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Effectiveness of reinforcement plates pertaining to pressure equipment

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ABSTRACT: *The aim of this paper is to provide additional insight into the effect of a geometric gap between a cylindrical- or spherical shell and a reinforcement pad on the stress intensity in the vicinity of the nozzle penetration when the pressure vessel is subjected to internal pressure. Current design codes do not provide a methodology for the stresses in the reinforcement region. As a result, the distribution and magnitudes of the local stresses induced by the geometric discontinuity and the internal pressure loading are not known. For a variety of reasons, perfect contact cannot be kept between vessel and pad which results in a gap. The effect of the gap on the stresses in the nozzle - reinforcement region is a matter of common interest to both designers and manufactures. This article emphasizes a practical solution to compensate for the gap between reinforcement pad and shell that does not achieve a completely "integral" connection. The solution provides for the application of an efficiency factor* η *that in fact leads to a substitute pad thickness and can additionally be used in nozzle compensation calculations according to various design codes. Finally, brief attention is also paid to the influence of a gap on nozzle loads.*

KEYWORDS: *Gap, stress intensity, discontinuity, reinforcement pad, efficiency factor, nozzle loads*

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I. INTRODUCTION

Pressure vessels are common in industrial process facilities, where the use of reinforcement plates is a well-known application especially in low and moderate pressure regimes. Their purpose is to strengthen nozzles, so that a higher load capacity can be achieved. What is unknown to many engineers is the effectiveness of the reinforcing plate associated with vessel nozzles. Typically a reinforcing plate is provided with a fillet weld along the outer edge of the respective pressure-bearing part. Despite the measures that have been taken to ensure a good fit between the reinforcing plate and the pressure-bearing part, it appears that in practice it is impossible to avoid a gap between these two components. Complete contact between the two components is difficult to achieve and is dependent upon the location of the pad and the geometry of the pressure retaining boundary. Therefore, the connection between reinforcing plate and pressure-bearing member cannot be considered as integral. The recognized codes do not take this gap into consideration. As a result there could be an overestimation of the load capacity. The reinforcement pad efficiency factor is to compensate for the reduction in load - carrying capacity due to the presence of a geometric gap between reinforcing pad and shell. In line with [1] an efficiency factor η of 0.75 is proposed. The cross-sectional area of the repad within compensation limits (A_{fp}) multiplied by η actually yield a reduction of A_{fp}. The η-factor has been adopted from the Rules for Pressure Vessels (NL) [1] which were officially been frozen from January 1, 2005 because it was incompatible with the development of the European (harmonized) Unfired Pressure Vessel Standard. Reference [1] is originally based on [2]. Finally, the research question can be formulated as follows: In analytical code calculations based on the pressure-area methodology, the question arises whether a reinforcement pad efficiency factor should be included to compensate for the gap between the reinforcing pad and the vessel wall. The gap is the reason that the combination of pad and vessel wall cannot be regarded as completely integral, so that the load capacity is lower than with an integral connection. Numerical analysis (FEM) will be used to model such a connection with a gap sufficiently realistic so that a well-founded opinion can be formed regarding the application of a pad efficiency factor in analytical code calculations.

Table 1: Nomenclature

Symbol	Description	Unit
A_{fs}	Cross-sectional area of shell within compensation limits	mm^2
Af_b	Cross-sectional area of branch / nozzle neck within compensation limits	mm^2
Af_w	Cross-sectional area of fillet weld between nozzle neck and pad and pad and shell	mm^2
Af_p	Cross-sectional area of pad within compensation limits	mm^2
Ap_s	Pressure loaded area in shell	mm^2
Ap_b	Pressure loaded area in branch (nozzle neck)	mm^2
D_e	Outside diameter of shell	mm
$D_{i,s}$	Inside diameter of shell	mm
d_{eb}	Outside diameter of nozzle neck fitted to shell	mm
d_{ib}	Inside diameter of nozzle neck fitted to shell	mm
$e_{a,s}$	Thickness of shell	mm
$e_{a,b} = d_{a,b}$	Thickness of branch or nozzle neck	mm
$e_{a,p}$	Thickness of reinforcing pad	mm
e_{fw}	Throat thickness of filled weld	mm
f	Nominal design stress shell	MPa
f_s	Nominal design stress shell material	MPa
f_{ob}	Nominal design stress branch / nozzle material	MPa
f_r	Nominal design stress reinforcing pad material	MPa
f_w	Nominal design stress fillet weld material	MPa
l_{so}	Shell boundary limit of reinforcement zone	mm
l_{bo}	Branch / nozzle neck boundary limit of reinforcement zone	mm
l_p	Width of reinforcing pad within boundary limits	mm
$F_R = F_r$	Allowable individual load	N
$M_C = M_c$	Allowable individual circumferential moment	Nmm
$M_L = M_l$	Allowable individual longitudinal moment	Nmm
$M_M = M_m$	Allowable meridional moment	Nmm
MAWP	Maximum allowable working pressure	MPa
P_{max}	Maximum allowable pressure	MPa
r_{is}	Inside radius shell	mm
ϑ_m	Design metal temperature	°C
η	Reinforcing pad efficiency factor	-

II. WORKED EXAMPLES

Model # 1 thru model # 6 shows the differences in pressure capacity between a flush pad-reinforced nozzle in a cylindrical shell with and without the η-factor, while for model # 7 and # 8 this is done for a pad -reinforced nozzle in a spherical segment. The calculations were performed according the pressure-area method [3],[4] and [8]. The principal of the pressure area method are illustrated in Figure 1 and 2, where a certain area of the vessel in the region of the opening multiplied by the design stress is equated to the cross-sectional area of the vessel in the same region multiplied by the pressure. This is a simplified limit load type with the yield stress factored to the design level. The strength condition is then:

$$\textit{Design pressure}\left[\frac{A_{ps}+A_{pb}}{(A_{fs}+A_{fb}+A_{fw}+A_{fp}\cdot\eta}+0.5\right]=f\leq\frac{\textit{yield stress}}{\textit{safety factor}}$$

Note: The main symbols used in the equations are shown in Figure 1.

It should be noted that the terms "model number" and "case number" used in this article are related to each other

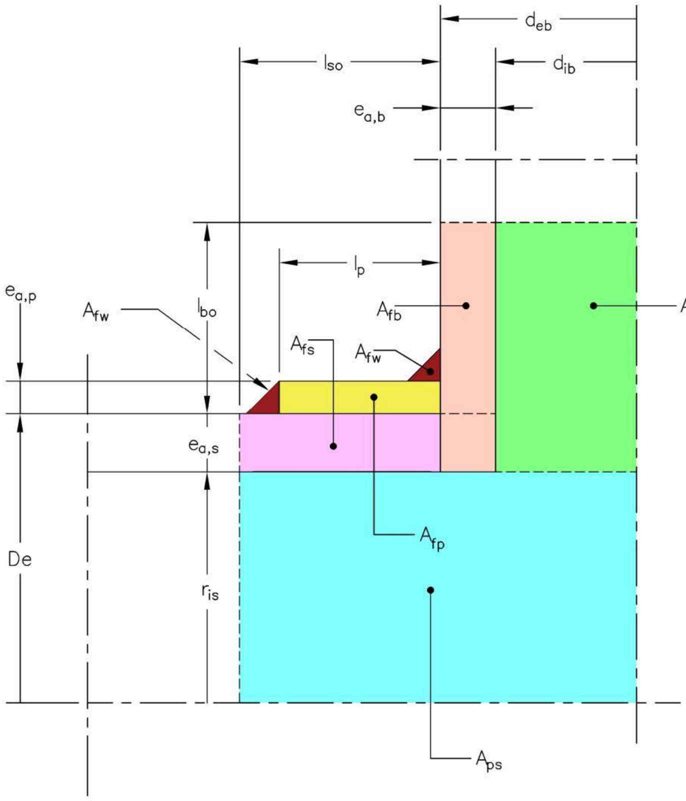


Figure 1: Illustration of set-in flush nozzle with reinforcing plate on a cylindrical shell

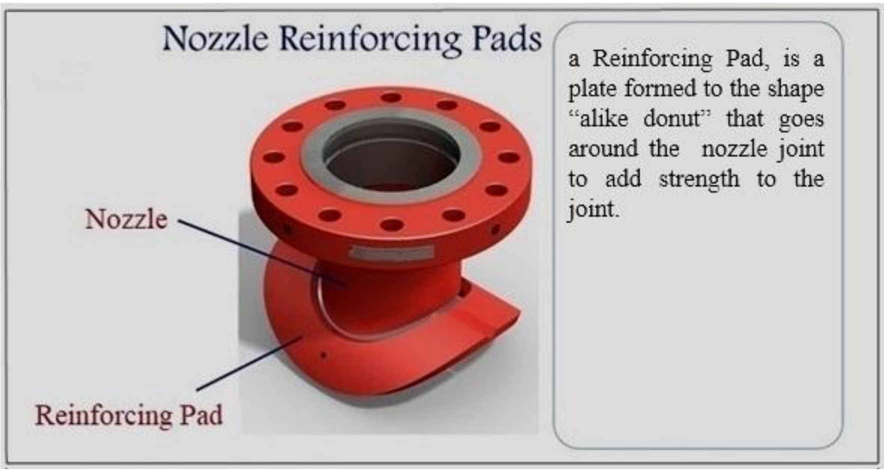
If $f_s = f_{ob} = f_r = f_w$ *the following general equations shall apply for the determination of the allowable pressure:*

$$P_{max} = \frac{f_s}{\left[\frac{A_{ps}+A_{pb}}{(A_{fs}+A_{fb}+A_{fw}+A_{fp}\cdot\eta)}+0.5\right]}$$

or $P_{max} = \frac{2\, f_s\, e_{a,s}}{(D_e - e_{a,s})} \equiv f_s \ln\left(\frac{D_e}{D_i}\right)$ for cylindrical shell w/o nozzle

respectively: $P_{max} = \frac{4\, f_s\, e_{a,s}}{(D_e - e_{a,s})} \equiv 2\, f_s \ln\left(\frac{D_e}{D_i}\right)$ for spherical shell w /o nozzle

IMAGE OF TYPICAL PAD REINFORCED NOZZLE



OVERVIEW CALCULATION MODELS

Table 2: Pad reinforced flush nozzles on cylindrical shell (Model # 1, 2, 3, 4, 5 and 6)
Pad reinforced flush nozzles on spherical shell (Model #7 and # 8)

MODEL NUMBER		# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8
Outside diameter cylindrical or spherical shell (mm)	D_e	1200	1200	1200	1200	3000	3000	4050	4050
Analysis thickness of the shell (mm)	$e_{a,s}$	12	12	15	15	8	8	25	25
Inside radius of cylindrical or spherical shell (mm)	r_{is}	588	588	585	585	1492	1492	2000	2000
Outside nozzle diameter (mm)	d_{eb}	323.8	323.8	323.8	323.8	609.6	609.6	609.6	609.6
Nozzle analysis thickness (mm)	$e_{a,b}$	8.34	8.34	18.76	18.76	8.34	8.34	15.295	15.295
Internal diameter of nozzle (mm)	d_{ib}	307.12	307.12	286.28	286.28	592.92	592.92	579.01	579.01
Width of reinforcing plate (mm)	l_p	110	110	90	90	145	145	150	150
Pad efficiency factor (-)	η	1.0	0.75	1.0	0.75	1.0	0.75	1.0	0.75
Thickness of reinforcing plate (mm) = η x $e_{a,p}$	$e_{a,p}$	12	9	15	11.25	12	9	25	18.75
Throat thickness of fillet welds (mm)	e_{fw}	6	6	6	6	6	6	0	0

Remark
Analytical calculations are performed for model numbers # 1, # 3, # 5 and # 7 according to the pressure-area method (η = 1.0), while for the model numbers # 2, # 4, # 6 and # 8 these are performed with a reduced pad thickness (η = 0.75) meant to compensate for the gap.

Design condition

MAWP (Max. allowable pressure (MPa)	P_{max}	As calculated
Design temperature (°C)	ϑ_m	200
Corrosion allowance (mm)	ca	0
Thickness tolerance shell	tol	0
Thickness tolerance nozzle neck		12.5%

Materials and properties

Part	Material	Yield strength (MPa)	Tensile strength (MPa)	Yield strength @ [ϑ_m] (MPa)	Design stress [f] (MPa)
Cylindrical shell	A515 Gr.65	240	450	207	138.00 = f_s
Nozzle neck	A106 Gr. B	240	415	207	138.00 = f_{ob}
Reinforcing plate	A515 Gr.65	240	450	207	138.00 = f_r

Note: Yield- and tensile properties are derived from ASME Section II - Part D [5]

Table 3: Manual calculations of cases #1, 3 and 5

Model # 1 (η = 1.0) & Model # 2 (η = 0.75) Elaborated calculation with nozzle neck thickness of 8.34 mm (0.875 x 9.53), shell thickness 12 mm, pad thickness 12 mm and pad width 110 mm		η **1.0**	η **0.75**
l_{so}	Maximum length of shell contributing to opening reinforcement $= [(D_e - e_{a,s})\, e_{a,s}]^{0.5}$	119.4	
l_{bo}	Maximum length of nozzle outside the shell for reinforcement $= [(d_{eb} - e_{a,b})\, e_{a,b}]^{0.5}$	51.3	
A_{ps}	Pressure loaded area within shell $= r_{is} (l_{so} + 0.5\, d_{eb})$	165404.4	
A_{pb}	Pressure loaded area within branch $= 0.5\, d_{ib} (l_{bo} + e_{a,s})$	9720.348	
A_{fb}	Cross-sectional area of branch within compensation limits $= e_{a,b} . (l_{bo} + e_{a,s})$	527.922	
A_{fs}	Cross-sectional area of shell within compensation limits $= e_{a,s} . l_{so}$	1432.8	
A_{fw}	Cross-sectional area of filled weld between nozzle and shell $= 2 . e_{fw}^{\ 2}$	72	
A_{fp}	Cross-sectional area of pad within compensation limits $= l_p . e_{a,p} . \eta$	1320	990

P_{max}	Maximum allowable pressure (MPa) : $$\frac{(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)\, f_s}{(A_{ps} + A_{pb}) + 0.5\,(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)}$$ $P_{max,\ shell\ w/o\ nozzle} = 2.788$ MPa	2.617	2.362
Model # 3 (η = 1.0) & Model # 4 (η = 0.75) Elaborated calculation with nozzle neck thickness of 18.76 mm (0.875 x 21.44), shell thickness 15 mm, pad thickness 15 mm and pad width 90 mm		**η**	
		1.0	**0.75**
l_{so}	Maximum length of shell contributing to opening reinforcement $= [(D_e - e_{a,s})\, e_{a,s}]^{0.5}$	133.3	
l_{bo}	Maximum length of nozzle outside the shell for reinforcement $= [(d_{eb} - e_{a,b})\, e_{a,b}]^{0.5}$	75.65	
A_{ps}	Pressure loaded area within shell $= r_{is}\,(l_{so} + 0.5\, d_{eb})$	172692	
A_{pb}	Pressure loaded area within branch $= 0.5\, d_{ib}\,(l_{bo} + e_{a,s})$	12975.641	
A_{fb}	Cross-sectional area of branch within compensation limits $= e_{a,b} . (l_{bo} + e_{a,s})$	1700.594	
A_{fs}	Cross-sectional area of shell within compensation limits $= e_{a,s} . l_{so}$	1995.5	
A_{fw}	Cross-sectional area of filled weld between nozzle and shell $= 2 . e_{fw}^{\ 2}$	72	
A_{fp}	Cross-sectional area of pad within compensation limits $= l_p . e_{a,p} . \eta$	1350	1012.5
P_{max}	Maximum allowable pressure (MPa) : $$\frac{(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)\, f_s}{(A_{ps} + A_{pb}) + 0.5\,(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)}$$ $P_{max,\ shell\ w/o\ nozzle} = 3.494$ MPa	3.752	3.508

Model # 5 (η = 1.0) & Model # 6 (η = 0.75) Elaborated calculation with nozzle neck thickness of 8.34 mm (0.875 x 9.53), shell thickness 8 mm, pad thickness 12 mm and pad width 145 mm	**η**	
	1.0	**0.75**
Maximum length of shell contributing to opening reinforcement $= [(D_e - e_{a,s})\, e_{a,s}]^{0.5} = l_{so}$	154.7	
Maximum length of nozzle outside the shell for reinforcement $= [(d_{eb} - e_{a,b})\, e_{a,b}]^{0.5} = l_{bo}$	70.81	
Pressure loaded area within shell $= r_{is}\,(l_{so} + 0.5\, d_{eb}) = A_{ps}$	685574	
Pressure loaded area within branch $= 0.5\, d_{ib}\,(l_{bo} + e_{a,s}) = A_{pb}$	23364	
Cross-sectional area of branch within compensation limits $= e_{a,b} . (l_{bo} + e_{a,s}) = A_{fb}$	657.27	
Cross-sectional area of shell within compensation limits $= e_{a,s} . l_{so} = A_{fs}$	1237.6	
Cross-sectional area of filled weld between nozzle and shell $= 2 . e_{fw}^{\ 2} = A_{fw}$	72	
Cross-sectional area of pad within compensation limits $= l_p . e_{a,p} . \eta = A_{fp}$	1740	1305
Maximum allowable pressure (MPa) : $$\frac{(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)\, f_s}{(A_{ps} + A_{pb}) + 0.5\,(A_{fs} + A_{fb} + A_{fw} + A_{fp} . \eta)}$$ $P_{max,\ shell\ w/o\ nozzle}$ 0.738 MPa $\Rightarrow \sigma_{\theta m} = P_{max} / \ln(D_e / D_i)$ $\sigma_{\theta m} = 0.738 / \ln(3000 / 2984) = 138$ MPa Ratio : $P_{max;\eta = 1.0} / P_{max;\eta = 0.75} = 1.132$	0.719	0.635

Note: reinforcing pads shall be fitted in close contact with the shell.

Table 4: Approximate method

Load-bearing lengths & auxiliary quantities	**Pressure loaded areas & load - carrying cross-sectional areas**	**Maximum allowable pressure @ nozzle intersection**
$l_{so} = [(2\, r_{is} + e_{a,s})\, e_{a,s}]^{0.5}$	$A_{ps} = 0.5\, r_{is}^2\, \dfrac{l_{so} + a}{0.5\, e_{a,s} + r_{is}}$	$P_{max} = \dfrac{f_s}{[\dfrac{A_{ps} + A_{pb}}{A_{fs} + A_{fb} + A_{fp} . \eta} + 0.5]}$
$l_{bo} = [(d_{eb} - e_{a,b})\, e_{a,b}]^{0.5}$	$A_{pb} = 0.5\, d_{ib}\,(l_{bo} + e_{a,s})$	**Approximate method**

$r_{ms} = (r_{is} + 0.5\, e_{a,s})$	$A_{fs} = l_{so} \,.\, e_{a,s}$	Pressure loaded area: $A_p = \frac{r_{is}}{2}(l_{so} + \frac{d_{eb}}{2}) + \frac{dib}{2}(l_{bo} + e_{a,s})$
$\delta = \frac{d_{eb}}{2\, r_{ms}}$	$A_{fb} = e_{a,b}\,(l_{bo} + e_{a,s})$	Load - carrying cross-sectional areas: $A_{fs} = l_{so} \,.\, e_{a,s}$ $A_{fp} = l_p \,.\, e_{a,p}$ $A_{fb} = e_{a,b}\,(l_{bo} + e_{a,s})$
$a = r_{ms} \,.\, \arcsin \delta$	$A_{fp} = l_p \,.\, e_{a,p}$	$\mathrm{MAWP} = \frac{f_s}{[\frac{A_p}{A_{fs} + A_{fb} + A_{fp}\,.\,\eta} + 0.5]}$

Note: The approximate method may be used at the option of the designer. Simple formulae for calculation of A_p, A_{fs}, A_{fp} and A_{fb} give acceptable results within the accuracy of the method

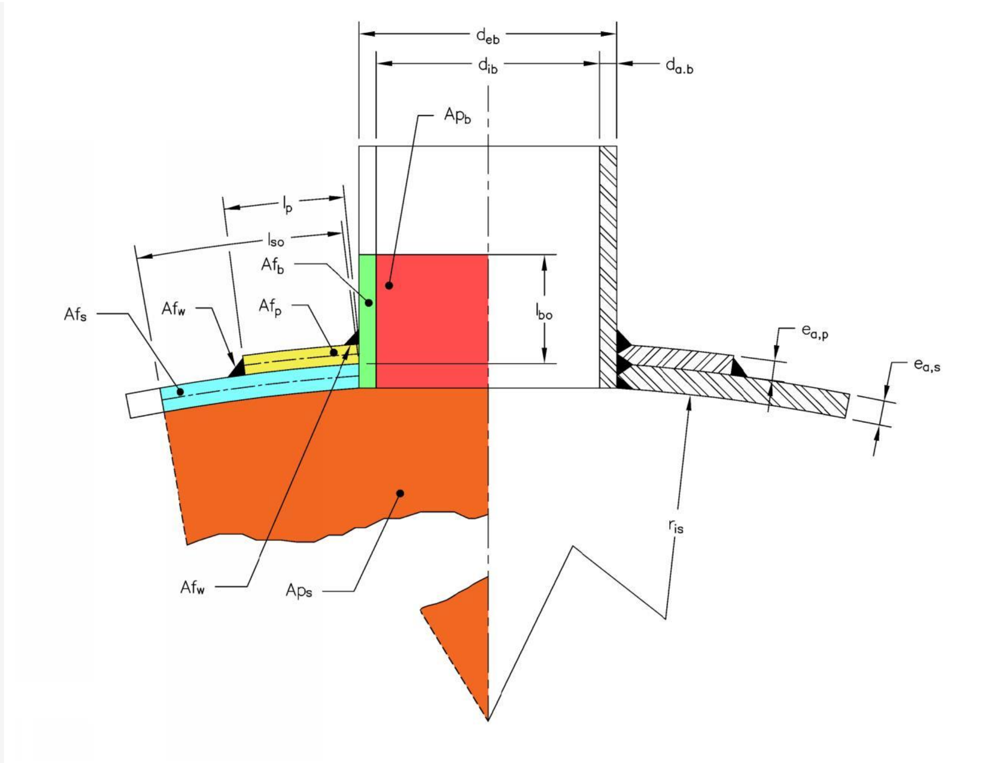


Figure 2: Illustration of set-in flush nozzle on spherical shell with reinforcing plate

Table 5: Manual calculations of case #7

Model # 7 (η = 1.0) & Model # 8 (η = 0.75) **Elaborated calculation with nozzle neck thickness of 15.295 (0.875 x 17.48), spherical shell thickness 25 mm, pad thickness 25 mm and pad width 150 mm**	**η**	
	1.0	**0.75**
Maximum length of shell contributing to opening reinforcement (l_{so}) (mm)	317.21	
Maximum length of nozzle outside the shell for reinforcement (l_{bo}) (mm)	95.34	
Mean radius of spherical part (r_{ms}) (mm)	2012.5	
Auxiliary quantity (δ) (-)	0.15145	
Segment length (a) (mm)	305.98	
Pressure loaded area within spherical shell (A_{ps}) (mm^2)	619321.13	
Pressure loaded area within branch (A_{pb}) (mm^2)	34835.51	
Cross-sectional area of branch within compensation limits (A_{fb}) (mm^2)	1839.78	

Cross-sectional area of spherical shell within compensation limits (A_{fs}) (mm^2)	7930.36	
Cross-sectional area of filled welds (A_{fw}) (mm^2)	0	
Cross-sectional area of pad within compensation limits (A_{fp}) (mm^2)	3750	2812.5
Maximum allowable pressure (MPa) : $$\frac{\left(A_{fs} + A_{fb} + A_{fw} + A_{fp} \cdot \eta\right) f_s}{\left(A_{ps} + A_{pb}\right) + 0.5\,(A_{fs} + A_{fb} + A_{fw} + A_{fp} \cdot \eta)}$$ Ratio : $P_{max;\eta = 1.0} / P_{max;\eta = 0.75} = 1.072$ In case the cross-sectional area of the filled welds with a throat thickness of 12.5 mm are taken into account, A_{fw} becomes $2 \times 12.5^2 = 312.5\ mm^2$ The MAWP becomes: 2.89 MPa (for η = 1.0) respectively 2.69 (for η = 0.75) Ratio : $P_{max;\eta = 1.0} / P_{max;\eta = 0.75} = 1.074 \Rightarrow P_{max,\ shell\ w/o\ nozzle} = 3.429$ MPa	2.82	2.63

Table 6: Elaboration based on approximate method

Load - carrying cross-sectional areas: $A_{fs} = l_{so} \cdot e_{a,s}$ $A_{fp} = l_p \cdot e_{a,p}$ $A_{fb} = e_{a,b}\,(l_{bo} + e_{a,s})$	317.21 x 25 = 7930.25 mm^2 150 x 25 = 3750 mm^2 15.295 (95.33 + 25) = 1840.45 mm^2	
Pressure loaded area: $A_p = \frac{r_{is}}{2}\,(l_{so} + \frac{d_{eb}}{2}) + \frac{dib}{2}\,(l_{bo} + e_{a,s})$	2000/2 (317.21 + 609.6 /2) + 579.01/2 (95.33 + 25) = 656846.14 mm^2	
	η = 1.0	η = 0.75
$\text{MAWP} = \frac{f_s}{\left[\frac{A_p}{A_{fs} + A_{fb} + A_{fp} \cdot \eta} + 0.5\right]}$	2.81 MPa	2.62 MPa

Observation: The results of the accurate method hardly differ from those of the approximate method!

III. FURTHER ANALYSIS

Overview of stress concentration factors (SCF's) and stress intensities pertaining to various calculation models:
Model # 1: Conforming elaborated calculation with pad efficiency factor η =1.0.
Model # 2: Conforming elaborated calculation with pad efficiency factor η = 0.75.
Model # 3: As model # 1 with increased shell and nozzle neck thickness and smaller pad width.
Model # 4: As model # 2 with increased shell and nozzle neck thickness and smaller pad width.
Model # 5: Conforming elaborated calculation of repad nozzle with pad efficiency factor η = 1.0.
Model # 6: Conforming elaborated calculation of repad nozzle with pad efficiency factor η = 0.75.
Model # 7: Conforming elaborated calculation of repad nozzle with pad efficiency factor η = 1.0.
Model # 8: Conforming elaborated calculation of repad nozzle with pad efficiency factor η = 0.75.

Table 7: Overview computed stress concentration factors , stress intensities and MAWP's

Model	**# 1**	**# 2**	**# 3**	**# 4**	**# 5**	**# 6**	**# 7**	**# 8**
Internal Pressure (MPa)	2.617	2.362	3.493	3.493	0.719	0.635	2.81	2.62
$SCF_{vessel\ shell}$	4.27	4.24	3.07	3.08	3.86	3.80	3.39	3.32
$SCF_{nozzle\ neck}$	5.07	5.27	3.00	2.98	4.81	4.49	4.38	4.26
Stress Intensity $_{vessel\ shell}$ (MPa)	328.15 (1.585)	327.16 (1.580)	268.08 (1.295)	298.05 (1.440)	261.01 (1.261)	259.38 (1.253)	256.51 (1.239)	254.31 (1.229)
Stress Intensity $_{nozzle\ neck}$ (MPa)	389.87 (1.883)	406.66 (1.965)	262.06 (1.266)	288.10 (1.392)	325.50 (1.572)	306.56 (1.481)	331.30 (1.600)	326.42 (1.577)
MAWP (MPa) based on maximum permitted stress intensity of 3f (2Sy) at junction	2.78	2.40	5.39	4.88	0.914	0.857	3.52	3.32
MAWP (MPa) Based on undisturbed shell w/o nozzle	2.788		3.494		0.738		3.429	

The ratios in brackets indicate the quotient of stress intensity and yield strength.

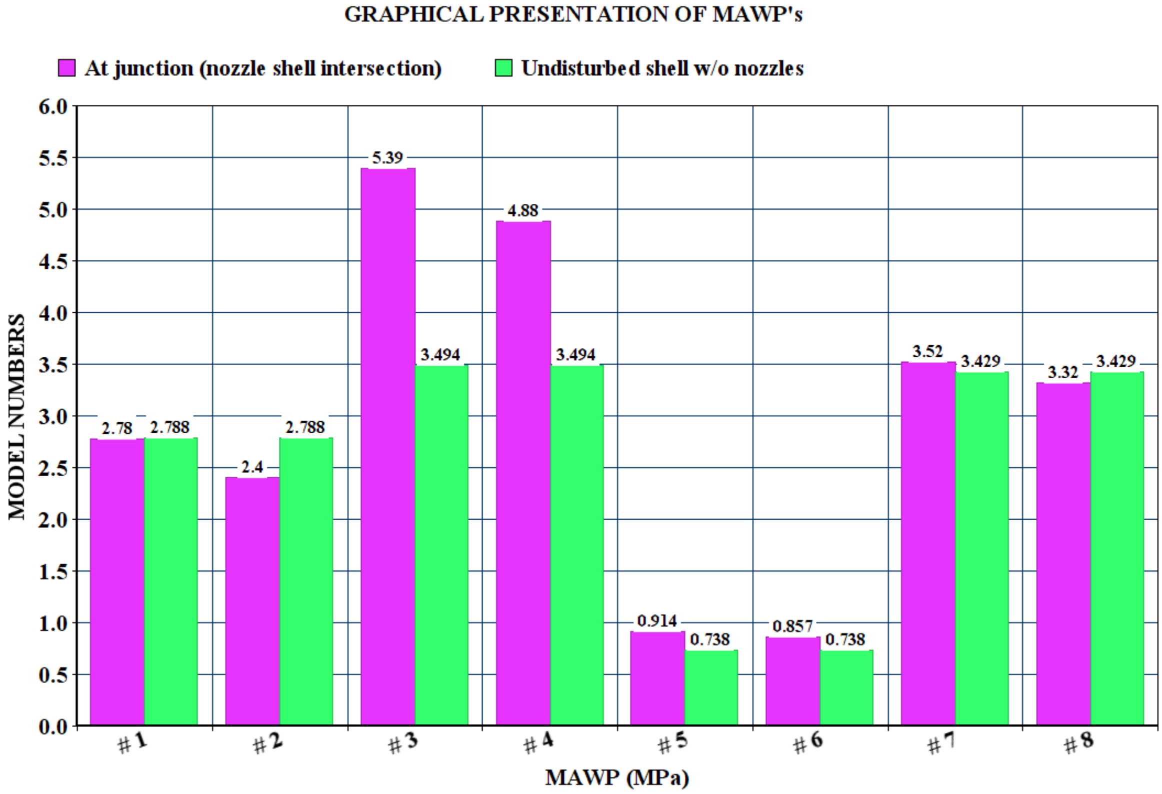


Figure 3: Graphical presentation of MAWP's

It should be emphasized that if the ratio is less than 2, there is margin left for piping reactions imposed to the nozzle. However, if this is not the case or if there is only a small margin, extra reinforcement of the nozzle should be considered so that sufficient margin is available for the stresses caused by the expected imposed piping loads. The factor of 2 x yield strength (2Sy) corresponds with the elastic shakedown criterion of 3 x design stress (3f).

Of course, it should be realized that at a lower design pressure, a larger margin is available to cope with higher nozzle loads.

Designers wishing to use finite element analysis (FEA) are advised to consult references [6]and [7] before modeling the reinforcing pad. Correct modeling of the pad is crucial for obtaining reliable results.

The results listed in Table 1 were obtained using software version 1.3 of NAM Specification for Pressure Vessels NSS 12-D-4-05 and are based on numerical analysis using FE / Pipe from PRG.

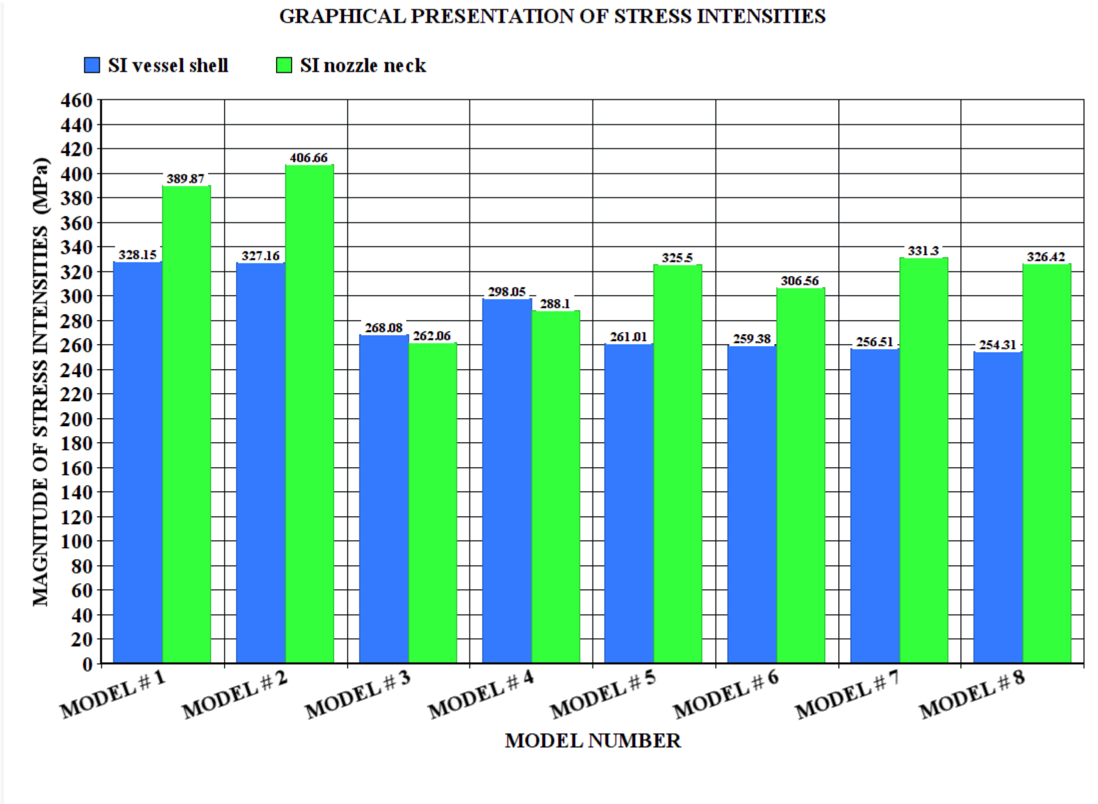


Figure 4: Graphical presentation of stress intensities

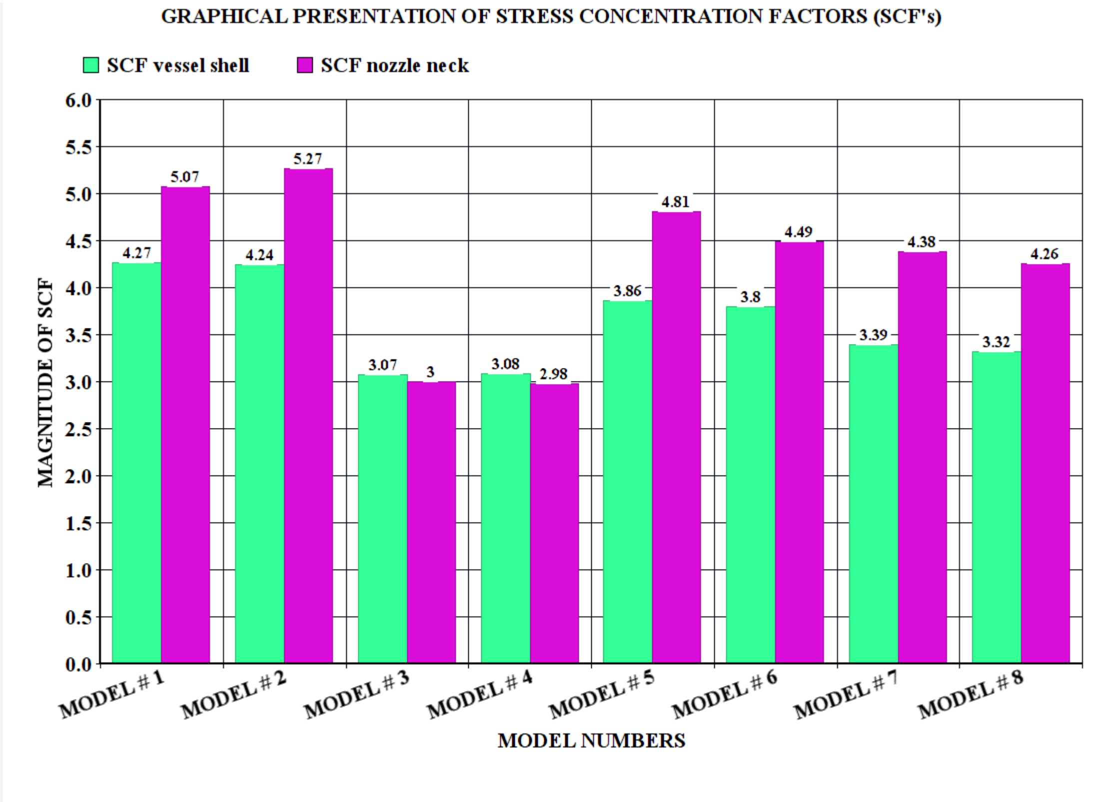


Figure 5: Graphical presentation of stress concentration factors (SCF's)

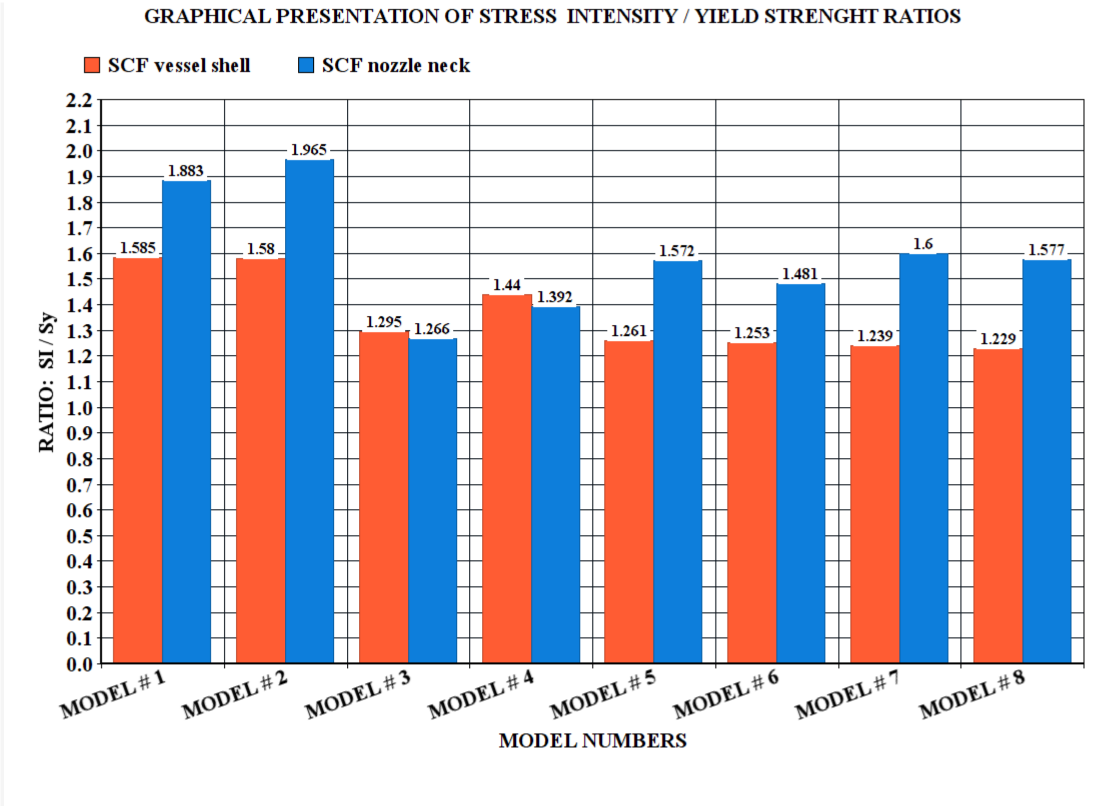
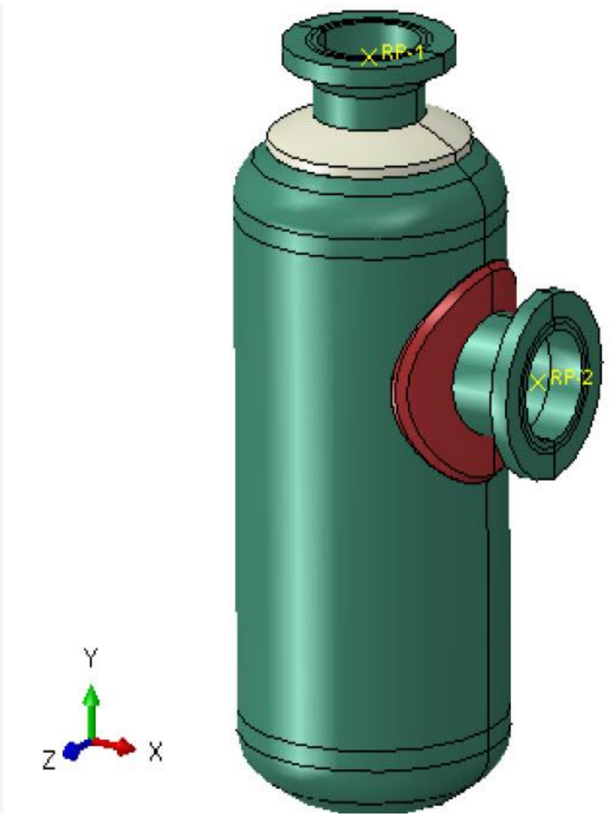


Figure 6: Graphical presentation of stress intensity / yield strength ratios

Image of typical FE model of pressure vessel with pad reinforced nozzles



IV. INFLUENCE OF PAD EFFICIENCY FACTOR ON INDIVIDUAL ALLOWABLE NOZZLE LOADS

Using the software package VES from P3 Engineering (NL), the individually permissible external nozzle loads have been computed for a number of models based on EN 13445-3. Here we will suffice with the results shown in Table 2 followed by a data analyse.

Table 8: Overview of the individual allowable nozzle loads for the nozzle configurations corresponding to models # 1 through # 8

MODEL Number	η Pad Efficiency Factor	F_R (N) Radial Force	M_C (Nm) Circumferential Moment	M_L (Nm) Longitudinal Moment	M_M (Nm) Meridional Moment
# 1	1.0	189831	31804	61083 ✶	
		97598 ✶	17656 ✶	66543	
# 2	0.75	152846	24640	53111 ✶	
		97598 ✶	17656 ✶	66543	
# 3	1.0	266843	46999	103993	
		130295 ✶	23825 ✶	82928 ✶	
# 4	0.75	214473	36336	83469	
		130295 ✶	23825 ✶	82928 ✶	
# 5	1.0	162899	44501	111308	
		53303 ✶	14386 ✶	60874 ✶	
# 6	0.75	125585	32808	95012	
		53303 ✶	14386 ✶	60874 ✶	
# 7	1.0	2361669			468511
		1265339 ✶			313902 ✶
# 8	0.75	1927904			374158
		1265339 ✶			313902 ✶

Key: Data marked in **red** applies to the location at the nozzle edge
Data marked in **blue** applies to the location at the repad edge
The determining allowable individual nozzle loads are marked with ✶

Data analysis and observations regarding Table 8

With the exception of model # 1 and # 2, the allowable individual load components are higher at the edge of the nozzle than at the edge of the repad. For models # 1 and # 2, the permissible individual load M_L is greater at the edge of the repad than at the edge of the nozzle. Argumentation for this is lacking.

The differences in the allowable individual load components at the edge of the nozzle compared to the edge of the repad are sometimes considerable. Hardly any differences can be observed in the individual allowable loads between η = 1.0 (without gap correction) and η = 0.75 (includes gap correction). Moreover the bar chart below visualizes the load ratios between the allowable individual loads at the nozzle edge and at the edge of the reinforcing pad.

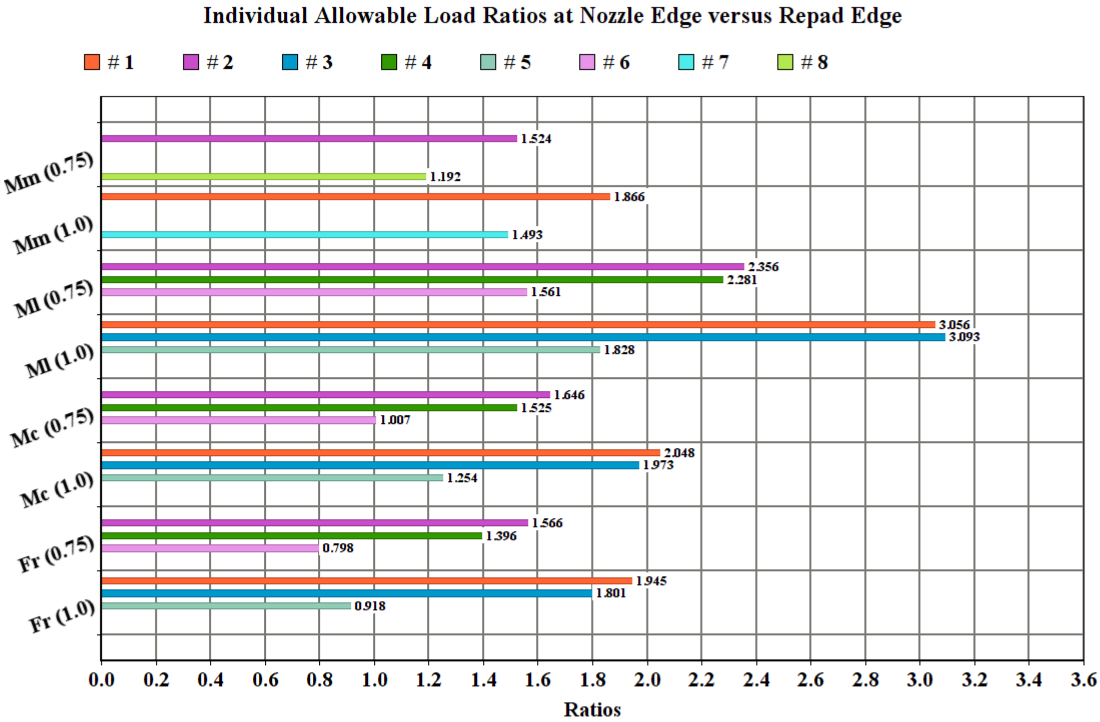


Figure 7: Individual allowable load ratios at nozzle edge versus repad edge

Note: In the appendix, a recommendation is made for taking into account a substitute (effective) pad thickness in case a FEA analysis is performed where both internal pressure and external loads are taken into account. This approach is taken from [9].

V. NUMERICAL (FEA) VALIDATION OF PAD EFFICIENCY FACTOR η

Finite element computations are performed, with software program Abaqus, to investigate if the pad efficiency factor (η) is necessary. Finite element models of different nozzle intersections are used to determine the actual MAWP. This MAWP will be compared to the MAWP obtained with the equations above (with and without η).

Different geometries of a nozzle intersection are evaluated to determine in which cases a pad efficiency factor is useful or not. The different analysed geometries are determined based on criteria in EN13445-3 [3] and PD 5500 [4]. This paper is focused on pressure vessels rather than piping. Therefore some of these criteria are narrowed down in order to obtain realistic cases. These criteria are:

$$400 < \frac{D_e - e_{a,s}}{e_{a,s}} < 100 \qquad \text{Equation 1}$$

$$0.125 < \frac{d_{e,b} - e_{a,b}}{D_e - e_{a,s}} < 0.4 \qquad \text{Equation 2}$$

$$1 \cdot e_{a,s} < e_{a,p} < 1.5 \cdot e_{a,s} \qquad \text{Equation 3}$$

$$0.3 < \frac{d_{i,b}}{D_{i,s}} < 0.8 \leftrightarrow 2 \leq e_{a,b}/e_{a,s} \leq 1 \qquad \text{Equation 4}$$

Determination by linear interpolation between 1 and 2

The vessel diameter (D_e) is set on 1200 mm. Based on this value and the criteria above different geometric cases can be defined.

The different nozzle intersections are modelled with a 1/4th (90°) shell model (element type S4). The outer edge of the reinforcement pad is tied with the vessel to simulate the weld between the repad and shell. The surface of the repad is not connected with the shell (i.e. no contact is present between the shell and repad). Symmetric boundary conditions are applied at the symmetric planes (YZ and XY plane). A reference point at the top of the nozzle is fixated in every direction. Only internal pressure is taken into account (i.e. no external force). An end cap force (corresponding with the internal pressure) is applied at the reference point which is coupled with the vessel's shell. This reference point is fixated in every direction/rotation except for the axial direction from the vessel's shell point of view. The mesh of the model is shown in Figure 7.

An elastic ideal plastic material model with a yield strength of 207 MPa is used. The MAWP of each geometry is determined based on a limit load assessment according to API 579-1 Annex 2D [10]. In this limit load assessment the pressure is increased until numerical instability is obtained. This moment corresponds with a significant region that is plastically deformed. The MAWP determined with the FEM model corresponds with the applied pressure, at approximate 20% plastic strain, divided by 1.5 (since the yield strength is used).

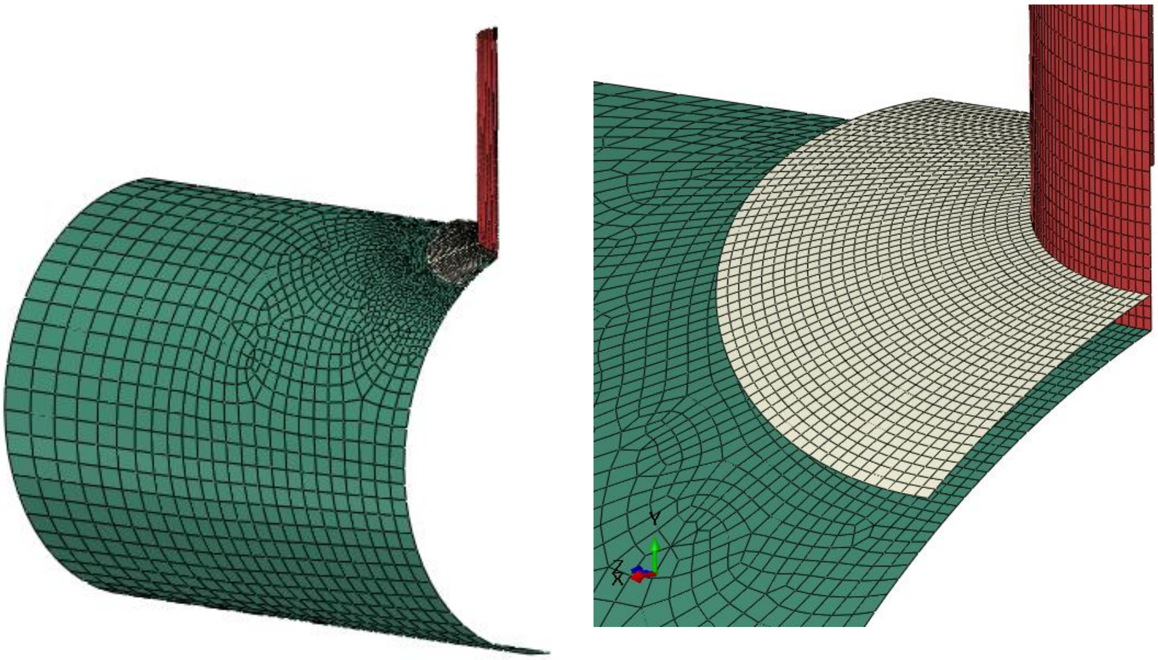


Figure 8: Mesh of the FEA model

First the geometries of the 8 case numbers defined in Table 2 are validated with FEM. This results:

CASE NUMBER	# 1 (η=1)	# 2 (η=0.75)	# 3 (η=1)	# 4 (η=0.75)	# 5 (η=1)	# 6 (η=0.75)	#7 (η=1)	# 8 (η=0.75)
MAWP (MPa)	2.617	2.362	3.494	3.494	0.719	0.635	2.82	2.63
MAWP FEA non integral (MPa)	2.78		3.97		0.768		2.61	

Note that all the MAWP values determined with FEA are more conservative then the results obtained with the manual calculations with the exception of cases 7 and 8 (cases of a nozzle in a spherical shell). For these cases the application of the pad efficiency factor do result in more or less the same MAWP as obtained with FEM. A conclusion should not be draw from these results since the cases above are arbitrary and limited in number. Therefore some more cases have been evaluated with FEM and compared to the manual calculations. Some of the results are reported in the table below. The different geometries in the table obey the criteria of equation 1 through equation 4. The figure below shows that the used cases covers a wide range of possible geometries.

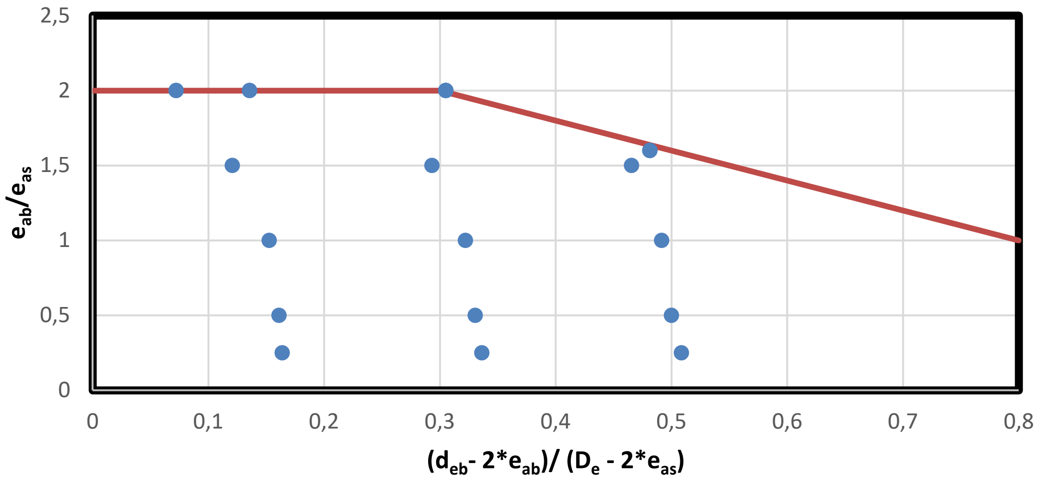


Figure 9: Illustration of the defined FEA cases in function of the used criteria

For each case the maximum allowable working pressure is determined for the vessel's shell, nozzle and the nozzle neck. The overall maximum allowable working pressure according to a level 1 assessment is therefore the minimum of these 3 values. The maximum allowable working pressure according to the FEA calculations is labelled as 'Limit Load'. The comparison between the results obtained with the level 1 assessment and the FEA computations is shown in Figure 10. In this figure the ratio of the FEM result is divided by the result obtained with the level 1 calculation. A value higher than 1 is conservative since the level 1 assessment underestimate the maximum allowable working pressure.

The first conclusion which can be made from Figure 10 and Table 9 is that in most cases the maximum allowable working pressure of the vessel's shell is the limiting factor rather than the nozzle neck. Only in 5 cases the nozzle neck is the limiting factor according to the level 1 assessment. Other geometries, for which the nozzle neck is the limiting factor, can be defined. However these geometries are in general more uncommon for pressure vessels (e.g. very small wall thicknesses, larger diameter ratio).

For 5 cases the nozzle neck is the limiting factor. From these 5 cases is 1 case not conservative (MAWP FEA/ MAWP level 1 ratio of 0.92). The level 1 assessment on this case would however be conservative when a pad efficiency factor of 0.75 is applied (ratio increases to 1.02).

Table 9: Geometry and results of the different cases

D_e	d_{eb}	e_{as}	e_{ab}	e_{ap}	L_p	MAWP (shell)	MAWP (nozzle)	MAWP (nozzle neck)	MAWP (manual)	MAWP (FEM)
1200	400	10	20	10	30	2.3	13.8	2.41	**2.30**	**2.38**
1200	400	10	20	15	150	2.3	13.8	3.89	**2.30**	**2.65**
1200	400	10	20	10	100	2.3	13.8	2.88	**2.30**	**2.64**
1200	400	10	10	10	100	2.3	6.9	2.10	**2.10**	**2.28**
1200	400	10	5	10	100	2.3	3.45	1.82	**1.82**	**1.82**
1200	400	20	5	20	100	4.6	3.45	3.87	**3.45**	**3.76**
1200	400	20	30	20	100	4.6	20.7	5.70	**4.60**	**5.17**
1200	200	10	20	10	30	2.3	27.6	3.15	**2.30**	**2.64**
1200	200	10	20	15	150	2.3	27.6	5.42	**2.30**	**2.65**
1200	200	10	5	10	100	2.3	6.9	2.68	**2.30**	**2.56**
1200	200	20	5	20	100	4.6	6.9	5.45	**4.60**	**5.02**
1200	200	20	30	20	100	4.6	41.4	7.47	**4.60**	**5.31**

1200	600	10	16	10	30	2.3	7.36	1.66	**1.66**	**1.88**
1200	600	10	16	15	150	2.3	7.36	2.76	**2.30**	**2.55**
1200	600	10	16	10	100	2.3	7.36	2.01	**2.01**	**2.22**
1200	600	10	10	10	100	2.3	4.6	1.62	**1.62**	**1.75**
1200	600	10	5	10	100	2.3	2.3	1.38	**1.38**	**1.28**

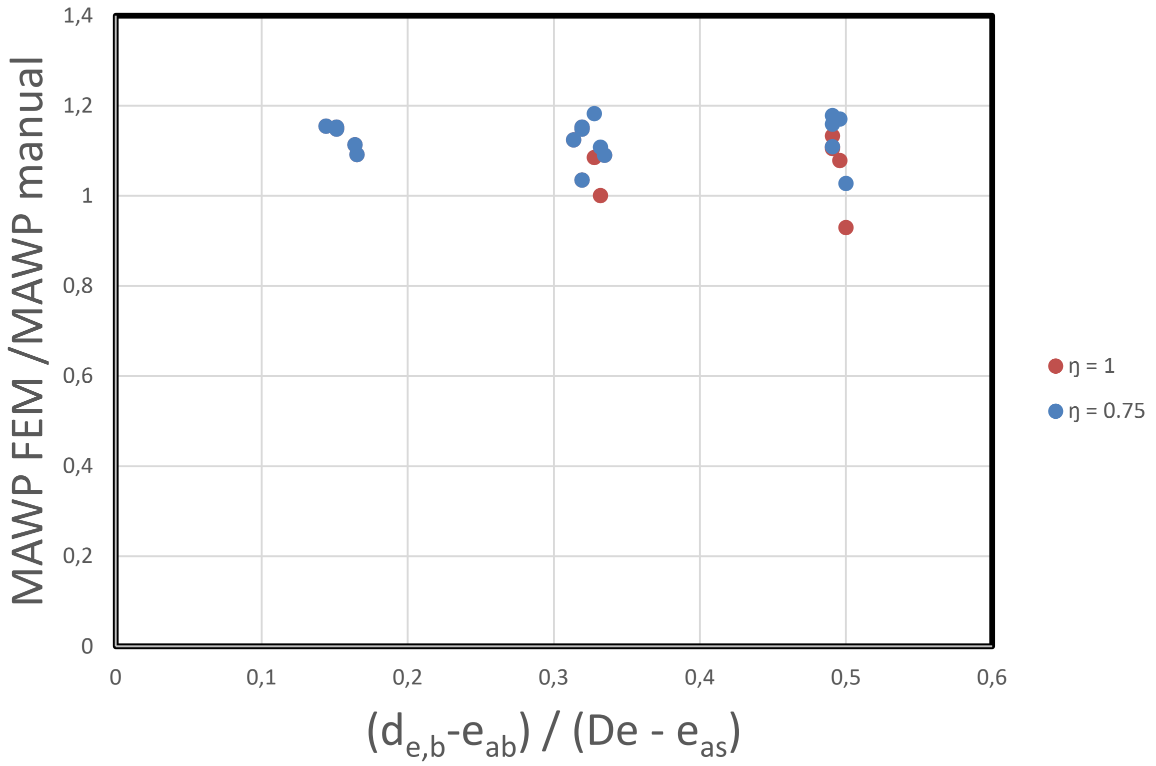


Figure 10: Comparison FEA results versus manual calculations

VI. DISCUSSION

To compensate the inevitable weakening effect of nozzle penetrations in pressure vessel walls, it is industry practice to incorporate reinforcing pads (repads) around the nozzle. The reinforcement pads have a stress-reducing effect on the local stresses at the nozzle-shell junction, and thereby lead to an increased internal pressure capacity. The use of reinforcing pads prevents the use of a thicker shell to achieve an equal pressure capacity, which can be economically unattractive. From an economic (cost) point of view nozzles with inherent reinforcement should be considered. Furthermore, it should be remembered that reinforcing pads should be avoided in situations where loads are not predominantly static such as dynamic loadings from agitators or mixers. The same caution should be considered for nozzles operating under low temperature where the risk of brittle fracture should be prevented. By including a pad efficiency factor in the calculation of the cross-sectional area of the pad within its compensation limits, the presence of a gap is eliminated by successively multiplying the pad thickness by this factor. This means that the total shell thickness is equal to the sum of shell thickness plus pad thickness x pad efficiency factor and to be considered as an integral insert shell plate.

As the ratio between A_{fp} and (A_{fs} + A_{fb} + A_{fw}) increases, the ratio $P_{max;\ \eta\ =\ 1.0}$ / $P_{max;\ \eta\ =\ 0.75}$ will also increase. For a number of investigated cases, the ratios vary between approximately 1.07 and 1.13. With the exception of the Model # 3 / # 4 configuration, where the undisturbed cylindrical shell determines the permissible internal pressure. As the contribution of A_{fp} relative to A_{fs}, A_{fb} and A_{fw} increases, the allowable pressure will also increase. The introduction of the pad efficiency factor η (if < 1.0) has a slightly reducing effect on the permissible pressure.

VII. FINDINGS

For model cases # 1, # 2, # 5, # 6, # 7 and # 8, the stress intensities in the nozzle neck are higher than in the cylindrical or spherical shell while for model cases # 3 and # 4, the stress intensities in the shell are slightly higher than in the nozzle neck.

For model cases # 1,# 2, # 5, # 6, # 7 and # 8, the nozzle intersection determines the allowable internal pressure. For model cases # 3 and # 4 the undisturbed shell is determining for the allowable internal pressure. Table 2 shows that there are significant differences between the individually allowable nozzle loads for a pad efficiency of 1.0 compared to 0.75. Apparently the gap has a significant negative effect on the permissible external nozzle load as opposed to the small effect on the internal pressure capacity.

VIII. CONCLUSION

FEM analysis are performed to compare the manual calculations with and without pad efficiency factor with numerical results. Due to the criteria described in equation 1 through equation 4 is the maximum allowable working pressure of the vessel in most cases the limiting factor. It is however possible that in some cases the manual calculation (w/o pad efficiency factor) is not conservative, see Figure 9. By applying a pad efficiency factor of 0.75 the non-conservatism is resolved for the cases which are analysed in this paper. It can be concluded that the application of a so-called efficiency factor η of 0.75 when using a reinforcing pad to compensate for the weakening of a nozzle penetration into a vessel wall is a responsible and safe choice and is recommended. Code writing bodies are advised to take a prudent approach to this issue, whereby this article could provide a valuable boost.

IX. CLOSING REMARKS

A nozzle assessment according to EN 13445-3[3] is not always conservative with respect to the FEA results. It is therefore proposed to apply an efficiency factor of 0.75 and in practice does this means that the pad area must be multiplied by that factor. In principle, the pad efficiency factor can be omitted if it is established that the stress in the nozzle neck is decisive.

An extra remark had to be made about the limited amount of cases, which has the nozzle neck as limiting factor, that are assessed in this paper. It is however allowed to perform the level 1 assessment on a broader range of geometries. The same comparison as in this paper should be made for the entire range of permitted geometries. Based on these results a better understanding of the conservatism of the level 1 assessment can be obtained. It is the intention to present this in a follow-up paper.

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Typical repad configuration

ANNEX

This annex contains a number of collected opinions of recognized experts who have dealt with this topic. Not all views on this matter are uniform. Typically, many rely on their personal experience because no failures have occurred in their practice. Few have attempted to substantiate their position through numerical analysis, albeit on a limited scale. When performing finite element analysis (FEAs), one often encountered difficulties in modeling the gap between the reinforcing pad and the vessel wall. It is important for the design engineer to consider the various points of view on this matter and therefore to be able to make a prudent decision regarding whether or not to take a pad efficiency factor into account in case of pad reinforced nozzles in pressure vessels.

The regions in *italics* presented below are a mix of quotes and personal interpretations of previous discussions with consulted experts.

Most of the codes in use today, such as EN13445, PD 5500, AD 2000 and ASME, do not use an efficiency factor.

The ASME VIII Div. 2 also uses the pressure - area method.

DBA (Design by Analysis) and DBF (Design by Formula or DBR (Design by Rule)) are nowadays almost 1 to 1 on top of each other in terms of conservatism. In other words, the DBF is almost as good as the DBA - although one cannot gain much more with the FEM method compared to the DBF method.

However, this DBF method sets requirements for the welding configuration that must meet the minimum required dimensions. Welding configuration also has requirements in EN13445.

As it looks, the design has evolved from the efficiency factor according to requirements regarding the welded joints to which the reinforcement ring is connected on the shell and on the nozzle and shell at the nozzle.

It cannot be confirmed that the resistance to collapse for gross plastic deformation of a pad reinforced shell is exactly equal to the resistance of a single wall shell whose thickness is equal to the sum of the pad and shell. This assumes welds can assure the proper transfer of load between shell, the pad and the nozzle. It can be assumed that the static pressure capacity will not be effected by the geometrical gap between the pad and the shell.

Firstly, the failure mode for nozzle connections is generally taken to be progressive plastic deformation due to the high local stresses at the nozzle to shell junction, rather than gross plastic deformation.

In PD 5500 the designer is permitted to use a design by analysis approach if it is considered that the design is not covered by the rules in section 3.The resistance to collapse for gross plastic deformation of a pad reinforced shell is exactly equal to the resistance of a single wall shell whose thickness is equal to the sum of the pad and the shell. This assumes the welds can assure the proper transfer of load between the shell, the pad and the nozzle. However the situation is different if we consider other failure modes, such as alternate plasticity or fatigue: in such cases of course stress concentrations and thermal stresses caused by the gap between the pad and the shell may cause problems, but certainly not a failure for gross plastic deformation. The only case in which an efficiency of the pad could be required, is the case where the pad itself has a weld perpendicular to the maximum circumferential stress. The main PV standards do not consider an efficiency caused by the gap.

Of course the fact that a gap exists between a pad reinforcement and the shell on which it is applied is a normal characteristic of this type of reinforcement. This is certainly a problem from some point of views:

possibility of weld defects in the root of the welds made from one side only, greater sensitivity to fatigue because of the notch effects due to the existence of the gap. In fact all the main Pressure Vessel standards (among them ASME VIII division 1 and 2 and the German standard AD 2000) do not consider a pad efficiency: but some of them impose a specific calculation of the weld connecting the above components. Also in EN 13445, although a specific calculation of the welds is not provided, there are limitations concerning the minimum weld dimensions aimed to assure a sufficient strength. What of course has to be taken into account (although not explicitly specified) is the negative tolerance of the reinforcing plate. Therefore, in my opinion, there is no need to consider a pad efficiency, which on the contrary would result in an unnecessary increase either of the pad thickness or of the pad width, with a possible worsening of the already mentioned problems.

When a repad is used, the nozzle is no longer recognized as being integrally reinforced because of uncertainty in determining the transfer of load between the shell, the repad, and the nozzle. Since this subject matter is not addressed by the recognized codes and standards, it is suggested to introduce a pad efficiency factor of 0.75 which is intended to compensate for the gap between pad and shell.

Although certain areas of the shell and reinforcing plate could be in contact the reinforcing plate is expected to only be in full contact with the shell upon radial expansion of the shell under hydrostatic loading.

In the thin shell analysis of welded pad reinforced nozzles in pressure vessels, contact between the pad and the vessel is often assumed. The significance of this contact force to the stress distribution in the structure is little known. With the aid of FEA software, the contact behavior between the vessel shell and the reinforcement pad under the internal pressure can be analyzed. Not only the contact deformation and the contact pressure can be analyzed, but also the different gaps between the shell and the pad, the stiffness factor and the influence of different diameters of shells on contact can be discussed as well. For welded pad reinforced nozzles, a better prediction of the stress distribution between the pad and the vessel can be made.

Reinforcing pads arranged around nozzle give a reinforcing effect. However, the gap between the pad and the vessel shell which is always physically present actually reduces the amplifying effect to some extent. Absence of local contact between pad and shell affects the stress distribution around the nozzle.

The effect of a geometric gap between the cylindrical shell and reinforcement pad on the local stresses in the area of the intersection under internal pressure on the nozzle. Finite element analyses was performed on a number of pad reinforced nozzle configurations with different gaps. The results indicate that for a pressure loaded nozzle the effect of a gap on the stresses respectively on pressure capacity is not one that would ordinarily be expected.

For those which considered pad reinforced intersections, the pad is assumed to be an integral part of the model, which does not represent the real condition.

In order to consider a more realistic model of pad reinforced cylindrical intersections, it is necessary to model the welding line between the reinforcing pad and the nozzle/vessel walls and to assume contact between the external surface of the vessel and the internal surface of the reinforcing pad. The problem becomes clearly of nonlinear nature and, consequently, the simulation is more complex and more time consuming. Stress analysis on vessel/nozzle intersections was performed for cylindrical pressure vessels both non-reinforced and pad reinforced using finite element models. On phase 1, results for non-reinforced vessel models were compared to experimental data from the literature, validating the 3D 20-node element model.

Models representing two conditions of pad reinforcement (bonded pad and partially welded) were then analyzed and results were compared. Pad reinforcement significantly reduced both tangential and radial peak stresses on the vessel/nozzle region, corroborating the ASME Code Criteria – Area Replacing Method requirements for vessel/nozzle intersection reinforcement.

Although negligible for the nozzle region, bonded and partially welded pad reinforced models, however, presented higher differences on stress levels for the vessel region for both tangential and radial stresses. The partially welded pad reinforced model presented tangential stresses 20% higher and radial stresses were significantly higher than the bonded pad reinforced model. This fact suggests that a more accurate representation of the real configuration of the structure should be attempted in order to make adequate predictions. Results can be used by designers as guidelines for modelling reinforced cylindrical vessel/nozzle intersections.

DBA shall not be confused with DBF. There are many different possibilities to model a reinforcing pad by FEM analysis: which kind of elements? Shell elements or brick elements? And what kind of analysis: classical elastic analysis (Annex C of EN 13445-3) or limit analysis (Annex B of EN 13445-3)? And how do you model the gap? And the welds joining the pad to the shell? Are you imagining a physical gap, defining the gap (constant?) thickness and the relevant radii? And how to simulate the weld defects in the root of the welds? According to the decision you take you will probably obtain substantially different results. All the main PV standards simply

assume that a pad reinforcement may be considered equivalent to a single shell whose thickness is given by the sum of two thicknesses. This assumption is simply based on experience, and I do not think that it can be verified by a FEM analysis, since different methods of analysis may lead to completely different results. Therefore the only verification is provided by experience: of course having clarified the numerous problems created by this type of reinforcement: thermal stresses, stress concentrations not adequate for cyclic loading, need of considering also a calculation of the weld strength (which sometimes is missing in the standard). At the end, a self-reinforced nozzle has less problems and (at least in the Western countries) is also more economic.
TRD 301 included the efficiency factor as mentioned (see attachment). AD 2000-Merkblatt B 9 has also considered implementing this factor. But, following a proposal of some interest groups, it was agreed some 7 years ago to abolish this factor. Regarding the reinforcement pads it was assumed – taking into account the geometric limitation - that the nozzle weld and the weld on the pad edge transfer enough tension to the material below (i. e. shell of pressure equipment).
The TRD - Code was finally replaced 2012-12-31 by the requirements of EN 12952 / 12953 Series, whereas some requirements of the TRD - Code have not been taken into account.
Where a pad is used, it is regarded as being as effective as if the extra material were provided by a thicker vessel plate, so a good fit to the vessel shell is required.

APPENDIX

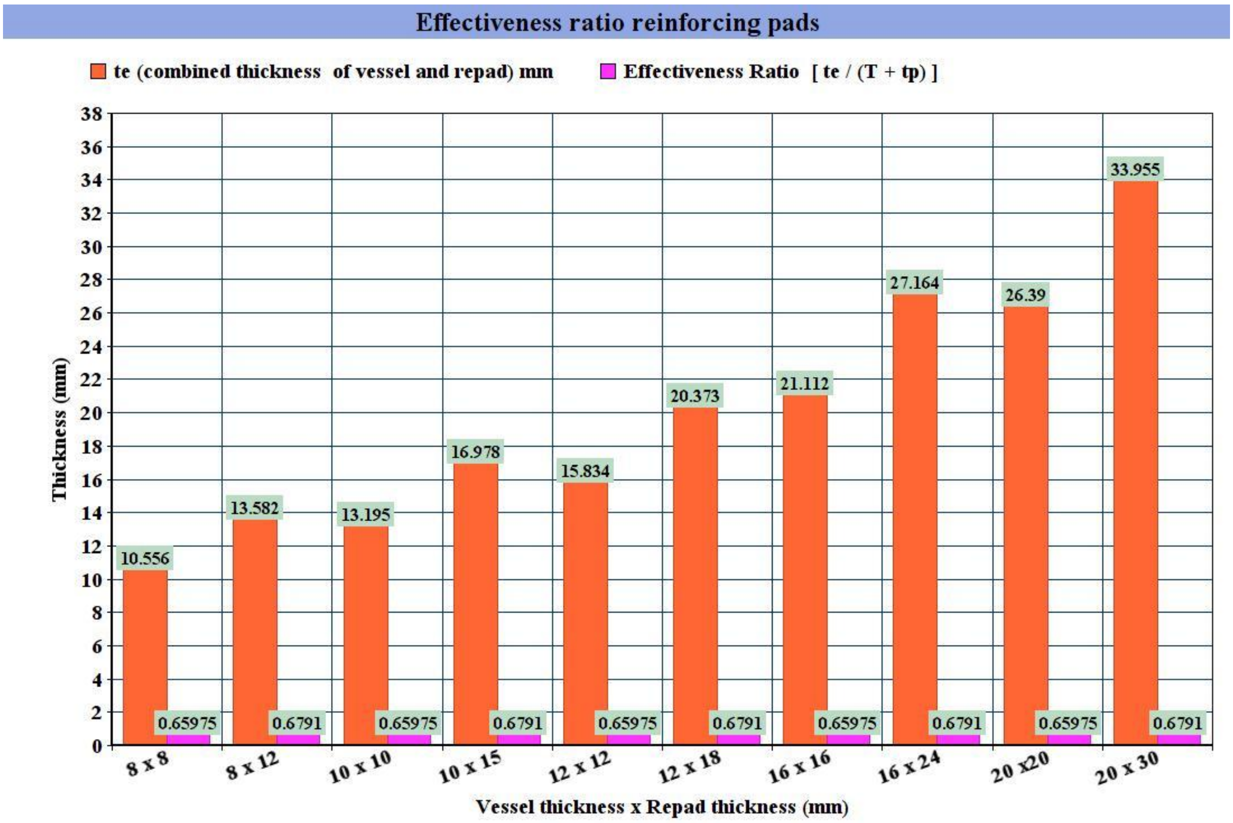
Vessel plus reinforcing pad combined thickness

When performing numerical FEA analysis, it is recommended to observe the following:
The effective combined vessel shell plus pad thickness (t_e) is given by the following equation:

$$\boldsymbol{t_e} = (\boldsymbol{T}^{2.5} + \boldsymbol{t_p}^{2.5})^{0.4}$$

T = vessel thickness (mm)	**t_p = thickness of reinforcing pad (mm)**	**t_e = combined thickness of vessel and repad (mm)**	**η = effectiveness ratio t_e / (T + t_p)**
8	8 (1 x T)	10.556	0.65975
8	12 (1.5 x T)	13.582	0.67910
10	10 (1 x T)	13.195	0.65975
10	15 (1.5 x T)	16.978	0.67910
12	12 (1 x T)	15.834	0.65975
12	18 (1.5 x T)	20.373	0.67910
16	16 (1 x T)	21.112	0.65975
16	24 (1.5 x T)	27.164	0.67910
20	20 (1 x T)	26.390	0.65975
20	30 (1.5 x T)	33.955	0.67910

Analysis should not based on a total pad plus vessel thickness, but on a thinner effective pad thickness that represents the bending stiffness of a separate pad and vessel shell, which is conservative for local membrane stress.
In case local membrane stresses govern the maximum loads, then use the full thickness in the analysis.
In most cases this means that if only internal pressure is considered, one can take into account the total thickness of vessel shell and reinforcing pad.
If both external load and internal pressure are to be considered (i.e. nozzle load analysis) it is recommended to take the equivalent thickness (t_e) into account.


Effectiveness ratio reinforcing pads
te (combined thickness of vessel and repad) mm
Effectiveness Ratio [te / (T + tp)]
Thickness (mm)
Vessel thickness x Repad thickness (mm)
8 x 8
8 x 12
10 x 10
10 x 15
12 x 12
12 x 18
16 x 16
16 x 24
20 x20
20 x 30
10.556
13.582
13.195
16.978
15.834
20.373
21.112
27.164
26.39
33.955
0.65975
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