process and actual testing setup, the RGB signal for every pixel on the obtained colored image of testing surface must be transformed into a respective color value using an automated image processing algorithm.

II. COLOR SPACE TRANSFORMATION

To convert the obtained color image of the testing surface using LCs from RGB color space to one in which the color information at each pixel level can be quantified, hue-based transformation algorithms are used. These algorithms ensure that this pure color information i.e. hue, is decoupled from the intensity. In the literature, such color spaces are hue-saturation-value (HSV), hue-saturation-intensity (HSI) and hue-saturation-lightness (HSL) in which the points in RGB color model are transformed using cylindrical-coordinates.

A. RGB to HSV/HSL Transformation

HSV and HSL models are referred to as single and double hexagon models, respectively. In the HSV model, the maximum intensity is available at the top plane of a hexagon whereas at the bottom, it converges to a single point indicating the color black. However, in HSL, lightness is the average of the smallest and largest color components present in RGB color model. A schematic representation of models is shown in figure 2 and the mathematical conversion algorithm as found in literature [6,7] is detailed in table 1 as ALG-6.

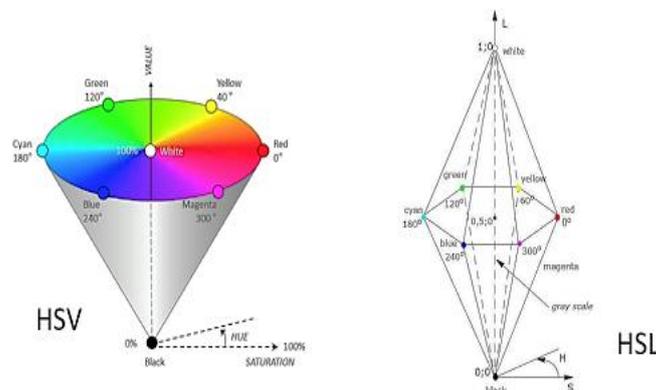


Fig 2: Schematic representation of HSV and HSL color models

B.RGB to HSI Transformation

The Hue, Saturation and Intensity model is also referred to as a double hexagon model. The bottom half of this color model resembles the HSV color model, however, the top half also converges to one point. The bottom and top convergence points correspond to black and white color respectively and the line joining them varies as different shades of grey. The maximum value for the saturation is obtained at medium grey intensity. The obtained color descriptions of the image are more acceptable and intuitive to human eye. The HSI planes can be defined and represented as hexagon, triangle or even a circle [8] as schematically shown in figure 3.

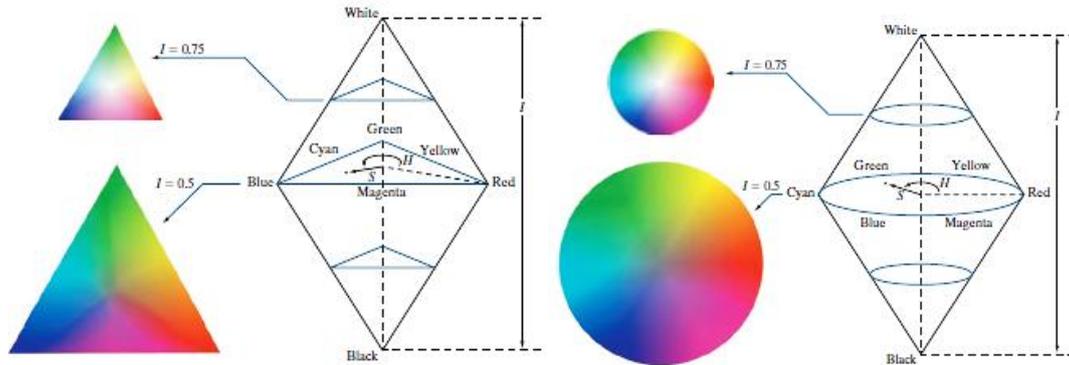


Fig 3: HSI model representation using triangular and circular color planes which are perpendicular to the vertical intensity axis [8].

The RGB to HSI transformation algorithms are named “ALG-1,2,3,…” for reference purposes as shown in table 1. These formulations are extracted from the research work related to quantitative measurement of shear stress on a testing surface using shear sensitive liquid crystals. ALG-1 was formulated by [8] and this formulation is extensively used by many researchers for calculation of Hue from RGB image [9-14]. A new chromaticity coordinates was defined by [15] and formulated a new equation for HSI calculation from RGB color space. The mathematical relation is mentioned as ALG-2 and is most used in hue calculations for quantifying shear stress using SSLCs [16-20]. Apart from these commonly used HSI transformations, a few others, extracted from image processing research, are also tabulated in table 1 and are referenced as ALG-3 [21], ALG-4 [22,23] and ALG-5 [24]. The R, G and B values used in ALG-1 to 5 must be normalized to have values in the range from 0 to 1 by dividing each by the sum of R, G and B values. Non dimensionalization formulation for ALG-6 is given at the end of table 1.

Table 1: Mathematical formulations of RGB to HSI / HSV color space transformation

Algorithm	Hue (H)	Saturation (S)	Intensity (I)
ALG-1	$H = \begin{cases} \theta, & G \geq B \\ 2\pi - \theta, & G < B \end{cases}$ $\theta = \cos^{-1} \frac{\frac{1}{2}[(R-G) + (R-B)]}{\sqrt{[(R-G)^2 + (R-B)(R-G)]}}$	$S = 1 - 3 * \frac{\text{Min}(R, G, B)}{R + G + B}$	$I = \frac{R + G + B}{3}$
ALG-2	$H = \begin{cases} \theta, & G \geq B \\ \theta + \pi, & G < B \end{cases}$ $\theta = \frac{\pi}{2} - \tan^{-1} \left(\frac{2R - G - B}{\sqrt{3}(G - B)} \right)$	$S = \frac{2\sqrt{(R-G)^2 + (R-B)(G-B)}}{\sqrt{6}}$	$I = \frac{R + G + B}{\sqrt{3}}$
ALG-3	$H = \begin{cases} \frac{\pi}{3} * \left(\frac{G-B}{\Delta} \right), & \text{if Max} = R \\ \frac{\pi}{3} * \left(\frac{B-R}{\Delta} \right) + \frac{2\pi}{3}, & \text{if Max} = G \\ \frac{\pi}{3} * \left(\frac{R-G}{\Delta} \right) + \frac{4\pi}{3}, & \text{if Max} = B \end{cases}$ $H = H + 2\pi, \quad \text{if } H < 0$	$S = \begin{cases} \frac{\Delta}{(\text{Max} + \text{Min})}, & 0 < I \leq \frac{1}{2} \\ \frac{\Delta}{2 - (\text{Max} + \text{Min})}, & I > \frac{1}{2} \end{cases}$	$I = \frac{1}{2} (\text{Max} + \text{Min})$
ALG-4	$H = \begin{cases} \left(\frac{G-B}{3(R+G-2B)} \right), & \text{if Min} = B \\ \left(\frac{B-R}{3(G+B-2R)} \right) + \frac{1}{3}, & \text{if Min} = R \\ \left(\frac{R-G}{3(R+B-2G)} \right) + \frac{2}{3}, & \text{if Min} = G \end{cases}$ $H = H * 2\pi$	<p>Same as ALG-1</p>	<p>Same as ALG-1</p>
ALG-5 (Kender's Algorithm)	<p>if ((R > B) and (G > B))</p> $H = \frac{\pi}{3} + \tan^{-1} \left(\frac{\sqrt{3}(G-R)}{(G-B) + (R-B)} \right)$ <p>else if (G > R)</p> $H = \pi + \tan^{-1} \left(\frac{\sqrt{3}(B-G)}{(B-R) + (G-R)} \right)$ <p>else if (B > G)</p> $H = \frac{5\pi}{3} + \tan^{-1} \left(\frac{\sqrt{3}(R-B)}{(R-G) + (B-G)} \right)$ <p>else if (R > B)</p> $H = 0$ <p>else 'achromatic'</p>	$S = 1 - 3 * \frac{\text{Min}(R, G, B)}{R + G + B}$	$I = \frac{R + G + B}{3}$
Algorithm	Hue (H)	Saturation (S)	Value (V)
ALG-6 (OpenCV)	$H = \begin{cases} 60(G' - B') / (V - \min(R', G', B')) & \text{if } V = R' \\ 120 + 60(B' - R') / (V - \min(R', G', B')) & \text{if } V = G' \\ 240 + 60(R' - G') / (V - \min(R', G', B')) & \text{if } V = B' \end{cases}$	$S = \begin{cases} (V - \min(R', G', B')) / V, & \text{if } V \neq 0 \\ 0, & \text{Otherwise} \end{cases}$	$V = \max(R', G', B')$

Note: $Max = Max(R, G, B)$; $Min = Min(R, G, B)$; $\Delta = Max - Min$; $R' = \frac{R}{255}$, $G' = \frac{G}{255}$, $B' = \frac{B}{255}$

III. COMPARISON OF TRANSFORMATION ALGORITHMS

In order to ascertain the accuracy and computational process of these transformation algorithms for obtaining the hue from RGB color space, a comparative study has been carried out, firstly, using a reference color chart, and then the same transformations were applied for an actual model coated with shear sensitive liquid crystals and subjected to aerodynamic flow condition of Mach 1 in a transonic wind tunnel.

A. Comparison Using Color Chart

A reference RGB color space rainbow image having 24 different sets of R, G and B values, named as cases 1-24, is transformed to respective hue values using most commonly used RGB to HSI algorithms, i.e. ALG-1-5 and given RGB to HSV transformation algorithm ALG-6. The hue values obtained from these transformations, which includes the normalization of R, G and B values as per algorithm requirement, corresponding to each case number are plotted in figure 4. The values obtained from these algorithms are compared with the one obtained from ALG-1 since it is the most commonly used transformation in liquid crystal studies. The hue values are mentioned in color chart of figure 4 separately as well for quantitative comparison, and the difference is plotted in figure 5.

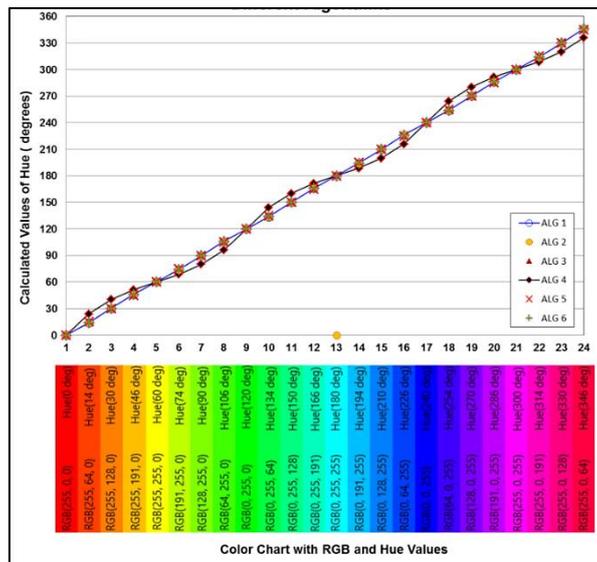


Fig 4: Hue Calculation from RGB values using transformation algorithms

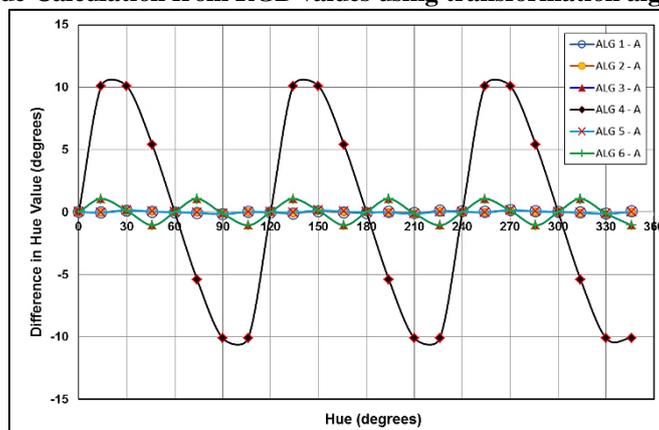


Fig 5: Comparison of hue values from algorithms with chart values of fig 4.

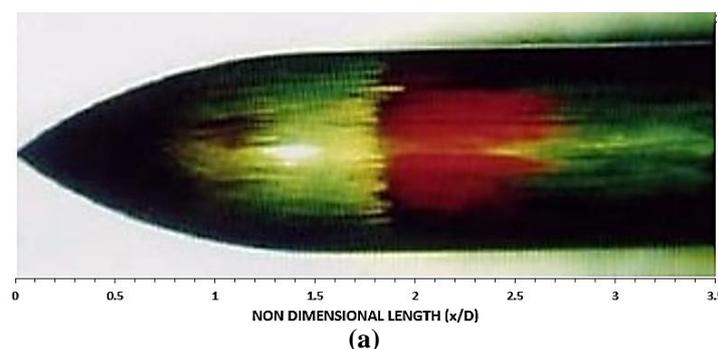
From these plots and the calculation process i.e the coding for these algorithms and the associated logical condition to be satisfied with each transformation, the following observations provide an indication of the accuracy of these algorithms.

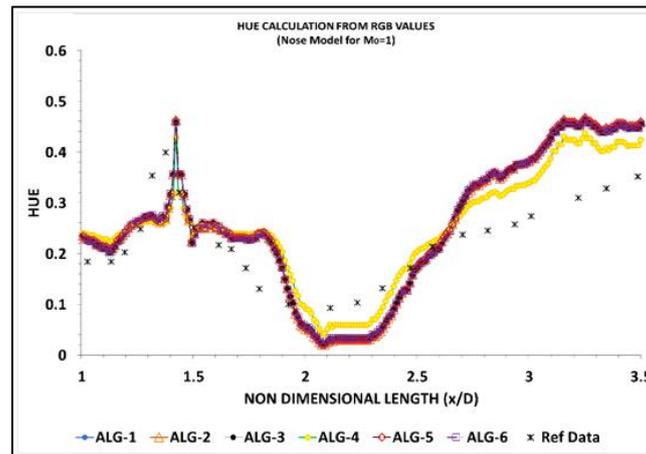
1. For ALG-5 (Kender's algorithm) the logical condition for using the respective transformation equation must be satisfied in the same order as mentioned. If the order of satisfying logical conditions to be satisfied is changed, the hue values will be highly erroneous and inaccurate. This is because, for few sets of R,G and B values, there may be more than one logical condition are satisfied and each giving different result. In such cases, the accurate hue value is only calculated by the first satisfied logical condition in order, as mentioned in table 1. Therefore, if the sequence of these associated logical conditions is not followed, the hue values will be erroneous.
2. The normalization formulation of R,G and B values to be used in hue transformation using HSV/HSL and HSI algorithms is different.
3. The arctan based algorithm (i.e., ALG-2) gives undefined values of hue for case number 1 and 13. This is because if, for these cases, $G=B$ resulting in zero in the denominator.
4. The hue values obtained from ALG-3, 4 and ALG-6 (HSV/HSL algorithm) have a sinusoidal pattern when compared with ALG-1. This may be attributed to the fact that these algorithms are linear transformations as compared to trigonometric function used in others.
5. The maximum variation or error in the amplitude of hue obtained from ALG-4 is approximately ± 10 degrees whereas that of ALG-3 and ALG-6 is approximately ± 1 degree which may be neglected.
6. Interestingly, ALG-3 and ALG-6 hue values and their variation with respect to reference algorithm are approximately similar.

B. Using Color image of Liquid Crystal coated Model.

The color values used in the color chart cannot be faithfully reproduced by the liquid crystal applied on a model surface for flow visualization and shear stress quantification. Therefore, for realistic comparison of transformation algorithms, the color image obtained from the experimental study of an ogive nose model mounted on an end of cylinder and exposed to flow conditions of Mach 1 in a transonic wind tunnel, is used. The surface of model was coated with BCN/165 liquid crystal for surface flow field visualization and quantification of shear stress after the experimental setup was calibrated using a separate mechanical rig. The image and the details of the experimental setup and calibration procedure can be found in [25]. It is noteworthy to mention here that liquid crystal cannot replicate the color values from the case number 19-24 because they are indistinguishable from previous cases. This is why the hue values obtained from RGB colored image of liquid crystal is generally restricted to 0-240 degrees in which R is located at 0-degree, G at 120 degrees and B at 240 degrees. The hue values are then normalized with 240 degrees and are then used for calibration and actual measurement purposes. Same procedure is also followed here, and normalized hue values are used for comparison purposes using transformation algorithms mentioned in table 1.

In the experimental study [25], the variation of hue, thus the shear stress, along the centerline of nose model was studied for flow field analysis and measurement of shear stress. The image of liquid crystal coated nose model under the influence of flow condition in the wind tunnel is taken from the reference published article and the R,G and B values are extracted along the centerline of nose model for each pixel which are used as input for respective hue transformation algorithms. The RGB colored image of the model along with its dimensions are shown in figure 6. The resulting non dimensional hue values calculated using afore mentioned transformation algorithms are plotted along the length of nose model which is non dimensionalized by the diameter of the cylinder. The reference hue values referred to are those published in the original article [25] and are only plotted to ascertain the trend in the variation of hue values along the centerline of the model. Moreover, it is possible that in original paper, the hue values may be corrected by considering the effect of incident lighting and observation angles as well as curvature of model.





(b)

Fig 6: (a) RGB image of the nose model coated with BCN/165 liquid crystal for Mach number 1. (b) Variation of Hue along the centerline of model calculated from different transformation algorithms along with the reference data points [25].

From these plots, it is evident that hue values from all transformation algorithms follow the trend of reference data points. The hue values for each pixel along the centerline of the model from all algorithms are plotted over each other except ALG-4. As the color values shift from red to green along the centerline, the oscillating nature of hue from ALG-4 about those obtained from ALG-1 and ALG-2 is observed in figure 6. Such behavior or trend of ALG-4 is very similar to the one recorded in figure 4 & 5 when the color values from the color chart is used. It is also observed that lowest hue values are recorded for the physical region of nose model which display the color red and the peaks of the hue plot correspond to region having greenish color. Infact, the hue values and its variation along the centerline of model, obtained from ALG-1,2,3,5, and 6, are more realistic than the reference data points once compared with the color distribution along the centerline in figure 6(a). Once the hue values are calibrated against the known shear stress using a calibration rig and are corrected for illumination and observation angles, these hue values can be converted into corresponding shear stress values. From the figure 6(a), it can be visualized that red color (lower hue values) correspond to the region of lower shear stress and represents the detached flow region. The flow then reattaches, and the shear stress again increases and so does the hue values. It is, therefore, important that transformation algorithm should provide accurate hue values which must corroborate with the physics of the flow field. From the result shown in figure 6(b), it can be concluded that all transformation algorithms except ALG-4 provide accurate hue values. Although ALG-1 and ALG-2 are mostly used in liquid crystal studies, the current results indicate that other transformation algorithms also produce reasonably accurate results. In this study, the RGB values are taken only along the centerline, therefore comments regarding computational time associated with each hue algorithm cannot be made, which can be a decisive factor in using respective algorithm for liquid crystal studies. As the basic advantage of using liquid crystal for surface flow field analysis is to achieve a holistic panoramic distribution of surface shear stress field on the model under consideration, the accurate calculation of hue values from different transformation algorithms can motivate the researchers to use any of these (except ALG-4) based on the less computational time requirement.

C. Influence of Spatial Resolution

In the previous section, the RGB values of each pixel along the centerline of the model is converted into corresponding hue values. However, in practice, calculation of hue at each pixel level is not viable computationally for liquid crystal studies because the area of interest is quite large enough for panoramic distribution and analysis of shear stress. Therefore, in most of the practice, the whole image is divided into square blocks of required resolution and average values of R, G and B are converted into corresponding averaged hue values that represent the single block of an image. In this section, the influence of spatial resolution for the accurate and realistic hue distribution is ascertained. Three different resolutions are analyzed namely; Case 1 (1x5 pixels = 0.2645mm x 1.3225mm), Case 2 (1x10 pixels = 0.2645mm x 12.645mm) and Case 3 (1x15 pixels = 0.2645mm x 3.9675mm). For each case, the averaged RGB values are fed into the transformation algorithms to get the corresponding hue values. The results along with the reference data points are plotted in figure 7.

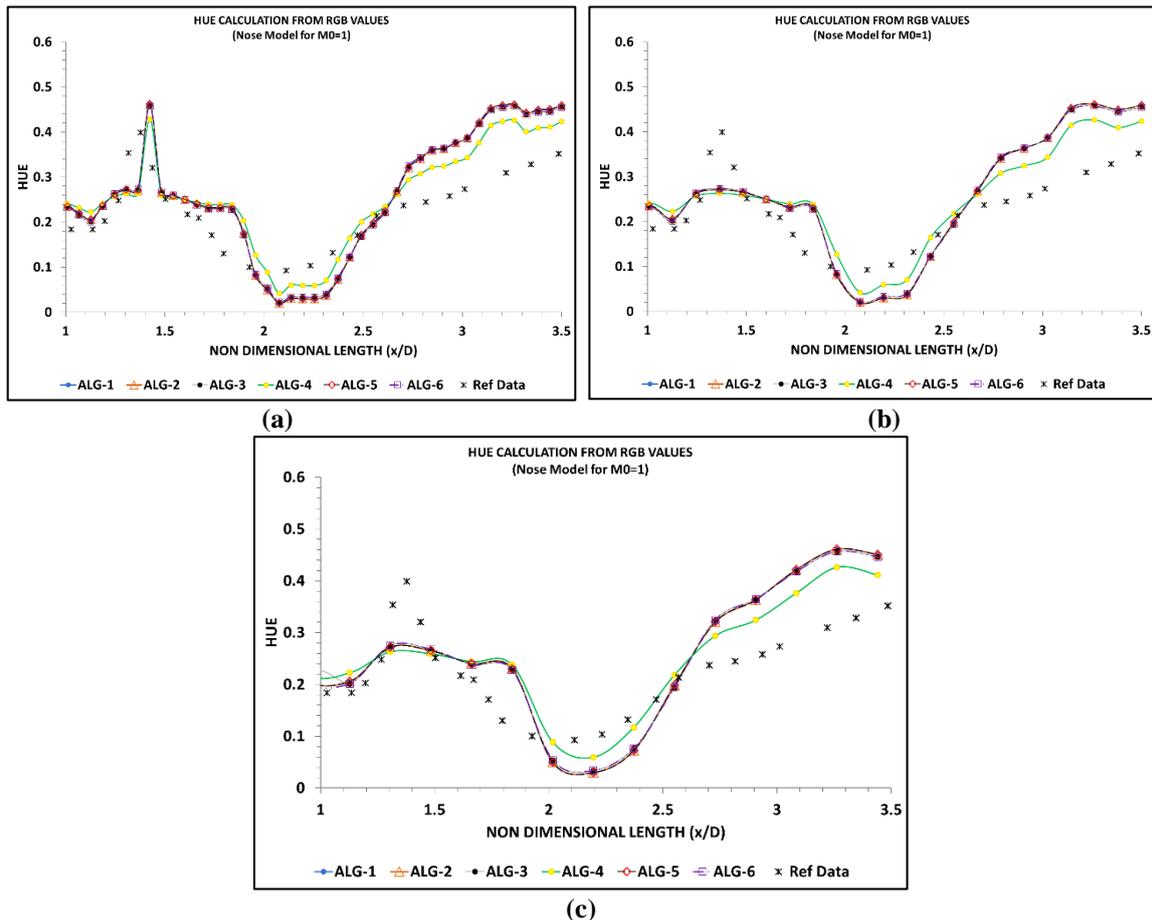


Fig 7: Variation of hue along the centerline of model for different spatial resolution (a) Case 1 (b) Case 2 (c) Case 3

It is observed in figure 7 that spatial resolution of Cases 2 and 3 are unable to capture the peak in the hue value corresponding to x/D of 1.4. Moreover, as the resolution is decreased, i.e. larger block size, the variation in hue profile is becoming smoother but fails to capture the abrupt changes locally. Also, in the vicinity of x/D of 1.0-1.2, the difference in the estimation of hue values using different algorithms becomes more obvious and here hue values from ALG-3 deviates from that of ALG-1 and ALG-2. From these plots it is very obvious that for every model or surface under testing using liquid crystal technique, an optimized block dimension must be selected for accurate and realistic representation and quantification of distribution of shear stress.

IV. CONCLUSION

In this research study, different RGB to hue transformation algorithms are compared and analyzed for their accuracy and viability for application in visualization and measurement of surface shear stress using shear sensitive liquid crystal technique. For this purpose, first color values from a standard color chart are used as input to these algorithms and then a realistic RGB image of a model from liquid crystal study is used for comparative study. It is observed that apart from the most commonly used hue transformation algorithms i.e. ALG-1 and 2 in liquid crystal studies, other algorithms except ALG-4 can also produce reasonably accurate results, and this finding can further be used for decision upon using an algorithm with less computational time requirement. The influence of spatial resolution of block size (for local RGB averaging) on accurate and realistic distribution of shear stress / hue on the model surface is also explored. It is observed that every model or surface under testing using liquid crystal technique, an optimized block dimension must be selected for accurate and realistic representation and quantification of distribution of shear stress.

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