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Simulation and Analysis of Active Damping System for Vibration Control

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Abstract: *Vibration is the mechanical oscillations of an object about an equilibrium point. Vibration occurs in most machines, structures and dynamic systems leading to many undesirable consequences. Vibration damping system of an automobile not only supports the vehicle body, engine and passengers but also absorbs shocks arising from irregularities of the road. In this paper, the mathematical model for the passive and active damping systems for quarter car model was obtained. The Linear Quadratic Regulator (LQR) Control technique was implemented to the active damping system for a quarter car model. Computer simulation using MATLAB/Simulinks software was performed to verify the efficiency of the active damping with the application of the LQR controller. Comparison between passive and active damping system was performed by using two types of road profiles. Profile 1 is assumed to have 3 bumbs and profile 2 is assumed to have a bumb and a hole. The study gave a control scheme for force control in an active vehicle suspension design using LQR Controller with a quarter car model for random input. The steady state of suspension travel, wheel deflection, body displacement, and the car body acceleration was fast. Other controllers can also be used with the same or different parameters.*

Keywords: *Vibration, Damping, Quarter Car, LQR Controller, Suspension,*

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I. INTRODUCTION

Vibration is a repetitive motion of objects in alternately opposite directions from the equilibrium position when that equilibrium has been disturbed [1]. It occurs in most machines, structures, and dynamic systems leading to many undesirable consequences. Vibration often becomes a problem due to unpleasant motions, noise and dynamic stresses that could lead to fatigue and failure of the structure or machine, energy losses, decreased reliability, and degraded performance [2]. Control of such vibrations is being widely studied in the engineering field.

All vehicles moving on irregular profiled road are exposed to vibrations which are harmful to passengers in terms of comfort and for the durability of the vehicle itself. Therefore, the main aim of a vehicle suspension is to ensure ride comfort and road-wheel continuous contact which directly contribute to the car safety. An active suspension system (ASS) consists of a sensor, actuator, a source of power and a compensator that can perform well under vibration. ASS has the capability to adjust itself continuously to changing road conditions resulting in a better set of design trade – offs compared to passive suspension where the parameters are fixed being to achieve certain level of compromise between road handling, load carrying and comfort. The parameters are non – modifiable by any mechanical part. ASS employs pneumatic or hydraulic actuators for additional energy. The actuator is secure in parallel with a spring and shock absorber [3]. ASS requires sensors to be located at different points of the vehicle to measure motions of the vehicle body. This information is fed as input for the controller in order to provide exact amount of force required through the actuator [4]. The passive damping system is a system of open loop control. It is designed to achieve certain conditions only. The passive damping problem is that, if the design is strongly damped, it will transfer much input from the road input or to lift the car on the road irregularities. Then, if it is lightly damped or flexible suspension it will reduce the stability of the vehicle which in turn change lane or it will swing the car [5]. In this paper, a linear quadratic regulator (LQR) control theory is used to develop ASS. The passenger feels highly comfortable if the weighted RMS acceleration is below $0.315 m/s^2$, but to cross the bump speed the vehicle must be less than10kmph which can be fairly comfortable to human body [22]. For the proper suspension travel minimum of 5 inches (0.127m) of suspension travel must be available in order to absorb a bump acceleration of one – half "g" without hitting the suspension stops [9]. For the proper road holding relative displacement between wheel and road must be in the range of 0.0508m [9].

II. RELATED WORK

Bart *et al.*, (2016), [6] offers an electromagnetic active suspension system that provides both additional stability and maneuverability by performing active roll and pitch control during cornering and braking, as well as eliminating road irregularities, hence increasing both vehicle and passenger safety and drive comfort. Prabhakar *et al.*, (2015), [7] simulate the passive suspension system for quarter car model with variable damping and stiffness parameters. Wissam *et al.*, (2015), [8] design an efficient control scheme for car suspension system. Mitre, *et al.*, (2014), [9] developed a MATLAB/SIMULINK model of full car to analyze the ride comfort and vehicle handling. Magdy, (2014), [10] study the application of PI/PID controller tuned by Adaptive Weighted Particle Swarm Optimization (AWPSO) algorithm to car suspension system. Kalidas, *et al.*, (2013), [11] present a car suspension system model which contains two parts. The first part deals with the formulation of a mathematical model for a conventional full car passive suspension system. The second part deals with simulation of the mathematical model of the suspension system. Simulation is carried out using MATLAB. Sathishkumar *et al.*, (2014), [12] discussed about the mathematical modeling and simulation study of two degree of freedom for a quarter car model. Swati, *et al.*, (2013), [13] studied the application of PI and PID controller to control the vibration occurred in the bus suspension system. Agharkakli *et al.*, (2012), [14] studied simulation and analysis of passive and active suspension system. Kumar (2012), [15] designed the PID controller was for an active suspension system. Shpetim *et al.*, (2012), [16] studied, design and optimization procedure for active and semi-activenon-linear suspension systems regarding terrain vehicles. Fayyad, (2012), [17] presented a control system for active Suspension systems with flexible leaf springs fixed at the four corners, to the modern automobiles with complex control algorithms. Ammar and Weiji (2012), [18] treated the vibration excited by road unevenness as a source of mechanical energy. It was converted into electrical energy to compensate for the energy consumption by the active suspension. Vladimir *et al.*, (2011), [19] applied the system approach and system engineering methods in the initial phase of vehicle active suspension development. Alexandru *et al.*, (2011), [20] represents a comparative analysis between the passive and active suspension systems of the motor vehicles. Amit *et al.*, (2011), [21] represent vehicle primary suspension system along with the analysis of a semi active suspension system with Bingham model for MR damper. Many controller designs ([8], [10], [15], [18] and [20]) have been proposed to develop ASSs.

III. SYSTEM MODEL

The dynamic model, which can describes the relationship between the input and output, enable one to understand the behavior of the system, figure 1 shows general principles of mathematical modeling. The purpose of mathematical modeling in this paper is to obtain a state space representation of the quarter car model. In this paper the suspension system is modeled as a linear suspension system. The state variable can be represented as a vertical movement of the car body and a vertical movement of the wheels. In order to obtain linear model, roll and pitch angles are assume to be small.

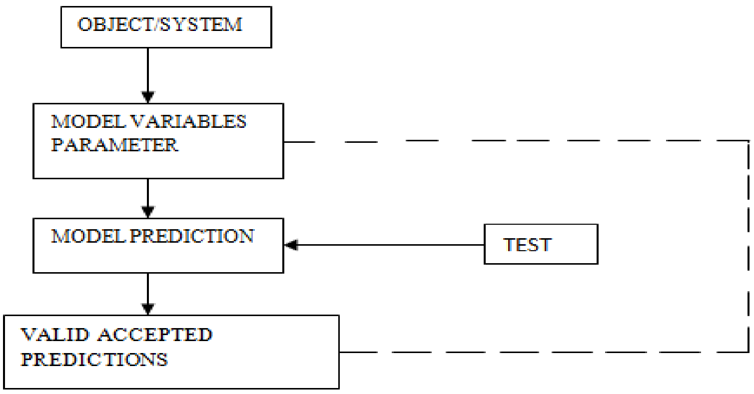

Figure 1: principles of mathematical modeling

A. **Mathematical Modeling of Passive Suspension for Quarter Car Model.**

A detailed of derivation for passive suspension is based on Robert and Kent, (1997) [23] and Mitra et al, (2014) [9]

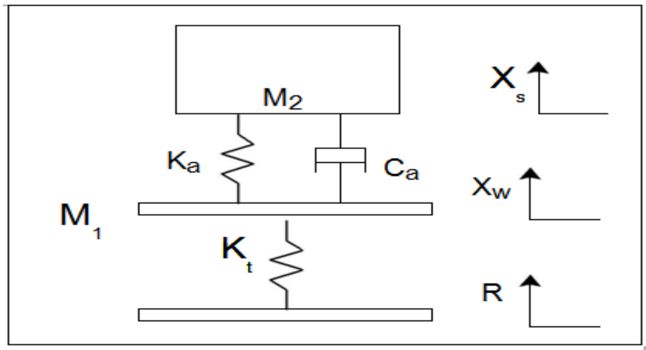

Figure 2: Passive Suspension for Quarter Car Mode

From Fig. 2, the mathematical equations are obtained

For M_1:

F = Ma

$$K_t(Xw - r) - K_a(Xw - Xs) - Ca(\dot{X}w - \dot{X}s) = M1\ddot{X}w$$

$$\ddot{X}w = \frac{Kt(Xw - r) - Ka(Xw - Xs) - Ca(\dot{X}w - \dot{X}s)}{M1} \qquad (1)$$

For M_2:

$$-K_a(Xs - Xw) - Ca(\dot{X}s - \dot{X}w) = M2\ddot{X}s$$

$$\ddot{X}s = \frac{-Ka(Xs - Xw) - Ca(\dot{X}s - \dot{X}w) = M2\ddot{X}s}{M2} \qquad (2)$$

Where,

M_1 = mass of the wheel /unsprung mass (kg)

M_2 = mass of the car body/sprung mass (kg)

r = road disturbance/road profile

X_w = wheel displacement (m)

X_s = car body displacement (m)

K_a = stiffness of car body spring (N/m)

K_t = stiffness of tire (N/m)

C_a = damper (Ns/m)

Let the state variables are

$$X_1 = X_s - X_w$$
$$X_2 = \dot{X}_s$$
$$X_3 = X_w - r \qquad (3)$$
$$X_4 = \dot{X}_w$$

Where

$X_s - X_w$ = suspension travel

$\dot{X}_s$ = car body velocity

$X_w - r$ = wheel deflection

$\dot{X}_w$ = wheel velocity

Therefore in state space equation, equation (3) can be written as;

$$\dot{X}(t) = Ax(t) + f(t) \qquad (4)$$

Where

$$\dot{X}_1 = \dot{X}_s - \dot{X}_w \approx X_2 - X_4$$
$$\dot{X}_2 = \ddot{X}_s$$
$$\dot{X}_3 = \dot{X}_w - \dot{r} \approx X_4 - \dot{r} \qquad (5)$$
$$\dot{X}_4 = \ddot{X}_w$$

Rewrite equation (4) into the matrix form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-K_a}{M_2} & \frac{-C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{-K_a}{M_1} & \frac{C_a}{M_1} & \frac{K_t}{M_1} & \frac{-C_a}{M_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \qquad (6)$$

B. Mathematical Modeling of Active Suspension for Quarter Car Model

Mathematical modeling for active suspension is derived from Fig. 2 and Fig. 3. There is slightly difference in the derivation of the mathematical modeling for active suspension from the passive suspension system. Derivation for M_1 (unsprung mass) and M_2 (sprung mass) are shown below:

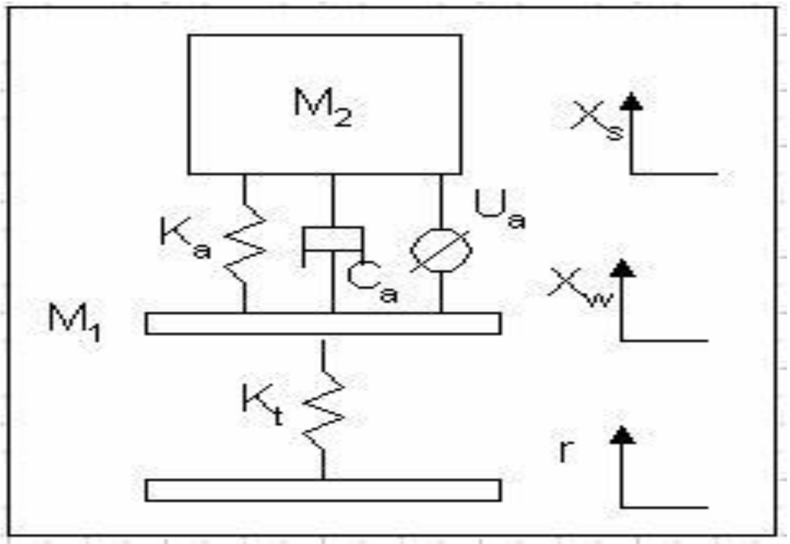

Figure 3: Active Suspension for Quarter Car Model

For M_1:

F = Ma

$$K_t(Xw - r) - K_a(Xw - Xs) - Ca(\dot{X}w - \dot{X}s) - U_a = M1\ddot{X}w$$

$$\ddot{X}w = \frac{Kt(Xw - r) - Ka(Xw - Xs) - Ca(\dot{X}w - \dot{X}s) - U_a}{M1} \text{ -------------------------- (7)}$$

For M_2:

$$-K_a(Xs - Xw) - Ca(\dot{X}s - \dot{X}w) + U_a = M2\ddot{X}s$$

$$\ddot{X}s = \frac{-Ka(Xs - Xw) - Ca(\dot{X}s - \dot{X}w) + U_a}{M2} \text{ -- (8)}$$

M_1 = mass of the wheel /unsprung mass (kg)
M_2 = mass of the car body/sprung mass (kg)
r = road disturbance/road profile
X_w = wheel displacement (m)
X_s = car body displacement (m)
K_a = stiffness of car body spring (N/m)
K_t = stiffness of tire (N/m)
C_a = damper (Nm/s)
U_a = force actuator

The state variables are established in equation (3) Therefore, equations (7) and (8) can be written as below

$\dot{X}(t) = Ax(t) + Bu(t) + f(t)$ -- (9)

Rewrite equation (4) into the matrix form

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-K_a}{M_2} & \frac{-C_a}{M_2} & 0 & \frac{C_a}{M_2} \\ 0 & 0 & 0 & 1 \\ \frac{-K_a}{M_1} & \frac{C_a}{M_1} & \frac{K_t}{M_1} & \frac{-C_a}{M_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \\ \frac{1}{M_1} \end{bmatrix} U_a + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{r} \text{ ---------------- (10)}$$

C. The parameters used

The parameters were based on [9]. The parameters used are;
Unsprung mass = mass of the wheel/tyre = 59kg = M_1
Sprung mass = mass of vehicle body = 290kg = M_2
Stiffness of the wheel/tyre (K_t) = 190000N/m
Stiffness of car body (K_a) = 16812N/m
Damper (C_a) = 1000Nm/s
Force actuator (U_a) = 10000N

D. LQR Controller Design

This study considered the following state variable feedback regulator.

$u = -kx$ --- (11)

Where k is the state feedback gain matrix

Optimization of control system consists of determining the control input u, which minimizes the performance index (J), which represents the performance characteristics requirement as well as controller input limitations. The performance index

$J = \frac{1}{2}\int_0^t (x'Qx + u'Ru)dt$ -- (12)

Where u = actuator force (N)

Q and R are positive definite weighting matrices.

Linear optimal control theory provides the solution of equation (12) in the form of equation (11)

$k = R^{-1}B'P$ --- (13)

Where matrix P must satisfy the reduced – matrix Riccati equation

$A'P + PA - PBR^{-1}B'P + Q = 0$ -- (14)

Then the feedback regulator u is given by

$u(t) = -(R^{-1}B')(t)$

$\quad = -kx(t)$

IV. RESULTS AND DISCUSSIONS

A. Road Profile 1 for Quarter Car and Full Car Simulation

Two types of road disturbance are assumed as the input for the system. The road profile 1 is assumed to have 3 bumps taken from [14]. The disturbance input representing in [14] is as shown below where a denotes the bump amplitude, and is characterized by;

$$r(t) = \begin{cases} \frac{a(1-\cos 8\pi t)}{2}, & 0.625 \le t \le 0.75 \\ & 3.375 \le t \le 3.5 \\ & 5.25 \le t \le 5.5 \\ 0, & otherwise \end{cases}$$

Where a = 0.05, 0.025, 0.05

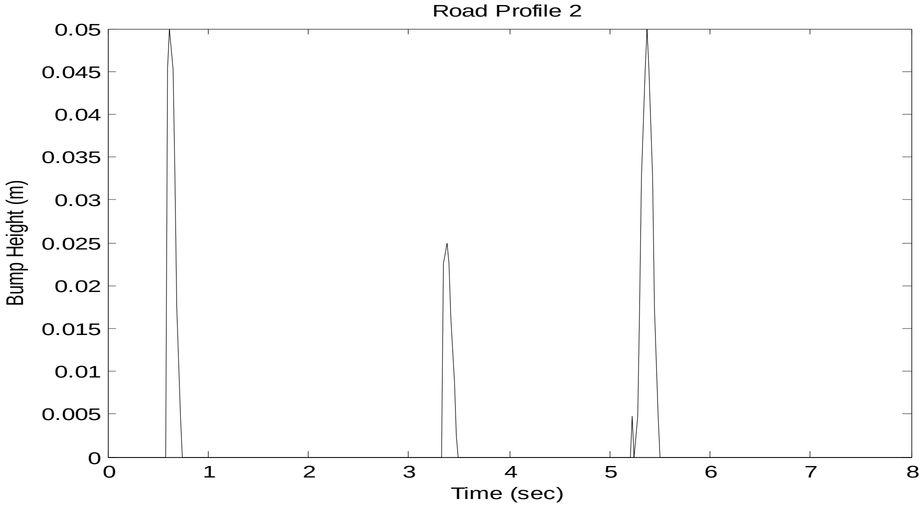

Figure 4: Road Profile 1 for Quarter Car model

3.2 Road profile 2 for Quarter Simulation

The road profile 2 is assumed to have a bump and a hole taken from (Sathishkumar 2014). The disturbance input is as shown below where a denote the bump amplitude. The road input is described by the following equation

$$r(t) = \begin{cases} \frac{a(1-\cos 8\pi t)}{2}, & 0.625 \le t \le 0.75 \\ & 3.375 \le t \le 3.5 \\ 0, & otherwise \end{cases}$$

where a = 0.05, -0.025

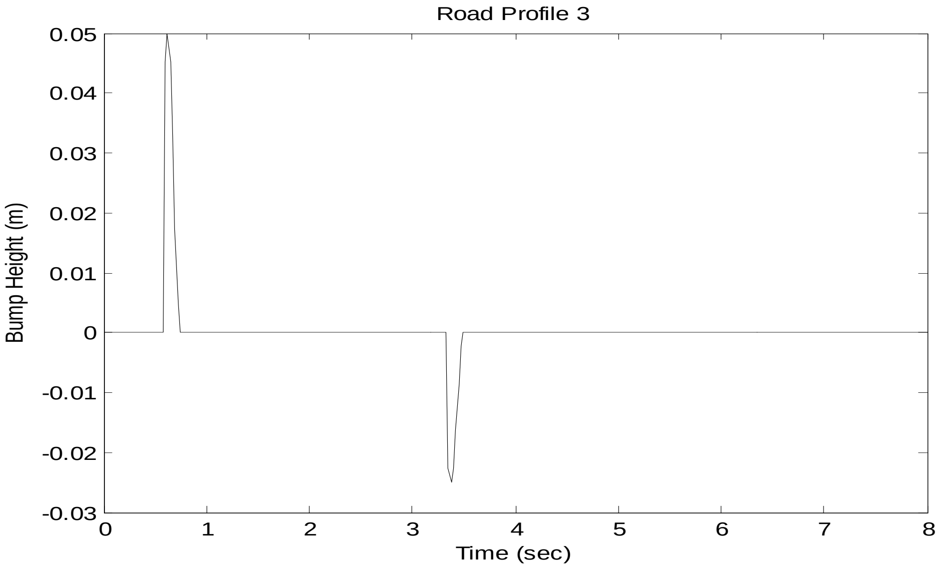

Figure 5: Road Profile 2 for Quarter Car Simulation

B. Comparison between Passive and Active Damping for Quarter Car

The computer simulation work based on equation (3.4) and (3.17) has been performed. Comparison between passive and active suspension for quarter car model was observed. For the LQR controller, the weighing matrix Q and weighing matrix R is set to be as below after turning of Q and R to obtain suitable feedback gain k.

Q =

33.6280	-33.6180	1.9990	-1.9990
-33.6180	33.6280	-1.9990	1.9990
1.9990	-1.9990	0.1189	-0.1189
-1.9990	1.9990	-0.1189	0.1189

K =

-1.0380 1.0390 0.0818 0.1076

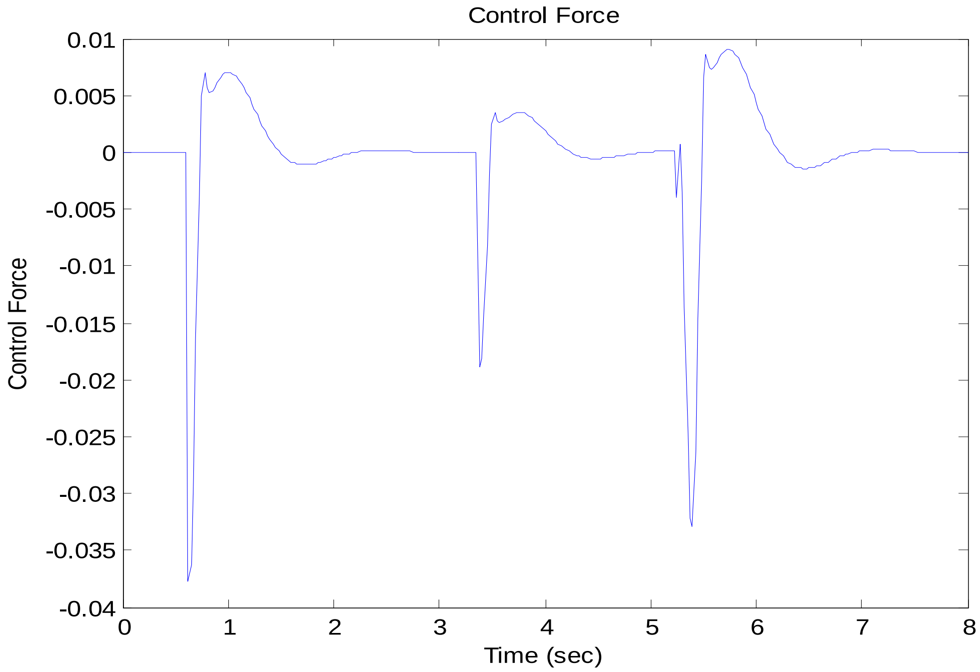

Figure 6: Force generated using LQR Controller with road profile 1

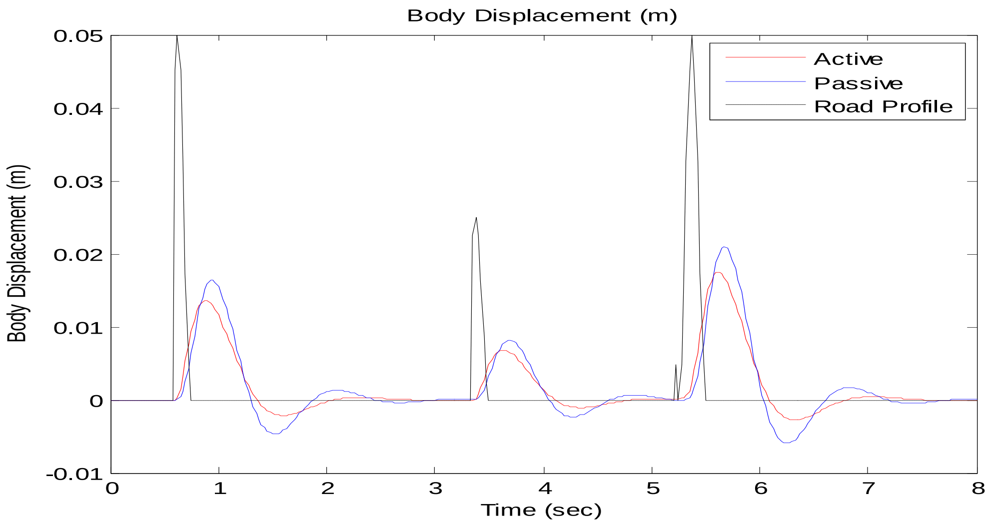

Figure 7: Body displacement with road profile 1

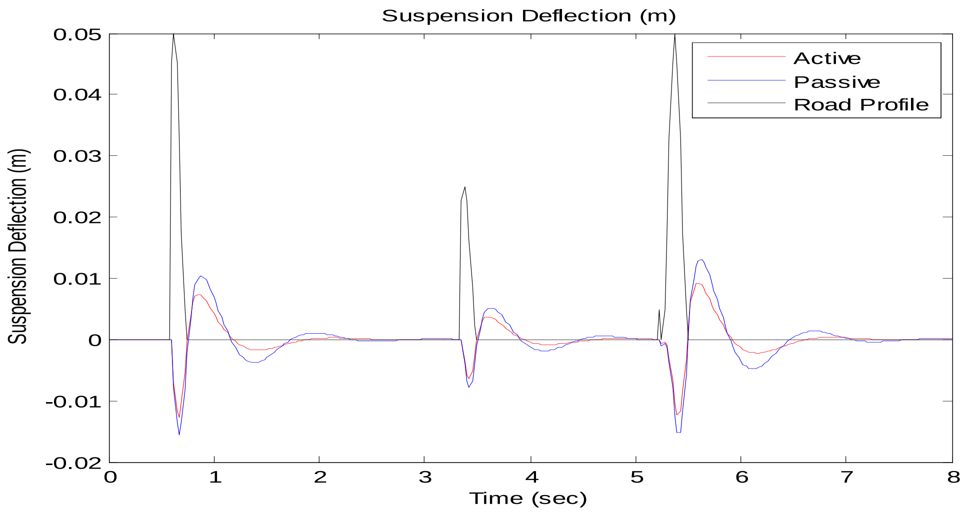

Figure 8: Suspension Deflection with Road Profile 1

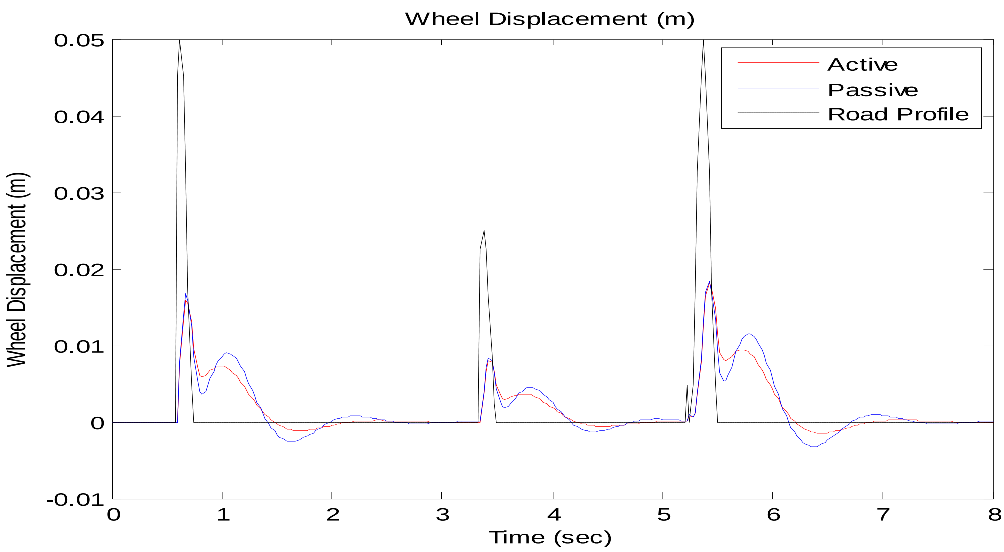

Figure 9: Wheel Displacement with road profile 1

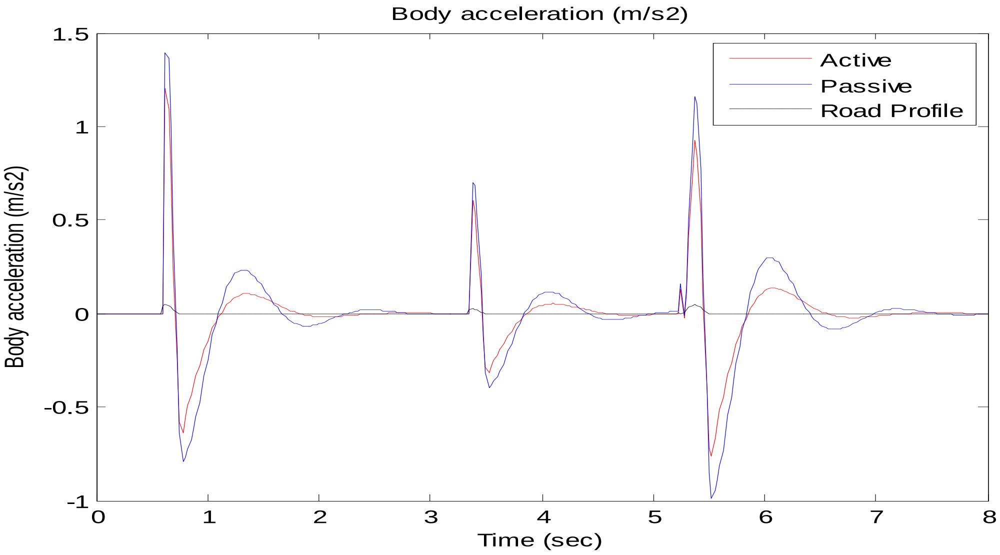

Figure 10: Body Acceleration with Road Profile 1

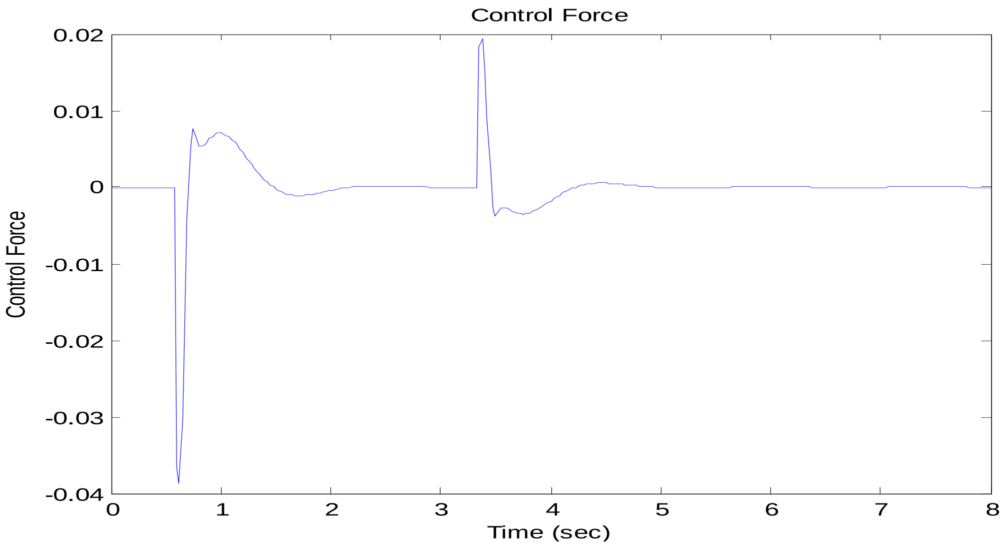

Figure 11: Force generated using LQR Controller with road profile 2

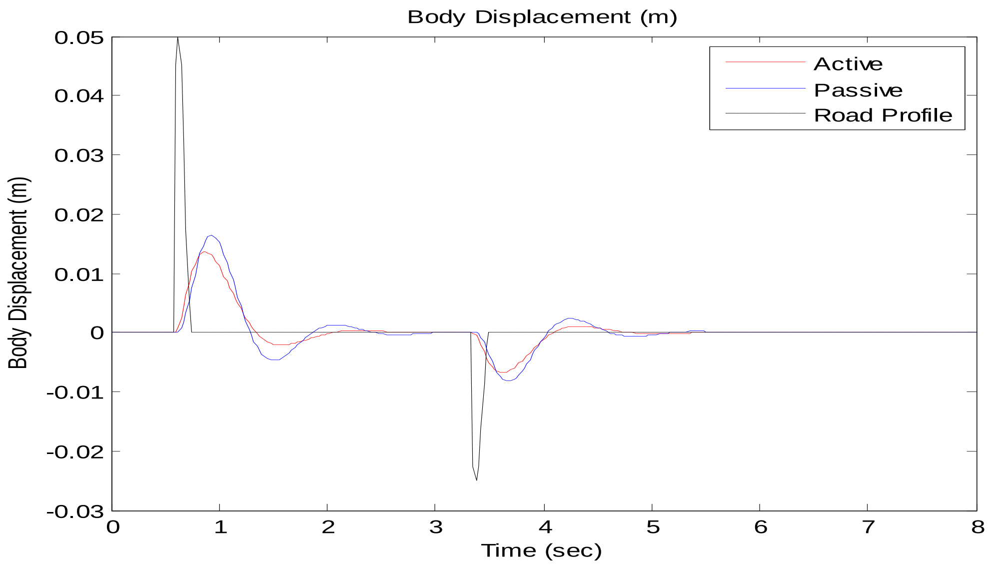

Figure 12: Body displacement with road profile 2

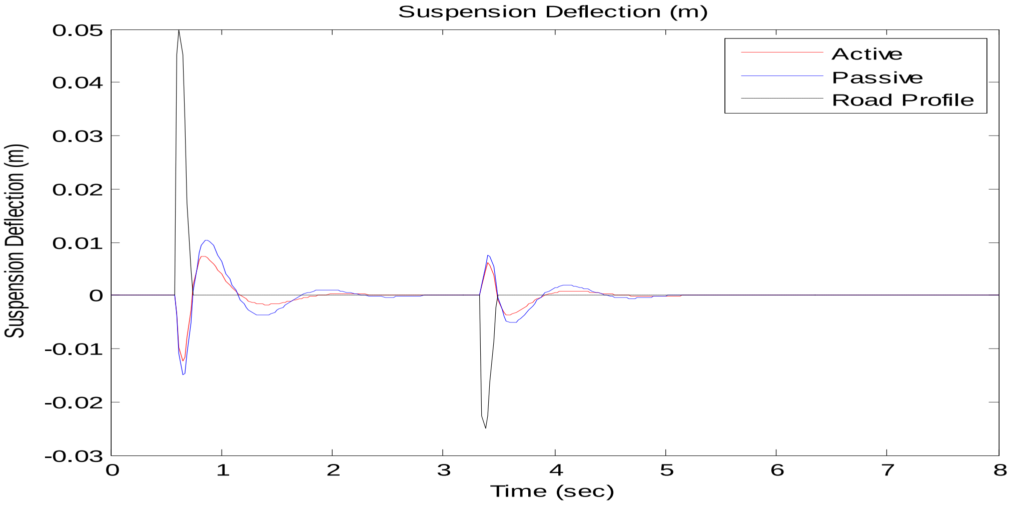

Figure 13: Suspension Deflection with road profile 2

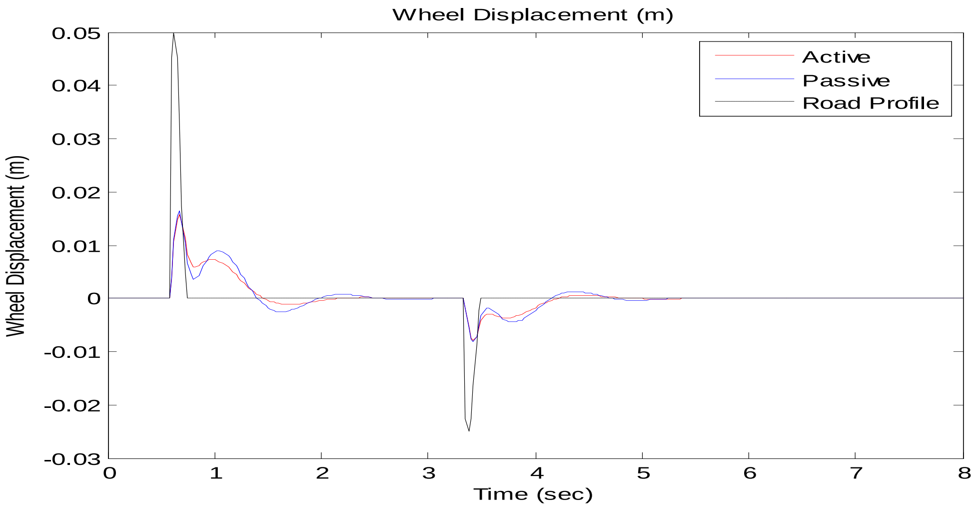

Figure 14: Wheel Displacement with Road Profile 2

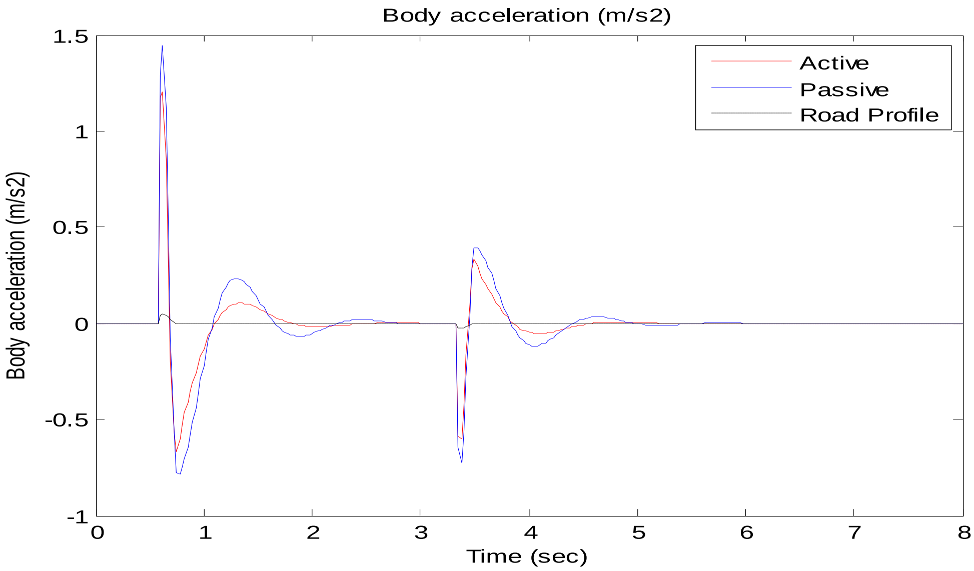

Figure 15: Body Acceleration with Road Profile 2

A. Discussion of Result: Quarter Car Model

The force generated by the actuator for the two road profiles are shown in Fig. 6 and Fig. 11. By comparison, it clearly shows that Active damping using LQR controller gives lower amplitude and faster settling time of less than 2 seconds for all the parameters. That is, the Body displacement, Suspension deflection, Wheel displacement and Body Acceleration for all the two road profiles compared to Passive damping system as can be seen in Fig. 7 to Fig. 15. Car body displacement represent ride quality and Wheel displacement represent car handling performance.

V. CONCLUSION

Mathematical model for Passive and Active damping system for quarter car was derived and validated. Simulation of the Passive and Active damping system with LQR controller was performed and comparison was made between the Passive and Active damping system. The Active damping with LQR controller gives lower amplitude and faster settling time of less than 2 seconds for all the parameters compared to the Passive damping system.

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