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"Construction Methodology and enumeration of the Steiner Triple Systems of Order 27"

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Abstract:*This paper describes the construction and enumeration of the Steiner Triple system of large order systems. It describes the methodology use to generate the triplets for the systems of order 27. We have presented the methodology of enumerated permutation of the diagonal matrices. Enumeration of the total Steiner Triple systems of order 27 has been deduced. In the proposed methodology, combinatorics has been applied and problems for the enumeration and calculation have also been discussed.*

Keywords:*Steiner triple systems, order, combinatorics, construction, permutation*

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I. INTRODUCTION:

Block design theory is an important branch of the combinatorial mathematics and plays an important role in experiment design, competition arrangement, digital communication areas. In 1850 a British mathematician Thomas P.Kirkman posed a famous 15 schoolgirls problem and solved this problem in the same year. In 1971 D.R. Ray-Chaudhari and R.M. Wilson published a paper with topic Solution of Kirkman's schoolgirl's problem to show how to construct Kirkman triple systems of order 6n + 3. In 2003 a method of constructing Kirkman triple systems of order $s \times t$ by using main matrix and subsidiary matrix is proposed. We will be presenting the enumeration and construction through the edge graphical approach and the symmetry problem has been resolved in this work.

The solution has been presented using the edge weight matrices constructed along, for the construction of the Steiner Triple systems following all the resolvability and the graphical permutation and the combinatorics approach.

II. PROPERTIES OF BLOCK DESIGN

[I] Theorem of Jordan:

Let G be a permutation group on n letters which is primitive, and let H be a transitive subgroup of G on m letters, fixing the remaining n-m letters, 2 <m <n. Then (1) if H is primitive, Gis n - m + 1 fold transitive; (2) in any event G is doubly transitive.

[II] Theorem -2: Let G be a Jordan group on n letters and suppose the subgroup H as large as possible, namely (i) H is not contained in a subgroup H' fixing k' letters 3 <k' <k and transitive on the remaining n - k' letters, and (ii) H contains all permutations of G fixing the k letters which H fixes. Then the sets of k letters fixed by H and its conjugates in G form the blocks of an incomplete balanced block design D with parameters v = n, b = n(n - 1)/k(k I), k k, r = (n - l)/(k - I), 11.= 1. The group G may be regarded as an auto-morphism group of D which is doubly transitive on the letters of D. The subgroup H fixes the letters of a block of D and is transitive on the remaining letters. Conversely if D is a block design with parameters v, b, 1', k, and A 1 which has an auto-morphism group G doubly transitive on the letters of D in which the subgroup H fixing the letters of a block is transitive on the remaining letters, then G is a Jordan group on the v letters of D and H is the subgroup of G fixing k letters and transitive on the remaining letters.

III. PROPERTIES OF STEINER SYSTEMS

[I] Let S be a Steiner triple system in which, for every paint x, there is an involution 0: of S which has x as itsonly fixed point. Then every triangle of S generates an S(9). Conversely suppose that S is a Steiner triple system in which triangle generates an 8(9). Then for every paint x of S there is an involution a of S which has x as its only fixed point.

[II] Let S be a Steiner triple system and suppose that for each triple of 8 there is an involution whose fixed points are precisely the three points of the triple. Then every triangle of S generates an S(7) or an 8(9).

IV. CONSTRUCTION:

When t = 1, the admissibility condition reduces to the statement that k must divide v, which is elementary anyway and it is equally elementary how to construct such systems. As an example, to construct an S(1, 5, 15) let the base set be {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o}; the blocks can then be chosen to be abcde, fghij and klmno. The first nontrivial case is when t = 2 and k = 3, the covering of pairs of elements by triples, and these systems are more often called Steiner triple systems. We begin by working out the admissibility condition on v. By theorem 1, if there exists S(2, 3, v) then there exists S(1, 2, v − 1). Hence 2 divides v − 1, i.e. v is odd and can be written in the form v = 2s + 1, where s is a positive integer. Now applying theorem 2 to S(2, 3, v), we see that 3C2 divides vC2, i.e. 3 divides 1 2 v(v − 1) = s(2s + 1).

Since 3 is prime then either 3 divides s or 3 divides 2s + 1, i.e. either s must be of the form 3r or of the form 3r + 1, where r is a positive integer. Hence v = 6r + 1 or 6r + 3. Note here what we have done and, 3 more importantly, what still remains to be done. We have not shown that there actually exist Steiner triple systems S(2, 3, v) when v = 6r+1 or 6r+3. We have merely shown that these are the only possible values of v for which such systems can exist. The work of showing whether the necessary admissibility condition is also sufficient is still to come and is more difficult.

What is required is a general construction or a number of constructions to produce Steiner triple systems. The first person to solve this problem was an Anglican clergyman living in the nineteenth century. The Reverend T. P. Kirkman (1806-1895) was Rector of Croft, near Warrington in what was then Lancashire. In 1847 (reference 4) he published a paper giving the complete solution to the problem of constructing Steiner triple systems. In the sense that he did not earn his living from mathematics, Kirkman belonged to the line of great amateurs whose contributions have so enriched and advanced the subject. By right the systems should be called Kirkman triple systems, but Kirkman's work was overlooked for many years.

However.Kirkman's contributions to the development of combinatorial mathematics were eventually recognized and the name Kirkman triple system is now given to a special type of Steiner triple system with additional properties. Since Kirkman's time many other different constructions of Steiner triple systems have been discovered and we give below our favourite which occurs in the work of the American mathematician

V. RESULTS & DISCUSSION

Finally, when the 30 blocks of the form LLLNN are considered, everything is forced, as the reader will find if the construction is followed through. Obtaining 6 the larger system S(5, 6, 12) is easy. First a twelfth element, say ∞, is adjoined to all the blocks of the S(4, 5, 11). Then 66 further blocks are created as the complements of the existing 66 blocks;

Final STS Isomorphic counts: 132 blocks so formed are a Steiner system STS (27)

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