

## A Moving Single Station Doppler Passive Ranging Method based on Frequency Shift and Path Difference

TaoYU

(China Academy of Management Science, Beijing, China)

**Abstract:** By using the difference approximation of differential, the interchangeable relation between Doppler frequency shift and path difference can be obtained, and the calculation method of path difference based on Doppler frequency shift measurement is obtained. Based on this, the airborne Doppler passive ranging formula can be obtained by constructing the virtual double base array and using the linear solution of the double base path difference positioning equation. Error analysis shows that the relative ranging accuracy of the solution based on frequency shift measurement is basically the same as that of the existing airborne passive ranging solution based on angle and frequency difference measurement under the condition of short wavelength and fast flight speed. The relevant research of the author gives a relatively clear application criterion for Doppler shift localization of moving single station in different frequency bands.

**Key words :** Single station location; Airborne passive positioning ; Doppler shift ; Doppler frequency difference; Doppler changing rate; Path difference equation; Ranging

Date of Submission: 10-11-2021

Date of acceptance: 25-11-2021

### I. INTRODUCTION

Existing research results show that, unless Doppler rate of change is directly used, it can achieve localization based on only one detection<sup>[1-3]</sup>. Otherwise, if based on Doppler frequency shift measurement, at least two detection are generally required. An earlier study of the authors about passive ranging method based on Doppler rate of change is that based on mathematical definition, Doppler rate of change is transformed into the ratio of Doppler frequency difference to time difference by using difference method, and then an airborne Doppler passive ranging method is derived by using velocity vector relationship, which requires two frequency shift measurement<sup>[4,5]</sup>. The advantage of this research result is that neither direct measurement of Doppler rate of change nor other measurement methods are needed. The disadvantage is that Doppler frequency shift needs to be measured twice.

A recent research result of the author is that an airborne passive ranging solution based on angle and frequency difference measurement is presented by using the reciprocal relation between frequency shift and path difference, and also by using the double base path difference locating method and the single base midpoint direction finding solution derived from this method<sup>[6]</sup>. On the basis of the study, this paper further analysis shows that the airborne single station passive ranging formula, only using the linear solution of double-base path difference positioning, can be obtained based on the two Doppler shift measurement without the use of direction finding technology if the frequency shift measurement is directly used on the basis of the exchange relationship between the frequency shift and the path difference.

The study shows that although the detection system based only on frequency shift measurement has the advantage of requiring less equipment, the range error is obviously worse than the airborne passive ranging method based on angle and frequency difference measurement when the detection signal wavelength is longer and the flight speed is slower. However, when the wavelength is short and the flight speed is fast, the relative ranging accuracy of the solution based on frequency shift measurement is basically the same as that of the existing airborne passive ranging solution based on angle and frequency difference measurement.

1 Transformation from frequency shift to path difference

1.1 Basic definitions

A Doppler receiver  $R$  is installed on the moving platform to detect the stationary or slow-moving target  $T$  on the ground, and the received Doppler frequency shift is

$$\lambda f_d = v \cos \beta \quad (1)$$

Where:  $f_d$  is Doppler frequency shift;  $\lambda$  the wavelength;  $v$  the moving speed of the moving platform;  $\beta$  frontangle.

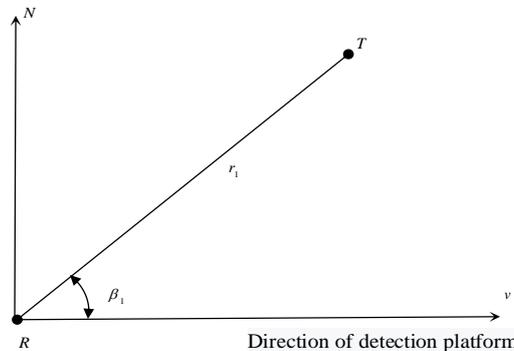


Fig. 1 Doppler frequency shift detection of moving single station

1.2 Differential treatment of change rate of radial distance

According to the relationship between the change rate of radial distance and the radial velocity, as well as between the radial velocity and the Doppler frequency shift, the relationship between the Doppler frequency shift and the change rate of radial distance can be obtained

$$\frac{\partial r(t)}{\partial t} = v_r = v \cos \beta = \lambda f_d \quad (2)$$

Assuming that the change of time is transient, by means of difference calculation, the differential of distance to time can be transformed into

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \quad (3)$$

Where,  $\Delta r$  is the path difference.  $\Delta t$  is the time difference taken by the platform to move from position 1 to position 2.

$$\Delta t = \frac{d}{v} \quad (4)$$

The Doppler frequency shift expression based on the path difference measurement is obtained

$$f_d = \frac{v \Delta r}{\lambda d} \quad (5)$$

If the Doppler shift is measured, the virtual path difference can be obtained

$$\Delta r = \frac{\lambda d}{v} f_d \quad (6)$$

2 Location solution with virtual array

2.1 Virtual path differences based on frequency shift measurement

Considering the application of double-base linear array, the two adjacent virtual path differences which is needed to construct double-base array are given by frequency shift measurement is

$$\Delta r_{12} = \frac{\lambda d}{v} f_{d1} \quad (7)$$

$$\Delta r_{23} = \frac{\lambda d}{v} f_{d2} \quad (8)$$

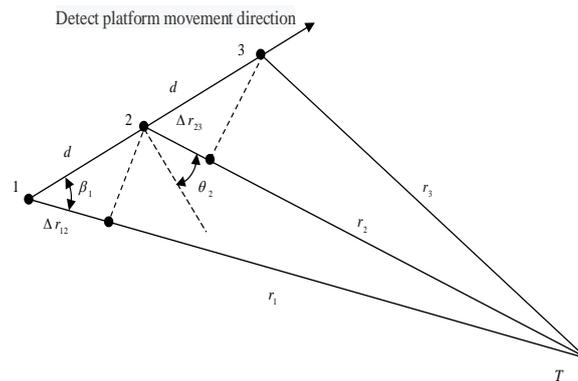


Fig. 2 Detection of fixed target by moving single station

2.2 Detection model

From the representation of path difference based on Doppler frequency shift measurement and the geometric model shown in Fig. 2, it can be seen that the two adjacent path difference required by double-base array can be obtained by using only one single-base receiving array in practical application. Therefore, the double-base linear array used in positioning calculation is actually a virtual array.

Corresponding to this mathematical geometric model, it can be assumed that there is a single station moving in a straight line, as shown in Figure 2, from position 1 to position 2. In this process, Doppler frequency shift detection is performed twice in a row at a single moving station.

$$\lambda f_{d1} = v \cos \beta_1 \quad (9)$$

$$\lambda f_{d2} = v \cos \beta_2 \quad (10)$$

2.3 Range solution

On the basis of frequency shift measurement and using the relationship between frequency shift and path difference to obtain two adjacent path difference equations which is required to construct virtual double basis array, the ranging formulabased on the double basis path difference method is directly used

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2(\Delta r_{12} - \Delta r_{23})} \quad (11)$$

2.4 Simulation calculation

During the simulation calculation, the radial distance  $r_1 = 600 \text{ km}$  from the detection platform to the target is preset at the starting position 1. In addition, the preset detection platform moving distance  $d = 100 \text{ km}$  and moving speed  $v = 300 \text{ m/s}$ , as well as the detection signal wavelength  $\lambda = 0.3 \text{ m}$ .

Then, the leading angle at the starting point 1 is linearly changed in the range of  $0^\circ < \beta_1 < 90^\circ$ , and the radial distance of the target at other locations and other geometric parameters are calculated by using trigonometric function relations. According to the definition (1) of Doppler frequency shift, the Doppler frequency shift value is calculated and obtained. Then the relation between frequency shift and range difference is used to give the virtual path difference of double base array.

On this basis, the calculated value of radial distance  $r_2$ , Eq.(11), obtained based on frequency shift detection is compared with the theoretical value of radial distance obtained by using trigonometric function.

Fig. 3 shows the relative calculation errors of ranging solutions at different moving distances, from which it can be seen that the calculation accuracy is inversely proportional to the moving distances. Among them, the curve pattern when the moving distance is only 10 meters is particularly eye-catching. It can be seen that the ranging solution based only on frequency shift measurement has good accuracy in short baseline by using the interchangeable relation between frequency shift and path difference. Fig. 4 shows the relative calculation errors of ranging solutions at different radial distances. It can be seen that the relative calculation error is inversely proportional to the radial distance. The simulation results also show that the values of flight speed and wavelength have little influence on the analysis of relative calculation errors.

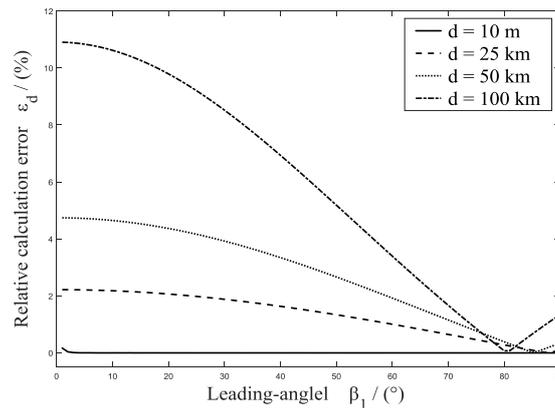


Fig. 3 Relative calculation errors at different moving distances

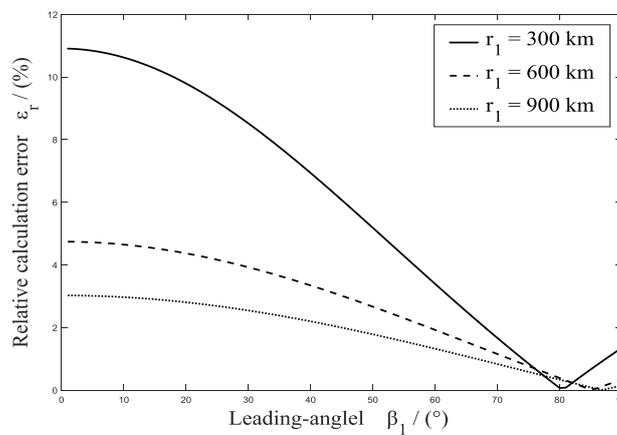


Fig. 4 Relative calculation errors at different radial distances

### 3 Range error

The relative ranging error is analyzed by total differential method. First set

$$r_2 = 0.5 \frac{P}{Q}$$

$$P = 2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2$$

$$Q = \Delta r_{12} - \Delta r_{23}$$

#### 3.1 Range error generated by frequency shift measurement $f_{d1}$

$$\frac{\partial r_2}{\partial f_{d1}} = \frac{0.5}{Q^2} \left( Q \frac{\partial P}{\partial f_{d1}} - P \frac{\partial Q}{\partial f_{d1}} \right)$$

$$\frac{\partial P}{\partial f_{d1}} = -2\Delta r_{12} \frac{\partial \Delta r_{12}}{\partial f_{d1}} - 2\Delta r_{23} \frac{\partial \Delta r_{23}}{\partial f_{d1}}$$

$$\frac{\partial Q}{\partial f_{d1}} = \frac{\partial \Delta r_{12}}{\partial f_{d1}} - \frac{\partial \Delta r_{23}}{\partial f_{d1}}$$

$$\frac{\partial \Delta r_{12}}{\partial f_{d1}} = \frac{\lambda d}{v}$$

$$\frac{\partial \Delta r_{23}}{\partial f_{d1}} = 0$$

#### 3.2 Range error resulting from frequency shift measurement $f_{d2}$

$$\frac{\partial r_2}{\partial f_{d2}} = \frac{0.5}{Q^2} \left( Q \frac{\partial P}{\partial f_{d2}} - P \frac{\partial Q}{\partial f_{d2}} \right)$$

$$\begin{aligned} \frac{\partial P}{\partial f_{d2}} &= -2\Delta r_{12} \frac{\partial \Delta r_{12}}{\partial f_{d2}} - 2\Delta r_{23} \frac{\partial \Delta r_{23}}{\partial f_{d2}} \\ \frac{\partial Q}{\partial f_{d2}} &= \frac{\partial \Delta r_{12}}{\partial f_{d2}} - \frac{\partial \Delta r_{23}}{\partial f_{d2}} \\ \frac{\partial \Delta r_{12}}{\partial f_{d2}} &= 0 \\ \frac{\partial \Delta r_{23}}{\partial f_{d2}} &= \frac{\lambda d}{v} \end{aligned}$$

3.3 Ranging error caused by moving distance

$$\begin{aligned} \frac{\partial r_2}{\partial d} &= \frac{0.5}{Q^2} \left( Q \frac{\partial P}{\partial d} - P \frac{\partial Q}{\partial d} \right) \\ \frac{\partial P}{\partial d} &= 4d - 2\Delta r_{12} \frac{\partial \Delta r_{12}}{\partial d} - 2\Delta r_{23} \frac{\partial \Delta r_{23}}{\partial d} \\ \frac{\partial Q}{\partial d} &= \frac{\partial \Delta r_{12}}{\partial d} - \frac{\partial \Delta r_{23}}{\partial d} \\ \frac{\partial \Delta r_{12}}{\partial d} &= \frac{\lambda}{v} f_{d1} \\ \frac{\partial \Delta r_{23}}{\partial d} &= \frac{\lambda}{v} f_{d2} \end{aligned}$$

3.4 Range error due to flight speed

$$\begin{aligned} \frac{\partial r_2}{\partial v} &= \frac{0.5}{Q^2} \left( Q \frac{\partial P}{\partial v} - P \frac{\partial Q}{\partial v} \right) \\ \frac{\partial P}{\partial v} &= -2\Delta r_{12} \frac{\partial \Delta r_{12}}{\partial v} - 2\Delta r_{23} \frac{\partial \Delta r_{23}}{\partial v} \\ \frac{\partial Q}{\partial v} &= \frac{\partial \Delta r_{12}}{\partial v} - \frac{\partial \Delta r_{23}}{\partial v} \\ \frac{\partial \Delta r_{12}}{\partial v} &= -\frac{d\lambda}{v^2} f_{d1} \\ \frac{\partial \Delta r_{23}}{\partial v} &= -\frac{d\lambda}{v^2} f_{d2} \end{aligned}$$

3.5 Error calculation

When the error of each observation is zero mean, independent of each other, relative ranging error

$$\sigma_r = \frac{1}{r_2} \left[ \left| \frac{\partial r_2}{\partial f_{d1}} \right| \sigma_f + \left| \frac{\partial r_2}{\partial f_{d2}} \right| \sigma_f + \left| \frac{\partial r_2}{\partial d} \right| \sigma_d + \left| \frac{\partial r_2}{\partial v} \right| \sigma_v \right] \tag{12}$$

Where:  $\sigma_\theta, \sigma_f, \sigma_d, \sigma_v$  are the root mean square errors of angle, frequency shift, moving distance and flight velocity measurement errors respectively.

The geometric parameters and Doppler shift are set and calculated in the same way as in the simulation. Parameter value used in calculation

$$r_1 = 600 \text{ km}, \quad d = 100 \text{ km}, \quad v = 300 \text{ m/s}, \quad \lambda = 0.3 \text{ m}, \quad \sigma_f = 50 \text{ Hz}$$

Fig. 5 shows the relative ranging error curves of the radial distance  $r_2$  at the midpoint of the double-base virtual array at different root mean square errors of frequency shifts measurement. The calculation results show that the root mean square error of frequency shift measurement has a great influence on the relative ranging accuracy. If the root mean square error of frequency shift measurement is large, the ranging error will deteriorate quickly.

Fig. 6 shows the calculation results of relative ranging error curves at different moving distances. It shows that only when moving distances are large, can the ranging accuracy be better.

Fig. 7 shows the relative ranging error curves at different radial distances  $r_1$ . The calculation results show that when the radial distance tends to increase, the relative ranging error will deteriorate rapidly.

Fig. 8 shows the calculation results of relative ranging errors at different wavelengths. The shorter the

wavelength is, the better the relative ranging accuracy is.

Fig. 9 shows the calculation results of relative ranging errors at different flight speeds. It shows that the faster the flight speed, the better the relative ranging accuracy.

In general, if the wavelength is not short and the flight speed is not very fast, the airborne passive ranging solution based only on frequency shift measurement needs to be within the range of  $\beta_1 > 40^\circ$  to meet the requirement that the relative ranging error is less than 5%.

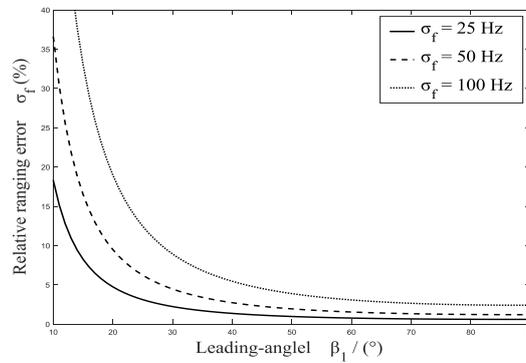


Fig. 5 Relative ranging errors with different root mean square errors

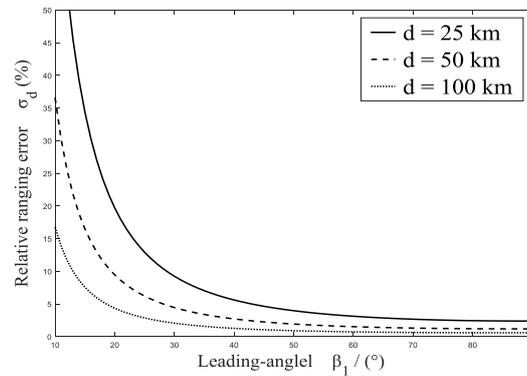


Fig. 6 Relative ranging errors at different moving distances

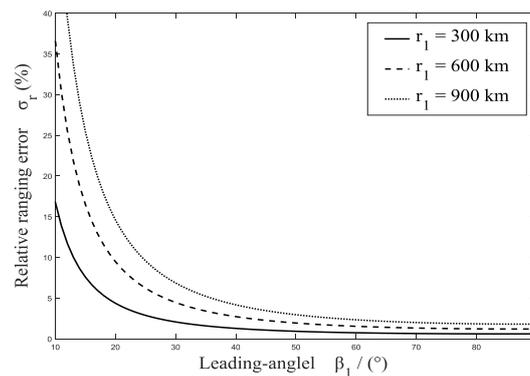


Fig. 7 Relative ranging errors at different radial distances

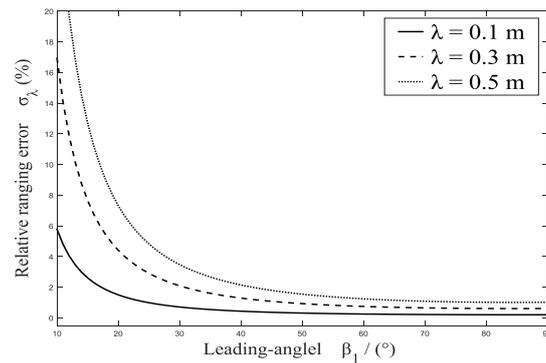


Fig. 8 Relative ranging errors at different wavelengths

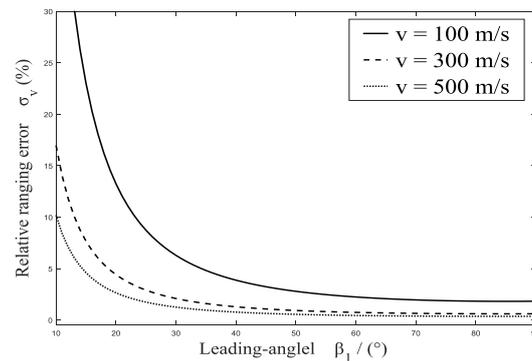


Fig. 9 Relative ranging errors at different flight speeds

3.6 Comparison with ranging solutions based on frequency difference and angle measurement

The relative ranging error of the ranging solution given in this paper is compared with that based on frequency difference and angle measurement in literature[6]. As can be seen from the error curve in Fig. 10, under the condition of large wavelength and low flight speed, the ranging error given by frequency shift measurement method is relatively large.

Parameter value used in calculation

$$r_1 = 600 \text{ km} , d = 100 \text{ km} , v = 300 \text{ m/s} , \lambda = 0.3 \text{ m} , \sigma_f = 50 \text{ Hz} .$$

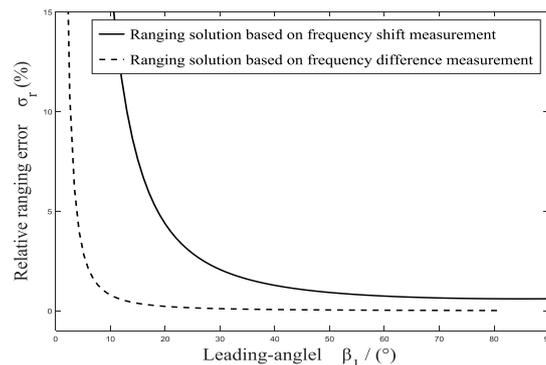


Fig. 10 Comparison of relative ranging errors of two ranging methods

Figure 11 shows that if shorter wavelengths and faster flight speeds are selected, the relative ranging errors of the two ranging methods tend to be the same. Parameter value used in calculation

$$r_1 = 600 \text{ km} , d = 100 \text{ km} , v = 800 \text{ m/s} , \lambda = 0.01 \text{ m} , \sigma_f = 50 \text{ Hz} .$$

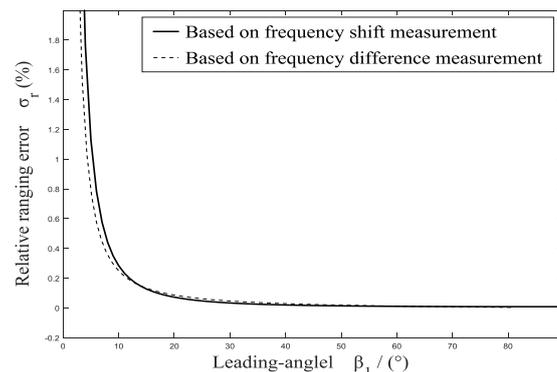


Fig. 11 Comparison of two ranging methods for shorter wavelength and faster flight speed

## II. CONCLUSION

The content of this paper shows that the ranging error of the method based only on Doppler frequency shift measurement is directly related to the length of wavelength and the speed of flight. The author's existing research results show that wavelength and flight speed have little influence on the ranging error of the method based on angle and frequency difference measurement. The simulation results in this paper show that the relative ranging errors of the two methods tend to be the same when the wavelength is short and the flight speed is fast. The relevant research of the author gives a relatively clear application criterion for Doppler shift localization of moving single station in different frequency bands.

## REFERENCES

- [1]. Li Zong-hua, XiaoYu-qin, Zhou yi-yu, Sun Zhong-kang, "Single-observer passive location and tracking algorithms using frequency and spatial measurements", *SYSTEMS ENGINEERING AND ELECTRONICS*, vol. 26, no. 5, pp. 613-616, May 2004.
- [2]. DIAO Ming, WANG Yue, "Research of passive location based on the Doppler changing rate", *SYSTEMS ENGINEERING AND ELECTRONICS*, vol. 28, no. 5, pp. 696-698, June 2006.
- [3]. TAN Xin-rong, GAO Xian-jun, LI Bao-zhu, WANG Yu, "An improved tracking filter algorithm of single observer passive localization based on Doppler changing rate", *Electronic Design Engineering*, vol. 22, no. 8, pp. 77-80, April 2014.
- [4]. YU Tao, "Airborne ranging principle based on Doppler frequency", *INFORMATION AND ELECTRONIC ENGINEERING*, vol. 9, no. 1, pp. 22-25, Feb. 2011.
- [5]. Tao Yu, *Technology of Passive detection location*, Beijing: National defense industry press, 2017.
- [6]. Tao YU. "An Airborne Passive Positioning Method Based on Angle and Frequency Difference Measurement", 2020 4th International Conference on Electronic Information Technology and Computer Engineering Proceedings, 2020, pp.296-301.

TaoYU. "A Moving Single Station Doppler Passive Ranging Method based on Frequency Shift and Path Difference." *American Journal of Engineering Research (AJER)*, vol. 10(11), 2021, pp. 60-67.