Algorithm and C Language Programming Concepts in Algebraic Structures and their Ideals

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Abstract: In this paper, We Introduce a Concept of Algorithm and Programming of C Language in Algebraic Structures and their Ideals, Sub-Programs have been Introduced to Some of the Algebraic Structures and their Ideals with Examples and Executed Programs.

Keywords: Algorithm, Programming SYNTAX of C Language, Keywords of C-Languages, Algebraic Structures, Ideals, Prime Ideals.

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I. INTRODUCTION

C-Language

The C programming language is a structure oriented programming language, developed at Bell Laboratories in 1972 by Dennis Ritchie. The C language is belonging to middle level programming language. Operating system programs such as Windows, Unix, Linux are written in C language.

C has been written in assembly language.

C is a general-purpose programming language, and is used for writing programs in many different domains, such as operating systems, numerical computing, graphical applications, etc. It is a small language, with just 32 keywords. It provides “high-level” structured programming constructs such as statement grouping, decision making, and looping, as well as “low-level” capabilities such as the ability to manipulate bytes and addresses.

C-Program

Syntax

#include <stdio.h>

int main()
{
    printf("Algorithm And C Language Programming Concepts in Algebraic Structures and their Ideals");
    getch 0;
}

Group Theory

Definition: A Set G is Said to be a Group with an Operation ‘o’ which Satisfies the Following Postulates.

(i) aob ∈ G ∀ a, b ∈ GClosure Axiom.
(ii) aoboc = aoboc ∀ a, b and c ∈ G Associative Axiom.
(iii) aoe = eoa = a ∀ a, e ∈ G Identity Law.
(iv) aoa⁻¹ = a⁻¹oa = e ∀ a, e ∈ G Inverse Law.

Algorithm / Sub Program in C Language

Step 1: Start
Step 2: Read a, b, c, e from a SET.
scanf("%d%d%d%d", a, b, c, e);
Step 3: Read Operator ‘o’ as + or * and Calculate \( a \circ b \).

printf("The Closure of \( a \circ b \) = %d, \( a \circ b \);

Step 4: if \( a \circ b \in G \) Closure Law Holds

printf("Enter the range of integers (only integers): \t");
flush(stdin);
scanf("%d", &n);
printf("nRange of integers n=%d: \t\n");
printf("nEnter any two numbers: \t");
scanf("%d%d", \&x, \&y);
printf("nNumber entered are a=%d b=%d: \t", a, b);
sum=a+b;
mul=a*b;
printf("nsum=%d, Multiply= %d :
");
if (sum<=n && mul<=n)
printf("nClosure Law Holds .");
else
printf("nClosure Law doesnot holds.");

Step 5: Check for Associativity Axiom.

printf("Enter the range of integers (only integers): \t");
flush(stdin);
scanf("%d", &n);
printf("nRange of integers n=%d: \t");
printf("nEnter any three numbers: \t");
scanf("%d%d", \&a, \&b, \&c);
printf("nNumber entered are a=%d b=%d:
");
sum1=a+(b+c);
sum2=(a+b)+c;
printf("nsum1=%d, sum2= %d :
");
if (sum1==sum2)
printf("nAssociative Law Holds .");
else
printf("nAssociative Law Doesn’t Holds.");*/

Step 6: Check for Identity.

Read e = 1 or e = 0 as per the Operation

printf("Enter the range of integers (only integers): \t");
flush(stdin);
scanf("%d", &e);
printf("nRange of integers n=%d: \t");
printf("nEnter any number: \t");
scanf("%d " \&a);
printf("nNumber entered is a=%d t", a);
sum=a+e;
mul=a*e;
if (sum==a && mul==a)
printf("nIdentity Holds .");
else
printf("nIdentity Axiom is not fulfilled.");

Step 7: Check for Inverse, when Identity ( e = 0 or o==+)

printf("Enter the range of integers (only integers): \t");
flush(stdin);
e==0;
printf("Enter any number:\t\n");
scanf("%d "&a);
printf("Enter other number:\t\n");
scanf("%d "&b);
if (a+b==e)
printf("Inverse Exists.");
else
printf("Inverse Axiom fails.");
/*
Check for Inverse, when Identity ( e = 1 or o==*)

printf("Enter the range of integers (only integers): \t\n");
fflush(stdin);
e=1;
b=(1%n);
printf("Enter any number:\t\n");
scanf("%d ",&a);
printf("Enter other number:\t\n");
scanf("%d ",&n);
if (a*b==e)
printf("Inverse Exists.");
else
printf("Inverse Axiom fails.");
*/

Step 8: Stop.

**Ring**

**Definition:** A Set \((R, +, .)\) is said to be a Ring if the Set Satisfies Certain Axioms.

(i) \(R\) is an Abelian Group (Group with Commutative Axiom) with Respect to the Operation ‘ + ’.

(ii) Closure Law Holds for \(a, b \in R\)

\[
(a \cdot b) \cdot c = a \cdot (b \cdot c) \quad \forall a, b, c \in R
\]

(iii) Distributive Law Holds

\[
\begin{align*}
(a + b) \cdot c &= (a \cdot b) + (a \cdot c) \\
(b + c) \cdot a &= (b \cdot a) + (c \cdot a)
\end{align*}
\]

**Ideals in a Ring**

**Definition [4]:** Let a Set \((R, o_1, o_2)\) be a Ring. A Subset \(I\) of \(R\) is Said to be a Left Ideal of \(R\) if

(i) \(r_0 \cdot a \in I\) and \(r_0 \cdot a \in I\) for all \(r \in R\) and \(a \in I\)

**Algorithm / Sub Program in C Language**

Step 1: Start.
Step 2: Read a Range for a Set \(R\) Called Ring \(n\) (Axioms Fulfilled).
Step 3: Choose a Subset \(I\) of \(R\), and Read a range for \(I\) \((m <= n)\).
Step 4: take any two Numbers from \(I\) and \(R\).
Step 5: Calculate their Compositions \((r_0 \cdot a \text{ and } r_2 \cdot a)\).
Step 6: Check the Compositions within the Set \(I\).
Step 7: Display the Result.
Step 8: Stop.

**Syntax**

```c
#include<stdio.h>
#include<math.h>
void main()
{
  int m, n;
  printf("Enter the Range of a Set Ring R: \t\n");
  flush(stdin);
  scanf("%d",&n);
```
Let a Set $R, a_1, a_2$ be a Ring. A Subset $I$ of $R$ is Said to be a Right Ideal of $R$ if 

(i) $a_1 r \in I$ and $a_2 r \in I$ for all $r \in R$ and $a \in I$

Syntax:

```c
if(m<=n)
{
 printf("the Composition for the Operator + is %d",m+n);
}
else
{
 printf("The Composition Doesn't Holds for '+'");
 return 0;
}
```

Example [3]: Modulo Divisor of $Z, \langle Z_{10} \rangle$ is an Ideal of the SET of Integers.

Syntax:

```c
void main()
{
 int count;
 int n;
 int m,k,r,d;
 printf("Enter that Range Integers");
 scanf("%d",\n);
 for(count = 0; count <= n-1; count++)// Display the numbers 0 -9//
 printf("%d ",count);
 printf("\n");
 printf("Please enter any the values for Modulo Z_{10}, = \n");
 scanf("%d %d",\n,m,&k);
 d=m/k;
 r=m%k;
```
if(r<=d)
{
    printf("Integer division: %i divided by %i is %i\n",m,k,d);
    printf("The remainder when %i is divided by %i is %i\n",m,k,r);
    return 0;
}

Example [3]: Let R be a Ring of \(2 \times 2\) Matrices over Integers

Let \(A = \left( \begin{array}{cc} x & y \\ 0 & 0 \end{array} \right) \parallel x, y \text{ integers}\) Then A is a Right Ideal of R.

Let \(p = \left( \begin{array}{cc} u & v \\ 0 & 0 \end{array} \right) \in A\) and \(q = \left( \begin{array}{cc} w & x \\ 0 & 0 \end{array} \right) \in A\)
\(p - q = \left( \begin{array}{cc} u & v \\ 0 & 0 \end{array} \right) - \left( \begin{array}{cc} w & x \\ 0 & 0 \end{array} \right) = \left( \begin{array}{cc} u - w & v - x \\ 0 & 0 \end{array} \right) \in A\).

Let \(r = \left( \begin{array}{cc} \alpha & \gamma \\ \beta & \delta \end{array} \right) \in R\)

Then \(pr = \left( \begin{array}{cc} u & v \\ 0 & 0 \end{array} \right) \left( \begin{array}{cc} \alpha & \gamma \\ \beta & \delta \end{array} \right) = \left( \begin{array}{cc} ua + vb & u\gamma + v\delta \\ 0 & 0 \end{array} \right) \in A\) Thus A is a Right ideal of R.

Algorithm/ Sub-Program: Example: Let R be a Ring of \(2 \times 2\) Matrices over Integers

Step1: Start
Step2: Declare Array to Read Matrices of Order \(2 \times 2\).
Step3: Read the Order of First Matrix \(m \times n\) and Second Matrix \(b\).
Step4: Enter the elements of First Matrix and make it a Loop Run to Form a Matrix.
Step5: Enter the elements of Second Matrix and make it a Loop Run to Form a Matrix.
Step6: Calculate the Sum of the two Matrices using \(\text{sum}[c][d] = \text{first}[c][d] + \text{second}[c][d];\)
Step7: Calculate the Multiplication of the two Matrices using \(\text{mul}[c][d] = \text{first}[c][d] * \text{second}[c][d];\)
Step8 : Stop.

#include <stdio.h>

int main()
{
    int m, n, c, d, first[6][6], second[6][6], sum[6][6];

    printf("Enter the number of rows and columns of matrix\n");
    scanf("%d%d", &m, &n);
    printf("Enter the elements of first matrix\n");
    for (c = 0; c < m; c++)
        for (d = 0; d < n; d++)
            scanf("%d", &first[c][d]);
    printf("Enter the elements of second matrix\n");
    for (c = 0; c < m; c++)
        for (d = 0; d < n; d++)
            scanf("%d", &second[c][d]);
    printf("Sum of entered matrices: \n");
    for (c = 0; c < m; c++)
        for (d = 0; d < n; d++)
            sum[c][d] = first[c][d] + second[c][d];
    printf("\n", sum[c][d]);
}

printf("\n");
Prime Ideal
Definition: An Ideal \( P \) of a Ring \((R, +, \cdot)\) is said to be a Prime Ideal if
\[ \forall a, b \in R \Rightarrow a \cdot b \in P \Rightarrow \text{either } a \in P \text{ or } b \in P. \]
Where ‘\( \cdot \)’ is a Binary Operation.

Example: Let \( Z \) be a Ring of Integers, Then \{0\} is an Ideal which is a Prime Ideal of \( Z \)
\[ \forall m, n \in Z \ \exists \ mn \in \{0\} \Rightarrow mn = 0 \Rightarrow \text{Either } m = 0 \text{ or } n = 0. \]

Cyclic Group
A group \( G \) is said to be Cyclic if there exists a \( a \in G \), Such that the Group Itself is generated by \( a \).
Group can be denoted by \(< a >\)

return 0;
}

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Sub-Program
Syntax:

```c
#include<stdio.h>
#include<math.h>

void main()
{
int count;
int n, k, result, a, b;
printf("Enter that Range Integers = ");
scanf("%d", &n);
printf("Enter that starting Integer = ");
scanf("%d", &k);
for(count = -k; count <= n; count++)       // Display the numbers from Negative Integers to Positive Integers/
print("d ", count);
print("n");
printf("Take two Integers a & b from the above Integer Set \n");
scanf("%d%*d", &a, &b);
result = a * b;
if(result == 0)
{
printf("The result a\*b = %d , Either a or b is Zero \n Hence \{0\} is a Prime Ideal of Z", result);
}
else
{
printf("Both a \& b zero / Both are non-zero \n Hence \{0\} is not a Prime Ideal of Z\n");
}
getch();
}
```

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Group can be denoted by \(< a >\)

Syntax:

```c
#include <stdio.h>

int main()
{
int n, i;
printf("Enter the Considered Generator: ");/* Consider any Number as Generator*/
scanf("%d", &n);
for(i=1; i<=10; ++i)
```
{  
    printf("%d * %d = %d \n", n, i, n*i);
}  
return 0;

II. CONCLUSION

This Research has a vast Advantages, as it contains Some Logical Programming Parts in C Language which can Further leads to the development of Applications in Programming Languages, and will Encourage Mathematical Logics to associate and Execute on the Front End of Computers in one go. In This Topics, we have Associated Algebraic Theory and Basic Concepts of Algorithm and Programming in C-Language with Some Executed Sub-Programs.

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REFERENCES
