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Loss Coefficients And Penalty Factor As Useful Tools In Coordinating Transmission Line Losses In The Optimal Scheduling Of Plants To Save Cost In Power System Operation.

*[1] M.N.Eleanya, [2] V.C.Ogboh, [3] U.E.Anionovo.
Department of Electrical Engineering, Nnamdi Azikiwe University Awka, Anambra State, Nigeria.

Abstract
In operating the power system for a load condition, the contribution from each unit within a plant or plant within a system, must be determined so that the cost of delivered power is a minimum. In this paper, two typical problems of economic dispatch are considered. In Problem 1, various loads of units within a plant are optimally scheduled without consideration of transmission losses. The saving in naira per hour is determined for each case considered through cost of generation for each unit. But it is well known that whenever power is transmitted or delivered, losses are bound to occur. And how these power losses could be coordinated rather than neglected in the optimal scheduling of plants within a system through the application of loss coefficients and penalty factor, is what problem 2 addressed. In both cases, the ultimate objective is to save cost in power system operation and this of course, is very evident from the results obtained.

Keywords: *Optimal scheduling, savings in cost, incremental fuel cost, loss coefficients, penalty factor.*

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I. INTRODUCTION:

An Engineer is always concerned with the cost of product and services. For a power system to return a profit on the capital invested, proper operation is very important. Rates fixed by regulatory bodies and the importance of conservation of fuel, place extreme pressure on power companies to achieve maximum efficiency of operation and to improve efficiency continually in order to maintain a reasonable relation between cost of a kilowatt-hour to a consumer and the cost to the company of delivering a kilowatt-hour in the face of constantly rising prices for fuel, labour, supplies and maintenance. By way of definition, optimal scheduling or loading is the process of apportioning the total load on a system between various plants within a power system and units within a plant to achieve the greatest economic of operation [1]. The main aim in the economic dispatch problem is to minimize the total cost of generating real power (production cost) at various stations while satisfying the loads and the losses in the transmission links. To determine the economic distribution of load between the various units consisting of a turbine, generator and steam supply, the variable operating costs of the unit must be expressed in terms of the power output. Fuel cost is the principal factor in fossil fuel plants and cost of nuclear fuel can also be expressed as a function of power output. In this paper, discussion is based on the economics of fuel with the realization that other costs which are a function of power output can be included in the expression for fuel cost.

II. ECONOMY OF OPERATION

It is assumed, that it is known a priori which generators are to run to meet a particular load demand on the station. Therefore given a station with K generators committed as well as the active power (demand) load, P_d, the real power generation, P_{gi}, for each generator has to be allocated so as to minimize the total cost. Hence;

$$C = \sum C_i\ (P_{gi}) \text{ ₦/hr} \tag{1}$$

Subject to the inequality constraint;

$$P_{gi\,min} \le P_{gi} \le P_{gi\,max} \quad \text{for } i = 1, 2 \text{ ---- } K \tag{2}$$

Where;
$P_{gi\,min}$ is lower real power generation
.
$P_{gi\,max}$ is upper real power generation.

Also;

$$\sum_{1=1}^{k} \mathrm{P_{gi\,max}} \geq \mathrm{Pd} \quad [2] \qquad (3)$$

However, consideration of spinning reserve, requires that;

$$\sum_{1=1}^{k} \mathrm{Pgi\ max} \geq \mathrm{Pd} \qquad (4)$$

In equation (1), it is assumed that the cost, C, is largely dependent on the real power generation P_{gi} and is insensitive to reactive power generation, Q_{gi} [3].

It is also assumed that the inequality constraint of equation (3) is not effective and hence;

$$\sum_{1=1}^{k} \mathrm{Pgi} - \mathrm{Pd} = 0 \qquad (5)$$

This problem can be solved by the method of Lagrange multipliers which is used for minimizing or maximizing a function with side conditions in the form of equality constraints. Using this method, an augmented cost function is defined as [3] ;

$$C^* = C - \lambda\left(\sum_{i=1}^{K} P_{gi} - P_d\right) \qquad (6)$$

Minimization is achieved by the condition;

$$\frac{\partial C^*}{\partial P_{gi}} = 0 \qquad (7)$$

Or;

$$\frac{\partial ci}{\partial Pgi} = \lambda \quad for\ \ i = 1\,,2,3, - - - - -, k \qquad (8)$$

Where;

$$\frac{\partial ci}{\partial Pgi} \quad is\ the\ incremental\ fuel\ cost$$

In the i^{th} generator with units as naira per megawatt-hour , (₦/Mwh);

$$\frac{\partial c_1}{\partial Pg_1} = \frac{\partial c_2}{\partial Pg_{\,2}} = - - - - - = \frac{\partial c_k}{\partial Pg_k} = \lambda \qquad (9)$$

Hence the optimal loading of generators corresponds to the equal incremental cost point of all generators. Equation (9) is the coordination equation numbering K and it is solved simultaneously with the load demand equation (ie. Eqn. 5) to yield a solution for the Lagrange multiplier, λ , and the optimal generation of K generators.

III. LOAD DISTRIBUTION BETWEEN UNITS IN A PLANT;

An early method to minimize the cost of power supply was to operate the most efficient unit at light load. As the load increased, power would be supplied by the most efficient unit until the point of maximum efficiency was reached. To take up further load, the next most efficient plant would start to feed power to the system. This method does not however minimize cost since the most efficient plant may not be the most economical [1]. To determine the economic distribution of load between various units in a plant (or various plants within a system) without consideration of transmission losses, the variable operating cost (mainly fuel cost) of the units or plants must be expressed in terms of power output as shown in figure 1.

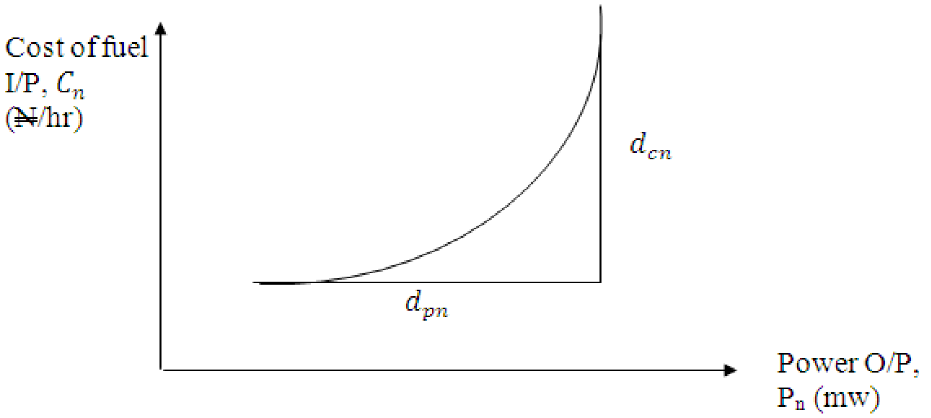


Fig 1.0: I/P Curve for a generating unit showing cost of fuel input Vs power output.

C_n = Input fuel cost to unit n (₦/hr)
P_n = Output power from unit n (MW)

The slope of the curve at any point (i.e . $\frac{dc_n}{dP_n}$), gives the increase in fuel cost (or operating cost) for a small increase in power output and is called the incremental fuel cost, λ. Approximately, the incremental fuel cost, λ, could be obtained by determining the increased cost of fuel (or operating cost) for a definite interval during which the power output is increased by a small amount. For instance, the approximate incremental cost at any particular output is the additional cost in naira per hour to increase the power output by 1MW. A plot of incremental fuel cost against power output, gives nearly a linear relationship.

Let us consider two units named n_1 and n_2, sharing a load at different incremental fuel cost, λ as illustrated in figure 2.

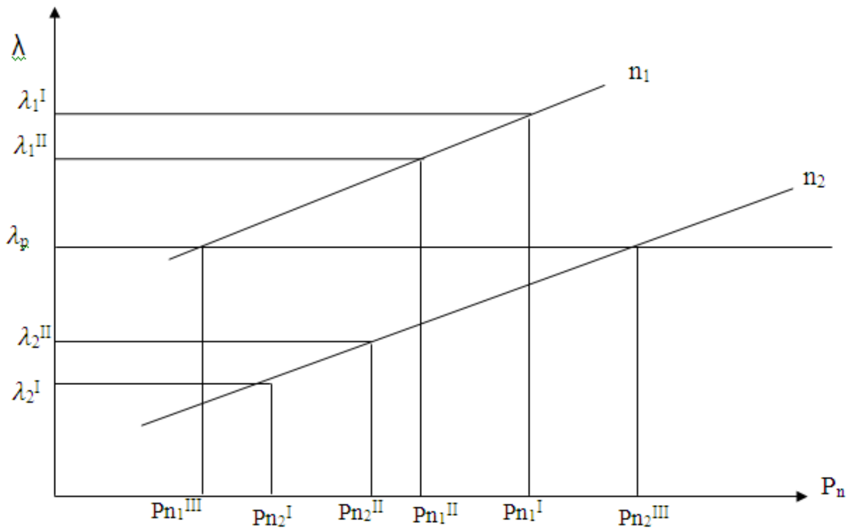


Fig. 2.0: Plot of incremental fuel cost, λ, vs Power output, P_n

From figure 2.0, reducing the load with higher incremental fuel cost, λ, (i.e unit 1) from Pn_1^{I} to Pn_1^{II}, reduces its incremental fuel cost from λ_1^{I} $to\ \lambda_1^{II}$. Now putting the same load on the unit with lower incremental fuel cost (i.e unit 2), raise its incremental fuel cost from $\lambda_2^{I}\ to\ \lambda_2^{II}$.

From the plotting, it is seen that reducing the load on the unit with higher incremental cost, will result in greater reduction of cost than the increase in cost for adding the same amount of load to the unit with lower incremental fuel cost. The transfer of load from one unit to the other can be continued with a reduction in total fuel costs until the incremental fuel cost of the two units are equal. The same reasoning can be extended to a plant with more than two units. Thus the criterion for economic division of load between units within a plant, is that all units must operate at the same incremental fuel cost [1]. The incremental fuel cost, λ, at this point of operation is called the plant incremental fuel cost (plant Lambda, λ_p).

IV. OPTIMAL SCHEDULING:

Optimal Scheduling based on what has so far been said, is illustrated by solving a typical economic dispatch problem referred to as problem 1 in this paper.

Problem 1: A plant consists of two units. The incremental fuel cost in naira per megawatt-hour for the units are approximated by the following equations:-

$$\frac{dc_1}{dp_{g1}} = 0.40p_{g1} + 80.00 \quad ; \quad \frac{dc_2}{dp_{g2}} = 0.80p_{g2} + 60.00$$

The generator limits are as follows:-

$$60MW \leq p_{g1} \leq 350MW \quad ; \quad 40MW \leq p_{g2} \leq 250MW$$

It is assumed that both units are operating at all times. The problem is therefore to determine how a total plant load of 180MW, 300MW, 420MW and 550MW will be optimally scheduled between the two units and then the savings in naira occasioned by the optimal scheduling.

Solution 1:

For unit 1:

λ_{min} = 0.40 X 60 + 80.0 = ₦$104.0/mwh$

λ_{max} = 0.40 X 350 + 80.0 = ₦$220.0/mwh$

For Unit 2:

λ_{min} = 0.80 X 40 + 60.0 = ₦$92.0/mwh$

λ_{max} = 0.80 X 250 + 60.0 = ₦$260.0/mwh$

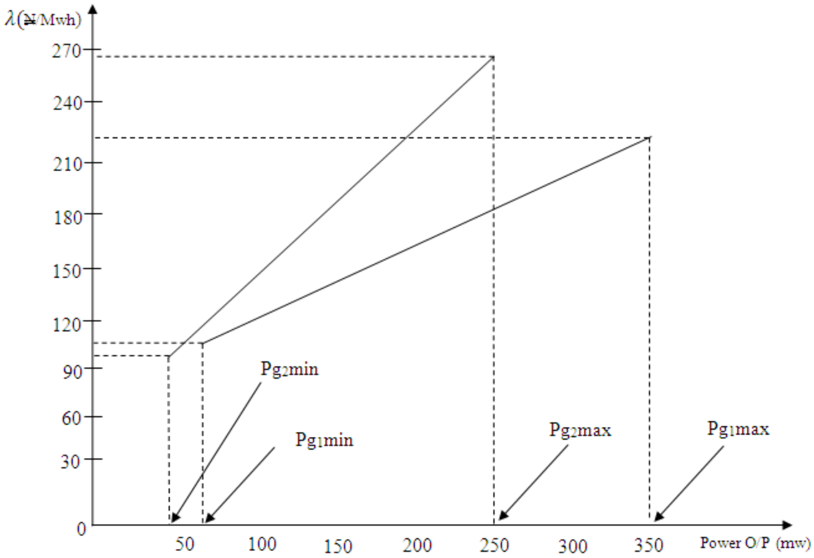


Fig. 3.0: Incremental fuel cost curve for the plant

$\lambda_2 < \lambda_1$ but for optimal scheduling, $\lambda_2 = \lambda_1$.
Therefore, as the plant load increases, the additional load should come from unit 2 until its incremental fuel cost equals that of unit 1; and until that point is reached, the incremental fuel cost of the plant, will be determined by unit 2 alone.

To determine the load at which λ_2 equals λ_1, we have that;
$0.80P_{g2} + 60.0 = 104$
This implies that P_{g2} = 55mw; and at this load, the total power demand, is; $P_d = P_{g1} + P_{g2}$ = 60 + 55 = 115mw.

Case1: Total load demand, P_d, of 180 mw:
Optimal scheduling demands that;
$0.40P_{g1} + 80.00 = 0.80\,(180 - P_{g1}) + 60.00$
Solving, gives;
P_{g1} = 103.33mw; P_{g2} = 76.67mw; λ_p = ₦121.33/mwh

Case 2: Total load demand, P_d, of 300 mw:
Optimal scheduling demands that;
$0.40P_{g1} + 80.00 = 0.80\,(300 - P_{g1}) + 60.00$
Solving, gives;
P_{g1} = 183.33mw; P_{g2} = 116.67mw; λ_p = ₦153.33/mwh

Case 3: Total load demand, P_d, of 420mw:

optimal scheduling demands that;
$0.40P_{g1} + 80.00 = 0.80\,(420 - P_{g1}) + 60.00$
Solving, gives;
P_{g1} = 263.33mw; P_{g2} = 156.67mw; λ_p = ₦220.00/mwh

Case 4: Total load demand, P_d, of 550MW:
Optimal scheduling demands that;
$0.40P_{g1} + 80.00 = 0.80\,(550 - P_{g1}) + 60.00$
Solving, gives;
P_{g1} = 350MW; P_{g2} = 200MW; λ_p = ₦220.00/mwh

The optimal loading of the two units is as displayed in table 1.0

Table 1.0: Optimal loading for the two units:

Case	Total load demand, P_d ($P_{g1} + P_{g2}$) MW	Unit 1 P_{g1} (MW)	Unit 2 P_{g2} (MW)	Plant incremental fuel cost, λ_p (₦/MWh)
1	180	103.33	76.67	121.33
2	300	183.33	116.67	153.33
3	420	263.33	156.67	185.33
4	550	350.00	200.00	220.00

V. SAVINGS IN OPERATING COST FROM OPTIMAL SCHEDULING:

To determine the savings in Operating cost resulting from optimal scheduling to see if at all, there is any justification for this method, the individual cases in section 4 are considered in turn.

Case 1: Total load demand of 180mw:
Optimal scheduling:- P_{g1} = 103.33mw; P_{g2} = 76.67mw
Equal Load distribution:- P_{g1} = 90mw; P_{g2} = 90mw

The cost of generation for unit 1 for a P_d of 180mw is;
$$c_1 = \int \frac{dc_1}{dp_{g1}}\, dp_{g1} = \int (0.40p_{g1} + 80.00)dp_{g1}$$
$= 0.20\,P_{g1}^2 + 80P_{g1} + K_1$ ₦/hr

Where K_1 is a constant
Hence the increase in cost for unit 1 is;
$C_1 = \left[0.20P_{g1}^2 + 80P_{g1} + k_1\right]_{103.33}^{90}$
= 8820 – 10401.82 = - ₦1,581/hr

While for unit 2, it is;
$$c_2 = \int \frac{dc_2}{dp_{g2}}\, dp_{g2} = \int \left(0.80p_{g2} + 60.00\right) dp_{g2}$$

$$c_2 = 0.40P_{g2}^2 + 60\, p_{g2} + k_2\ ₦/hr$$

Where K_2 is a constant

Hence the increase in cost for unit 2 is;

$$C_2 = \left[0.4oP_{g2}^2 + 60P_{g2} + k_2\right]_{76.67}^{90}$$

= 86.40 – 6951.52 = ₦1,688.48/hr
Therefore, the net saving caused by optimal scheduling is;
- ₦1,581.82/hr + ₦1,688.48/hr = ₦106.66/hr

Case 2: Total load demand of 300MW:
Optimal scheduling:- P_{g1} = 183.33mw; P_{g2}= 116.67mw
Equal load distribution:- P_{g1} = 150mw; P_{g2} = 150mw
The cost of generation for unit 1 for a P_d of 300mw is;
$$c_1 = \int \frac{dc_1}{dp_{g1}}\, dp_{g1} = \int \left(0.40p_{g1} + 80.00\right) dp_{g1}$$

= $0.20P_{g1}^2$ + $80P_{g1}$ + K_1

The increase in cost of generation for unit 1 is;

$C_1 = \left[0.20P_{g1}^2 + 80P_{g1} + k_1\right]_{183.33}^{150}$
= 16500 – 21,388.38 = - ₦4,888.30/hr
.
For unit 2, it is;

$$C_2 = \left[0.40P_{g2}^2 + 60P_{g2} + k_2\right]_{116.67}^{150}$$

=18,000 - 12,444.96 = ₦5,555.04/hr
The net saving caused by optimal scheduling is;
- ₦4,888.38/hr + ₦5,555.04/hr = ₦666.66/hr

Case 3: Total load demand of 420MW:
Optimal scheduling:- P_{g1}=263.33MW ; P_{g2}=156.67
Equal load distribution:- P_{g1} =210MW; P_{g2}=210MW
The cost of generation for unit 1 for P_d of 420MW, is;
$$c_1 = \int \frac{dc_1}{dp_{g1}}\, dp_{g1} = \int \left(0.40p_{g1} + 80.00\right) dp_{g1}$$

= 0.20 P_{g1}^2 + $80P_{g1}$+K_1 ₦/hr

The increase in cost for unit 1 is;
$c_1 = \left[0.20P_{g1}^2 + 80P_{g1} + k_1\right]_{263.33}^{210}$

= 25,620 - 34,934.94 = - ₦9,314.94/hr
For unit 2, it is;

$$c_2 = \int \frac{dc_2}{dp_{g2}}\, dp_{g2} = \int \left(0.40p_{g2} + 80.00\right) dp_{g2}$$

=0.40 P_{g2}^2 + 60pg$_2$ + K$_2$ ₦/hr
The increase in cost for unit 2 is;

$$C_2 = \left[0.40P_{g2}^2 + 60P_{g2} + k_2\right]_{156.67}^{210}$$

= 30,240 – 19,218.40 = ₦11,021.60/hr
Net saving caused by optimal scheduling is.
- ₦9,314.94/hr + ₦11,021.60/hr = ₦1,706.66/hr

Case 4: Total load demand, Pd, of 550MW:
Optimal scheduling:- P_{g1}=350MW; P_{g2}=200MW
Equal load distribution : P_{g1}=275MW; P_{g2}=275mw

The cost of generation for unit 1 for P_d of 550MW is;

$$c_1 = \int \frac{dc_1}{dp_{g1}}\, dp_{g1} = \int \left(0.40p_{g1} + 80.00\right) dp_{g1}$$

= 0.20P_{g1}^2 + 80P_{g1} + K$_1$ ₦/hr

The increase in cost for unit 1, is;

$$c_1 = \left[0.20P_{g1}^2 + 80P_{g1} + k_1\right]_{350}^{275}$$

=37,125. 00 - 52,500.00 = - ₦15,375. 00 /hr
For unit 2, the cost of generation is;

$$c_2 = \int \frac{dc_2}{dp_{g2}}\, dp_{g2} = \int \left(0.80p_{g2} + 60.00\right) dp_{g2}$$

=0.40 P_{g2}^2 + 60P_{g2} + K$_2$ ₦/hr
Hence the increase in cost of unit 2 is;

$$c_2 = \left[0.40P_{g2}^2 + 60P_{g2} + k_2\right]_{200}^{275}$$

= 46,750 - 28,000 = ₦18,750 00
The net saving caused by optimal scheduling is;
- ₦15,375.00/hr + ₦18,750.00/hr = ₦3,375.00/hr

The savings in naira in operational cost as a result of optimal scheduling for one year of continuous operation for all the cases considered, are displayed in table 2.

Table 2: Savings in Naira in operational cost as a result of optimal scheduling.

Case	Total load Demand, P_d. (MW)	Unit 1 P_{g1} (MW)	Unit 2 P_{g2} (MW)	Plant Incremental Fuel Cost, λ_p (₦/MWh)	Savings In Cost Due To Optimal Scheduling For One Year Of Continuous Operation (₦)
1	180	103.33	76.67	121.33	934,341.60
2	300	183.33	116.67	153.33	5,839,941.60
3	420	263.33	156.67	185.33	14,950,341.60
4	550	350	200	220.00	29,565,000.00

Table 2 has shown the savings in operational cost as a result of optimal scheduling. But the computations that gave rise to table 2, has been done without the consideration of transmission line losses. However, economic distribution of load between plants, requires the consideration of transmission line losses.

VI. LOAD DISTRIBUTION BETWEEN PLANTS AND LOSS COEFFICIENTS:

The losses in transmission from the plant having the lower incremental fuel cost, may be so great that economy may dictate lowering the load at the plant with the lower incremental fuel cost and increasing it at the plant with higher incremental fuel cost.

To take account of transmission line losses in the problem of determining economic loading of plants, there is the need to express the total transmission loss of the system as a function of the plant loadings.

Consider the transmission loss in a simple system of two plants and one load as shown in figure 4

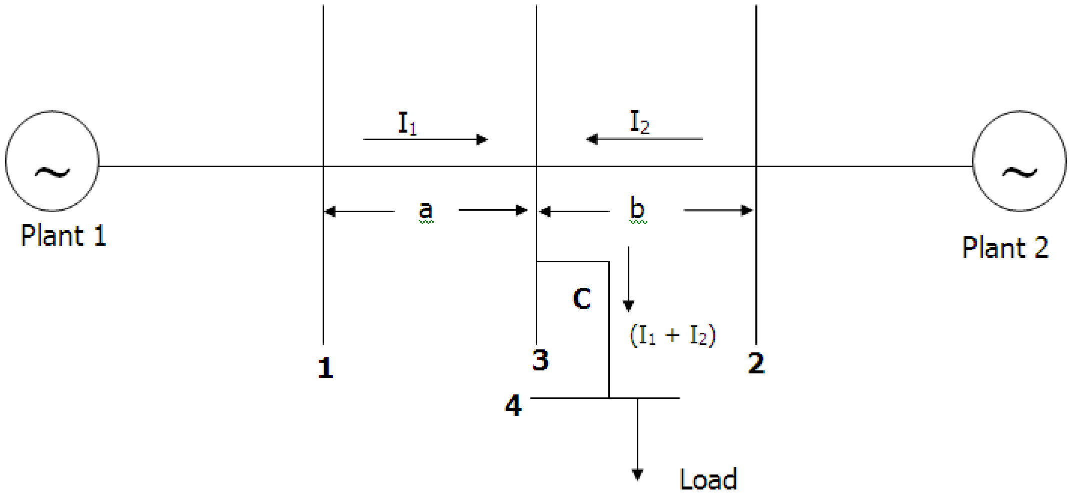


Fig 4.0: A simple system connecting two generating plants to one load.

Let R_a, R_b and R_c, be the resistance of lines a, b and c respectively. Then the total power loss of the 3 - Φ transmission system is given by [1];

$$P_L = 3|I_1|^2 R_a + 3|I_2|^2 R_b + 3|I_1 + I_2|^2 R_c \qquad (10)$$

I_1 and I_2 are assumed to be in phase so that;

$$|I_1 + I_2| = |I_1| + |I_2| \qquad (11)$$

$$(I_1 + I_2)^2 = (I_1 + I_2)(I_1 + I_2) = I_1^2 + 2I_1 I_2 + I_2^2 \qquad (12)$$

If equation (12) is substituted into equation (10) and simplified, we have that;

$$P_L = 3|I_1|^2 (R_a + R_c) + 6|I_1||I_2|R_c + 3|I_2|^2 (R_b + R_c) \qquad (13)$$

P_1 and P_2 are the power outputs at plant 1 and plant 2 respectively at power factors of $\cos\Phi_1$ and $\cos\Phi_2$ and V_1 and V_2 are their bus voltages, so that;

$$|I_1| = \frac{P_1}{\sqrt{3}\,|V_1|\cos\phi_1} \quad \text{and} \quad |I_2| = \frac{P_2}{\sqrt{3}\,|V_2|\cos\phi_2} \qquad (14)$$

Putting equation (14) into equation (13), gives;

$$P_L = \frac{P_1^2(R_a + R_c)}{|V_1|^2(\cos\phi_1)^2} + \frac{2P_1P_2R_c}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} + \frac{P_2^2(R_b + R_c)}{|V_2|^2(\cos\phi_2)^2} \qquad (15)$$

$$P_L = P_1^2 B_{11} + 2P_1P_2B_{12} + P_2^2 B_{22} \qquad (16)$$

Where;

$$B_{11} = \frac{R_a + R_c}{|V_1|^2(\cos\phi_1)^2}$$

$$B_{12} = \frac{R_c}{/V_1 \; //V_2 \; /(Cos\,\phi_1)(Cos\phi_2)}$$

$$B_{22} = \frac{R_b + R_c}{/V_2/^2 (Cos\phi_2)^2}$$

B_{11}, B_{12} and B_{22} are known as loss coefficients or B-Coefficients and are constant with variations of P_1 and P_2 if the bus voltages and power factor at the plants, remain constant. Equation (16) is the bus coefficient method employed to take account of the transmission loss in the economic distribution of loads between plants. Its units are expressed in reciprocal megawatts (Mw^{-1}).
The general form of the loss equation for any number of sources is;

$$P_L = \sum_m \sum_n P_m B_{mn} P_n$$

Where;
$\sum_m$ and $\sum_n$ indicate independent summations to include all sources.

The matrix form of the transmission loss equation is [1] ;

$$P_L = P^T BP \qquad (19)$$

Where for a total of S sources, we have;

$$P = \begin{bmatrix} P_1 \\ P_2 \\ : \\ : \\ P_s \end{bmatrix} \quad \text{and } B = \begin{bmatrix} B_{11} & B_{12} & B_{13}\text{----}B_{1S} \\ B_{21} & B_{22} & B_{23}\text{---}B_{2S} \\ : & : & : \quad : \\ : & : & : \quad : \\ B_{s1} & B_{s2} & B_{s3}\text{----}B_{ss} \end{bmatrix} \qquad (20)$$

VII. PENALTY FACTOR:

The method developed to express transmission loss in terms of plant outputs, enable us to coordinate transmission loss in scheduling the output of each plant for maximum economy for a given system load. The mathematical treatment is similar to that of scheduling units within a plant except that transmission loss is now included as an additional constraint.

Given a system for instance, the total cost of all the fuel for the entire system, is given by the express;

$$C_T = C_1 + C_2 + \text{------} + C_K = \sum_{n=1}^{k} c_n \qquad (21)$$

Where;
C_T is the total cost representing the sum of fuel cost of the individual plants C_1,C_2,---,C_K.

The total power input to the network from all the plants is;

$$P_T = P_1 + P_2 + \text{-----} + P_k = \sum_{n=1}^{k} p_n \qquad (22)$$

Where;
P_1, P_2, ---- P_K are the individual plant power inputs to the network.
Now the total fuel cost of the system is a function of the power input while the constraining relation on the minimum value of C_T, is;

$$\sum_{n=1}^{k} P_n - P_L - P_R = 0 \qquad (23)$$

Where;
P_L is the transmission loss
P_R is the total power received by the loads on the system.

$dP_R = 0$ since P_R is a constant and this reduces equation (23) to;

$\sum_{n=1}^{k} dP_n - dP_L = 0$ (24)

Minimization is achieved by;

$dC_T = 0$

Hence;

$$dC_T = \sum_{n=1}^{k} \frac{\partial C_T}{\partial P_n} dP_n = 0 \qquad (25)$$

Transmission loss, P_L, is dependent upon plant outputs and dP_L is expressed by;

$$dP_L = \sum_{n=1}^{k} \frac{\partial P_L}{\partial P_n} dP_n \qquad (26)$$

Substituting equation (26) into equation (24) ; multiplying by λ and subtracting the result from equation (25), gives;

$$\sum_{n=1}^{k} \left(\frac{\partial C_T}{\partial P_n} + \lambda \frac{\partial P_L}{\partial P_n} - \lambda\right) dP_n = 0 \qquad (27)$$

Equation (27) is for every value of n, satisfied provided that;

$$\left(\frac{\partial C_T}{\partial P_n} + \lambda \frac{\partial P_L}{\partial P_n} - \lambda\right) = 0 \qquad (28)$$

Rearranging equation (28), gives;

$$\frac{\partial C_n}{\partial P_n}\left\{\frac{1}{1-\frac{\partial P_L}{\partial P_n}}\right\} = \lambda \qquad (29)$$

OR

$$\frac{\partial C_n}{\partial P_n} L_n = \lambda \qquad (30)$$

Where;

$$L_n = \frac{1}{1-\frac{\partial P_L}{\partial P_n}} \qquad (31)$$

L_n is called the penalty factor of plant n.

The multiplier , λ, is in naira per megawatt-hour when fuel cost is in naira per hour and power is in megawatts. When transmission losses were not taken into account, the economic dispatch problem was solved by making the incremental fuel cost at each unit the same. We can still use the concept by observing that minimum fuel cost is obtained when the incremental fuel cost of each plant multiplied by it's penalty factor is the same for all plants in the system.

The products are equal to the system incremental fuel cost, λ, which is approximately the cost in naira per hour to increase the total delivered load by 1MW.

For a system of four plants for instance;

$$\frac{\partial C_1}{\partial P_1} L_1 = \frac{\partial C_2}{\partial P_2} L_2 = \frac{\partial C_3}{\partial P_3} L_3 = \frac{\partial C_4}{\partial P_4} L_4 = \lambda \qquad (32)$$

Hence the resulting set of equations is of the form [5] ;

$$L_n \frac{dC_n}{dP_n} = \lambda \quad for\ all\ P_{nmin} \leq P_n \leq P_{nmax} \qquad (33)$$

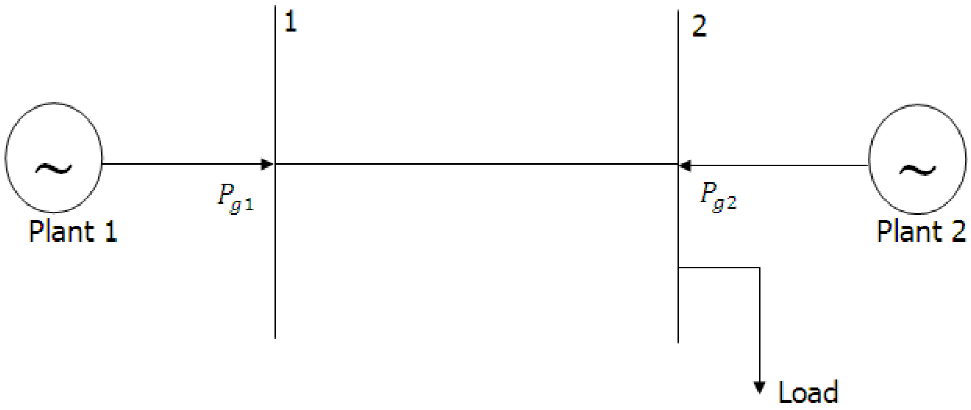
Equation (33) like equation (9) , is called the coordination equation.

The application of loss coefficients and penalty factor in coordinating power transmission losses in the optimal scheduling of plants to save cost in power system operation is illustrated by solving problem2.

Problem 2: A system consists of two generating plants connected by a transmission line. The incremental fuel cost for the generating plants are;
$\lambda_1 = 0.008P_{g1} + 8.00$ ₦/Mwh
$\lambda_2 = 0.0012P_{g2} + 9.00$ ₦/Mwh
The only load of the system is located at plant 2. When 500Mw is transmitted from plant 1 to plant 2, power loss in the line is 20Mw. We are required to find;
(a) The generation for each plant when the incremental fuel cost, λ, for the system is ₦15.00 /Mwh.

(b) The savings in naira per hour obtained by coordinating rather than neglecting the transmission loss in determining the load of the plants with the power received.

Solution for the two – plant system;



[a]

$\lambda_1 = 0.008P_{g1} + 8.00$ ₦/Mwh
$\lambda_2 = 0.0012P_{g2} + 9.00$ ₦/Mwh

For a two – bus system;
$$P_L = P_{g1}^2 B_{11} + 2P_{g1}P_{g2}B_{12} + P_{g2}^2 B_{22} \qquad \text{(i)}$$

Because all the loads in the system is at plant 2, varying P_{g2} can not affect the power loss, P_L.
This therefore implies that $B_{12} = B_{22} = 0$ (ii)

And so when P_{g1}=500MW, and P_L =20MW, equation (i) becomes;

$$20 = 500^2 B_{11} + 0 + 0$$
$$\Rightarrow B_{11} = \frac{20}{500^2} = 0.0008 \qquad \text{(iii)}$$
$$\frac{\partial P_L}{\partial P_{g1}} = 2P_{g1}B_{11} + 2P_{g2}B_{12}$$

$$\frac{\partial P_L}{\partial P_{g1}} = 2P_{g1}(0.008) + 0 = 0.00016P_{g1} \qquad \text{(iv)}$$

$$\frac{\partial P_L}{\partial P_{g2}} = 2P_{g1}P_{g2} + 2P_{g2}B_{22} = 0 \qquad \text{(v)}$$

The penalty factors are;

$$L_1 = \frac{1}{1-\frac{\partial P_L}{\partial P_{g1}}} = \frac{1}{1-0.00016P_{g1}} \qquad \text{(vi)}$$

$$L_2 = \frac{1}{1-\frac{\partial P_L}{\partial P_{g2}}} = \frac{1}{1-0} = 1.0 \qquad \text{(vii)}$$

For optimal scheduling;

$$\lambda_1 L_1 = \lambda_2 L_2 = \lambda_P \qquad \text{(viii)}$$

Hence for λ_P = ₦15/Mwh;

$$(0.008P_{g1} + 8.00) \text{ x } \frac{1}{1-0.0016P_{g1}} = 15 \qquad \text{(ix)}$$

Solving equation (ix), gives; P_{g1} = 673MW.
Also substituting the values of λ_2, L_2 and λp into equation (viii), gives;
0.012P_{g2} + 9.00 = 15
$\Longrightarrow P_{g2}$ = 500MW.

Power loss in transmission is;

$P_L = P_{g1}^2 B_{11}$ = 673^2 X 0.00008 = 36MW

Power received (power delivered), $P_R = P_{g1} + P_{g2} - P_L$
$\Longrightarrow P_R$ = 673 + 500 – 36 = 1137MW,

[b]

If transmission loss is neglected, the incremental fuel cost are equated to give;

0.008P_{g1}+ 8.00 = 0.012 P_{g2} + 9.00 (x)

Power delivered to the load is;

$P_R = P_{g1} + P_{g2} - P_{g1}^2 B_{11}$ = 1137MW (xi)

But from equation (x);

$$P_{g2} = \frac{0.008P_{g1} - 1.00}{0.012}$$

$\Longrightarrow P_{g2}$ = 0.67P_{g1} - 83.33 (xii)

Substituting equation (xii) into equation (xi) and solving for P_{g1} and P_{g2}, gives the following values for plant generations with losses not coordinated:-

P_{g1} = 758.50MW and P_{g2} = 422.50MW

This calculation implies that the load on plant1 is increased from 673MW to 758.50MW while that on plant 2 is decreased from 500MW to 422.50MW.

Hence the increase in fuel cost for plant 1 is;
$$c_1 = \int_{673}^{758.50} (0.008\,P_{g1} + 8.00)\,dp_{g1} = \left[0.004P_{g1}^2 + 8.00P_{g1}\right]_{673}^{758.50}$$
$$c_1 = (8{,}369.29 - 7{,}195.72) = ₦\,1{,}173.57/hr$$

And the decrease in fuel cost for plant 2, is;

$$c_2 = \int_{500}^{422.50} (0.012\,P_{g2} + 9.00)\,dp_{g2} = \left[0.006P_{g2}^2 + 9.00P_{g2}\right]_{500}^{422.50}$$
$$c_2 = (4{,}873.50 - 6{,}000) = -₦\,1{,}126.50/hr$$

Therefore the net saving in naira per hour by accounting for transmission loss in scheduling the received load, P_R of 1137MW through the application of loss coefficients and penalty factor is; ₦1,173.57/hr - ₦1,126.50/hr = ₦47.07/hr.

VIII. CONCLUSION

In this paper, two economic dispatch problems were considered. In problem 1, loads were optimally scheduled between units within a plant without consideration of the transmission losses. In each case of this problem considered, the negative sign indicates a decrease in cost as is expected for a decrease in output. Also the saving in naira per hour in each case considered, seems small but the amount saved every hour for one year of continuous operation would reduce fuel cost considerably as displayed in table 2. Solution of problem 2 clearly demonstrates how the application of loss coefficients and penalty factor can effectively coordinate transmission losses rather than neglected in the optimal scheduling of loads between plants within a system aimed at saving cost in the operation of power system.

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