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Invariant Stabilization Algorithms in a Control System with Rotating Operating Device

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ABSTRACT: *The publication suggests how to significantly improve the spacecraft center of mass movement stabilization accuracy in the active phases of trajectory correction during interplanetary and transfer flights, which in some cases provides for high navigation accuracy, when rigid trajectory control method is used. The required stability conditions obtained are consistent with the known criteria in the invariant theory.*

Keywords -*Space probe (SP), stabilization controller (SC), on-board computer (OC), gyro-stabilized platform (GSP), propulsion system (PS), angular velocity sensor (AVS), operating device (OD), space vehicle (SV), feedback (FB), control actuator (CA), control system (CS), angular stabilization (AS), center of mass (CM)*

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I. INTRODUCTION

The thriving space technology is characterized by an increasing complexity of the tasks to be solved by modern space vehicles (SV). The efficiency in solution of such tasks significantly depends upon technical characteristics of the on-board systems ensuring the functioning of the spacecraft. In some cases, when using a control system built according to the principle of program control (the "robust trajectories" method) the efficiency of task solution is much influenced by the accuracy of the spacecraft stabilization system in the powered portion of flight. This concerns, for example, the trajectory correction phases during interplanetary and transfer flights, when the rated impulse execution errors during trajectory correction resulting from various disturbing influences on the spacecraft in the active phase, greatly affect the navigational accuracy. Hence, reduction of the cross error in the control impulse on the final correction phase during the interplanetary flight, facilitates almost proportional reduction of spacecraft miss in the "perspective plane". For example, in some space probes (SP) like Deep Impact [1], [2] and Rosetta missions [3], [4] reduction of cross error by one order during the execution of correction impulse (for modern stabilization systems this value shall be $0.5\ m/s$) results in reduction of spacecraft miss in the "perspective plane" from 200 to 20 km. Such reduction of the miss accordingly increases a possibility of successful implementation of the flight plan, as well as the accuracy of the research and experiments conducted [5].

The Martian Moons Exploration (MMX) mission is scheduled to launch from the Tanegashima Space Center in September 2024. The spacecraft will arrive at Mars in August 2025 and spend the next three years exploring the two moons and the environment around Mars. During this time, MMX will drop to the surface of one of the moons and collect a sample to bring back to Earth. Probe and sample should return to earth in the summer 2029 [6].

Besides improvement of the navigational accuracy, reduction of spacecraft stabilization cross errors in the active phase, it also results in lower total characteristic velocity of corrective impulses, and, consequently, in reduction of fuel required for the correction. So, when the correction speed impulse reaches 30 m/s reduction of gross error during the correction maneuver results in proportional reduction of the required characteristic velocity during the next correction. The data referred to in [7], [8] show that improved accuracy of roll stabilization in the active phase by one order results in reduction of total characteristic correction velocity for Mars interplanetary probe (Mars-96, Russian Federation) from about 20 to 2 m/s, which corresponds to fuel savings approximately by 30 kg, or to increase of the payload mass by 4%. Due to the relatively small weight of modern scientific instruments (about 3-8 kg), even such seemingly small increase of payload weight can significantly extend the program of research and experiments implemented by the spacecraft.

Objectives: to solve the task of significant increase in stabilization accuracy of center of mass tangential velocities during the trajectory correction phases when using the "rigid" trajectory control principle.

Since the time of the active phase in correction maneuvers, which is to be determined by the required velocity impulse, shall not be clearly determined in advance, and quite limited, and because a guaranteed approach enabling to estimate the accuracy, is always used in practice for solving the targeting tasks, we shall understand the maximum dynamic error of the transition process as concerns the drift velocity of the spacecraft to mean the accuracy of the spacecraft center of mass movement stabilization [9].

Subject of research: The center of mass movement stabilization system in the transverse plane, which is used during the trajectory correction phases.

II. STATEMENT OF THE PROBLEM

In order the control actions could be created during the spacecraft trajectory correction phase, a high-thrust service propulsion system with a tilting or moving in linear direction combustion chamber shall be used.

Functioning of the spacecraft movement stabilization channel in the transverse plane is based on the feedback principle, and together with the spacecraft this channel forms a closed deviation control system. We can consider two channels in this control system: an angular stabilization channel and center of mass movement stabilization channel.

The angular stabilization channel facilitates angular position of the spacecraft when exposed to disturbing moments. The center of mass movement stabilization channel is to ensure proximity to zero of normal $\dot{y}$ and lateral $\dot{z}$ velocities of the spacecraft under the influence of disturbing moments and forces. In most of the known (model) spacecraft stabilization systems [10],[11],[12] the control signal in the center of mass movement stabilization channel is generated according to proportional plus integral control law based on the measurements of tangential velocity of the center of mass $\dot{y}(\dot{z})$ and its integral-linear drift $y(z)$. In the angular stabilization channel, the control signal shall be generated in proportion to the spacecraft deviation angle in the transverse plane $\vartheta(\psi)$ and the angular velocity of the spacecraft rotation in this plane $\dot{\vartheta}(\dot{\psi})$.

Distance Improvement of control accuracy increases chances for successful implementation of the flight program. However, a significant reduction in the correcting impulse lateral error leads to reduction in fuel required for corrections, and thus increases the payload [5], [6].

The publication addresses spacecraft which use high-thrust PS for correcting impulses and control at active phases. During the active phase, the spacecraft shall be exposed to disturbances caused mainly by working PS. These disturbances create components of the spacecraft center of mass velocity in the normal and lateral directions (the drift velocity), and the spacecraft center of mass stabilization system is to provide center of mass lateral drift velocities close to zero during active phases. Since the time of the active phase T, which is determined by specified velocity impulse is not known and quite limited during correction maneuvers [7], [8] and in view of the fact that a guaranteed approach evaluating accuracy is always used to solve a guidance task in practice, in this publication, we shall understand the maximum dynamic error of the transition process $\dot{y}_{\max}(\dot{z}_{\max})$ with normal (lateral) drift velocity of the spacecraft as the accuracy of spacecraft center of mass movement stabilization in transverse directions.

Consequently, our purpose is to significantly increase stabilization accuracy of the spacecraft center of mass tangential velocities (reduction of the maximum dynamic error in the drift velocity of the spacecraft in the transition process). This shall be done by synthesis of highly accurate stabilization algorithms in the rigid trajectory control system on the trajectory correction phases outside the atmosphere when using high-thrust engines.

The spacecraft center of mass movement stabilization system in the normal (lateral) plane applied in the trajectory correction phases shall be the subject of research. A high-thrust sustainer PS provided either with deviating or linearly moving combustion chamber shall be used in the correction phase to control motions of the spacecraft [5].

III. SYNTHESIS OF STABILIZATION ALGORITHMS

We study motions of the spacecraft in the normal plane of the inertial coordinate system XOY (Fig. 1) [5]. The center O of the inertial coordinate system at the beginning of the active phase is the same as the center of mass of the spacecraft; the axis OX coincides with the direction of the required correction impulse $\Delta\vec{V}_{cor}$, axis OY together with axis OX form a normal plane. The angular position of the spacecraft in the normal plane is determined by an angle ϑ between axis OX of the inertial coordinate system and X- axis O_cX_c of the bound coordinate system. Control of the spacecraft in the active phase shall be done by deflection of combustion chamber of PS at an angle δ between X-axis $O_c X_c$ of the spacecraft and X-axis of the nozzle symmetry of PS [9].

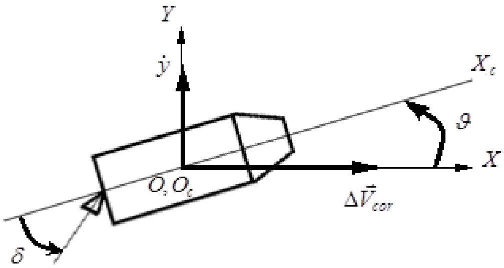

Fig. 1. Spacecraft diagram in the inertial coordinate system

The following assumptions and conditions were used in the process of synthesis of the stabilization algorithms [5]:

1. We assume that the spacecraft is subject to disturbances in the active phase (force F and moment M), which are mainly caused by working PS (tilt and thrust misalignment). Because of their nature, these parameters shall slowly change in time throughout the active phase (except for the period from the start of PS till switching to the nominal operation mode $\approx 0.2s$). For this reason, the disturbances may be considered permanent within the active phase with a reasonable degree of accuracy: $F = const; M = const$. We shall consider the work of the stabilization system within the entire possible range of disturbances: $0 < |F| \le F_{\max}; 0 < |M| \le M_{\max}$ (experience shows that the maximum force and moment are respectively about 0.3^0 and 3.5^0 in the equivalent deviation angles of PS).
2. The motion of the spacecraft is considered as movement of the absolute rigid body in vacuum relative to the reference trajectory in the normal plane of the inertial coordinate system.
3. A high-thrust chemical engine is used to control the spacecraft in the active phase. Control is provided by deflecting PS combustion chamber. The servo control, which deflects the combustion chamber, includes a feedback control actuator.

To stabilize the angular position of the spacecraft we shall use the information about deviation of the spacecraft body-fixed axes from the axes of the inertial coordinate system implemented in the gyro stabilized platform (CST) on board the spacecraft and the angular velocity sensors (AVS). The information on the deviation of the tangential velocities shall be taken from the accelerometers installed on CSP.

3.1. Mathematical Model

Taking in consideration the above assumptions and suppositions we can set down a system of equations (1) describing the behavior of the spacecraft center of mass motion stabilization system under study:

$$\begin{cases} \ddot{y} - C_{y\vartheta}\vartheta - C_{y\delta}\delta = F_y \\ \ddot{\vartheta} + C_{\vartheta\delta}\delta = M_z \\ \dot{\delta} = K_{CA}(W_{AS}\vartheta + W_{CM}\dot{y} - K_{FB}\delta), \end{cases} \qquad (1)$$

where $y-$ is the center of mass drift coordinate in the inertial coordinate system; $C_{y\vartheta}, C_{y\delta}, C_{\vartheta\delta}$ – are dynamic coefficients of the spacecraft; $C_{y\vartheta} = C_{y\delta} = \frac{P}{m}$, where $P-$ is PS thrust, $m-$ is mass of the spacecraft; $C_{\vartheta\delta} = \frac{Pl}{I_z}$, where $l-$ is the distance from the gimbal assembly of PS to the center of mass of the spacecraft, I_z – is momentum of inertia of the spacecraft relative to the axis $0z_c$ of the bound coordinate system; K_{CA} – is a velocity performance index of the control actuator; K_{FB} – is a control actuator feedback index; W_{AS} – is a response function of the angular stabilization controller; W_{CM} – is a response function of the stabilization controller through the center of mass channel.

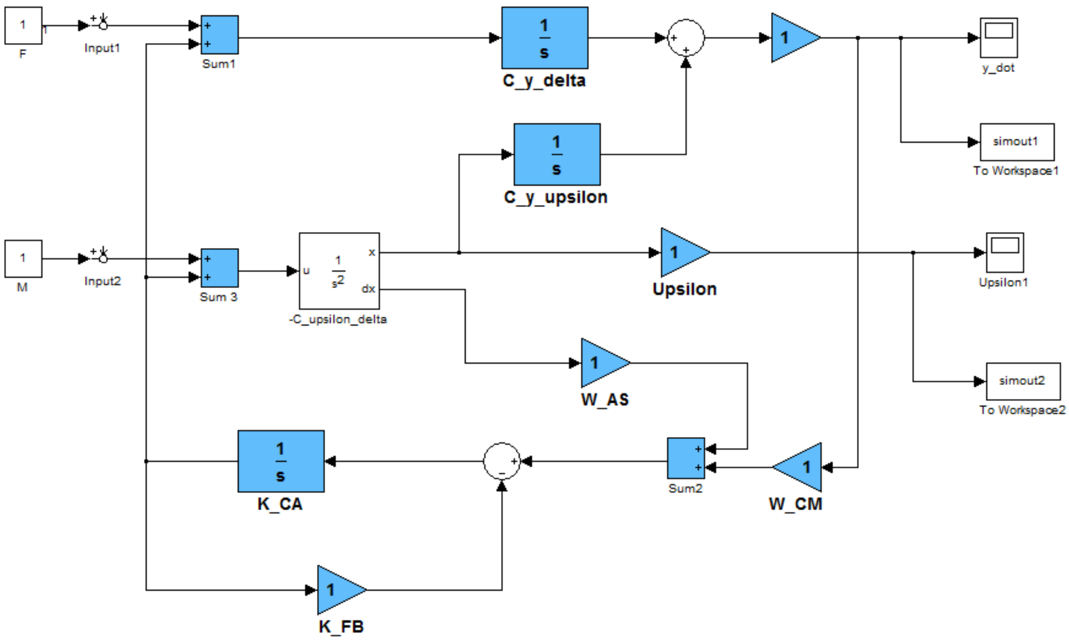

Fig. 2. Block diagram of the spacecraft center of mass motion stabilization system under study

According to the above mathematical model, a block diagram of the stabilization system under study shall be as follows (Fig. 2) [5], [9].

In order to improve accuracy of stabilization while using synthesized algorithms, a model of a model of a standard stabilization system shall be made. It is to be used as a reference model for comparison. The standard stabilization model uses a known stabilization controller [13],[14],[15], which provides control proportionally to the angle ϑ, of the spacecraft angular rotation velocity in the normal plane $\dot{\vartheta}$, linear drift y and the drift velocity $\dot{y}$. A block diagram of the standard stabilization system is shown in Fig. 3 [5], [9].

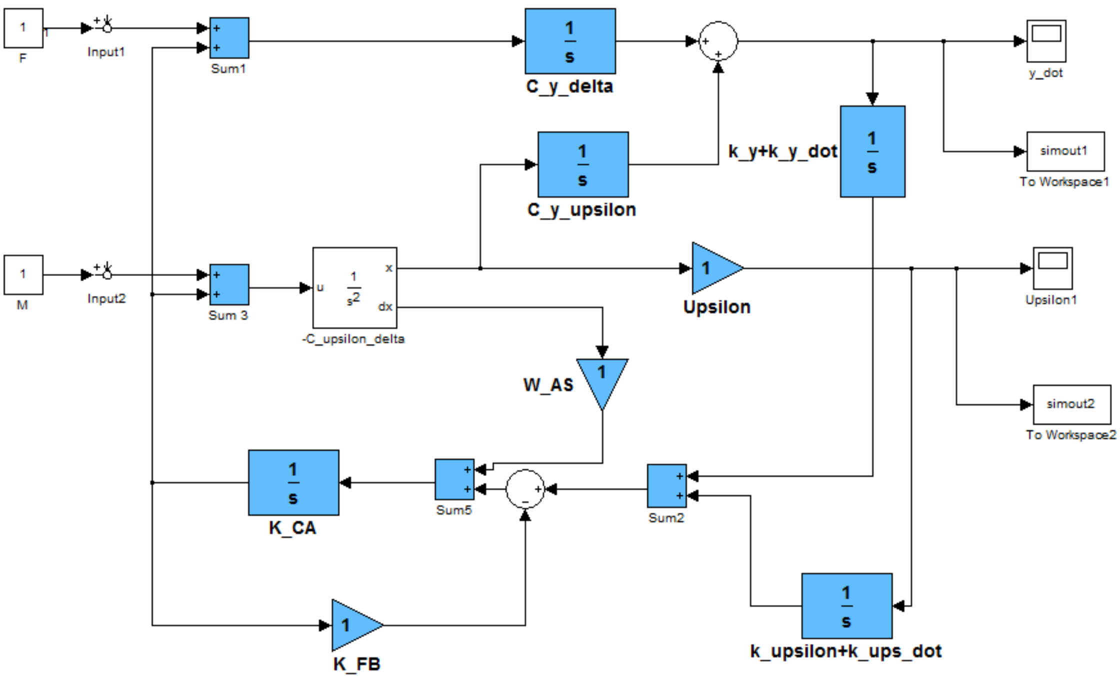

Fig. 3. Block diagram of a standard center of mass motion stabilization system of a spacecraft

3.2. Method to Solve an Invariant Problem

As mentioned above, usage of methods of the invariant theory [16],[17],[18],[19],[20],[21],[22],[23] is seen as a way to improve the accuracy of the automatic regulation system. In the present case, it is not possible to synthesize the invariant stabilization system using the method of combined regulation, which is traditional for invariant systems because actual measurements of the disturbing effects are not available. However, publications [24],[25] observe that it is possible to build an invariant system without use of combined regulation methods, if we apply the principle of dual-channel impact distribution in the controlled object. The principle of dual-channel impact distribution resides in the fact that if the controlled object has two distribution channels of the same impact, we may achieve mutual compensation of the impact transferred through the above channels by selecting a respective law of control so that the regulated value becomes invariant (independent) of the said impact.

Fig. 2 shows that the controlled object under study has two channels of distribution of disturbing moment M [5]. Therefore we can improve the accuracy of the stabilization system by using the invariant theory principle. So we select synthesis of high-precision stabilization systems based on the principles of the invariant theory as a method helping us to solve the problem set.

IV. SYNTHESIS OF INVARIANT STABILIZATION ALGORITHMS IN A CONTROL SYSTEM WITH ROTATING OPERATING DEVICE

Based on general provisions and the formulae of the invariant automatic control system theory [17],[18],[19],[20],[21],[22],[23],[24], [25], we shall synthesize an invariant center of mass stabilization system of the spacecraft, and compare the accuracy and quality of transition processes in the standard and invariant stabilization systems [5], [9].

4.1. Analysis the Physical Implementation of a Stabilization System Invariant to the Destabilizing Force and to the Disturbing Moment

Let's consider the system of equations (1), which describes the system to be explored, the stabilization and the functional diagram (Fig 2) [5] in terms of the choice of response functions $W_{AS}(s), W_{CM}(s)$ and $K_{FB}(s)$, providing invariance for coordinates $\dot{y}$ under influence F and M. The values F and M cannot be measured directly as supposed in combined regulation systems.

To analyze invariant conditions, we shall write equations (1) in operator form:

$$\begin{cases} s\dot{y} - C_{y\vartheta}\vartheta - C_{y\delta}\delta = F \\ s^2\vartheta + C_{\vartheta\delta}\delta = M \\ W_{CM}(s)\dot{y} + W_{AS}(s)\vartheta + \left(-K_{FB}(s) - \dfrac{s}{K_{CA}}\right)\delta = 0, \end{cases} \quad (2)$$

According to the basic provisions of the invariant theory [18],[19], [20] it is necessary that (2) to ensure invariance $\dot{y}$ under influences F and M

$$\dot{y} = \frac{\Delta\dot{y}_M}{\Delta} + \frac{\Delta\dot{y}_F}{\Delta} = 0 \quad (3)$$

Whence $\Delta\dot{y}_M = \Delta\dot{y}_F = 0$, where $\Delta\dot{y}_M, \Delta\dot{y}_F$ are invariant minors, and Δ is the main determinant of the closed system (2).

By substituting the determinants $\Delta\dot{y}_M, \Delta\dot{y}_F$ in (3) and Δ from the equations (2) we shall have the following necessary invariance conditions in the operator form [26]:

$$\begin{cases} s^2\left(K_{FB} + \dfrac{1}{K_{CA}}s\right) + W_{AS}C_{\vartheta\delta} = 0 \\ C_{\vartheta\delta}\left(K_{FB} + \dfrac{1}{K_{CA}}s\right) + W_{AS}C_{y\delta} = 0 \end{cases} \quad (4)$$

whence

$$\Delta W_{AS}, K_{FB} = -\frac{s}{K_{CA}}. \quad (5)$$

Rather than focusing so far on the meaning of the conditions obtained, let's consider the physical implementation of an invariant system providing the condition (4).

We know that the requirement to ensure the open system's absolute invariance is a criterion for the physical implementation of the invariant system proposed by academician B. Petrov [27].

The expression for Δ_{OS} shall be as follows [26]:

$$\Delta_{OS} = s\left[s^2\left(K_{FB} + \frac{1}{K_{CA}}s\right) + W_{AS}C_{\vartheta\delta}\right]. (6)$$

It is easy to see that when conditions (5) are met, the expression (6) identically becomes zero, whence it follows that an absolutely invariant in moment M and force F system can't be implemented [26].

Let's consider physical implementation of the system for each of the disturbances individually.

4.2. Physical Implementation of a Stabilizing System, which is Invariant under Disturbing Moment

In order to ensure invariance of the coordinate $\dot{y}$ under influence F it is necessary that the condition $\Delta\dot{y}_M = 0$ is met, that is

$$C_{y\vartheta}\left(K_{FB} + \frac{1}{K_{CA}}s\right) + W_{AS}C_{y\delta} = 0. (7)$$

It is obvious that in general the expression (6) doesn't become zero in case the invariance condition (7) is met, which makes it clear that the condition for physical implementation of the center of mass stabilization system under disturbance M is met.

We shall demonstrate that the transfer function of the open system under disturbing moment M is equal to zero if the invariance conditions are met. Hence $\dot{y} = 0$. Let's write down an open system determinant:

$$\Delta_{OS} = \begin{vmatrix} s & -C_{y\vartheta} & -C_{y\delta} \\ 0 & s^2 & C_{\vartheta\delta} \\ 0 & W_{AS} & \left(-K_{FB} - \frac{1}{K_{CA}}s\right) \end{vmatrix} (8)$$

The invariance minor Δ_{21} shall be obtained by cancellation of the first column and the second line in (8):

$$\Delta_{21} = \begin{vmatrix} -C_{y\vartheta} & -C_{y\delta} \\ W_{AS} & \left(-K_{FB} - \frac{1}{K_{CA}}s\right) \end{vmatrix} = C_{y\vartheta}\left(K_{FB} + \frac{1}{K_{CA}}s\right) + W_{AS}C_{y\delta}. (9)$$

The open system transfer function shall be as follows under disturbing moment M:

$$\frac{\dot{y}(s)}{M(s)} = W_M = \frac{\Delta_{21}}{\Delta_{OS}}. (10)$$

Because Δ_{21} meeting the invariance conditions (7) is zero, and Δ_{21} is a minor of absolute invariance, so $W_M = 0$, and therefore $\dot{y} = W_M M = 0$. It is not possible to achieve a minor value Δ_{21} exactly equal to zero, that is the actual value obtained $\dot{y}$ can be only close to zero [26]. It is significant that if the equality $C_{y\delta}\delta + C_{y\vartheta}\vartheta = 0$, and therefore the equality $\vartheta = -\delta$, are fulfilled, there are two channels for transmission of the same disturbance M in the stabilization object itself, as can be seen from the block diagram (Fig. 2) [5] since $C_{y\delta} = C_{y\vartheta}$ achieves absolute invariance under M.

4.3. Physical Implementation of a Stabilizing System, which is Invariant under Disturbing Force

In order to ensure invariance of the coordinate $\dot{y}$ under influence F it is necessary that the condition $\Delta\dot{y}_F = 0$ is met, or

$$s^2\left(K_{FB} + \frac{1}{K_{CA}}s\right) + W_{AS}C_{\vartheta\delta} = 0. (11)$$

It is easy to see that the expression (11) is the same as the main determinant (8), if the system is open at $\dot{y}$. Consequently, when the condition of absolute invariance is met, the open system determinant becomes zero, indicating physical inability to implement absolute invariance of the system under influence F. The obtained results may be physically interpreted as follows: the system under consideration conforms to the principle of dual-channel impact transmission under disturbing moment M, while lacking such a characteristic under destabilizing force F [26].

Fig. 1 [5] shows two channels transferring an impact from the origin of the disturbing moment M to the controlled condition $\dot{y}$, while there is only one channel between the origin of destabilizing force F and the controlled condition $\dot{y}$. This explains the earlier conclusion about the physical inability to implement a system which would be invariant both for M, and F.

Therefore, an analysis of the possibility of an invariant stabilization system shows that such a system can be implemented only under one of the influences, i.e. disturbing moment [5], [26]. As this impact is a determining one, it is useful to consider a possibility to build such a system [9].

4.4. Synthesis of a Stabilization Invariant under Disturbing Moment

Practical building of invariant systems shows that it is generally not possible to implement absolute invariant conditions. Likewise, in the case under consideration [26], it is evident that it is not possible to achieve the condition

$$\Delta\dot{y}_M = C_{y\vartheta}\left(K_{FB} + \frac{1}{K_{CA}}s\right) + W_{AC}C_{y\delta} = 0\cdot$$

In such cases, it is usually a task to build a system partially invariant or invariant to the point of ε [27]. Having in mind that the object is subject to a slowly changing influence, we shall assume that disturbance is $M = const$ and try to build a simple invariant system. In this case, we shall only compensate for the disturbance itself $M = const$, without claiming compensation of its derivatives. Meeting these requirements means that a free member in the expression for $\Delta\dot{y}_M = 0$, or

$$C_{y\vartheta}K_{FB} + k_\vartheta C_{y\delta} = 0, \quad (12)$$

where k_ϑ is the gain of the stabilization controller according to spacecraft angle of deflection.

The relation (12) demonstrates that the feedback of the control actuator ensuring invariance shouldn't be negative as usually but a positive one (because the signs in the expression (12) correspond to the earlier assumption that a feedback sign should be negative) [26]. Its gain should be:

$$K_{FB} = -k_\vartheta \frac{C_{y\delta}}{C_{y\vartheta}}. \quad (13)$$

If $C_{y\delta} = C_{y\vartheta}$, direct signal gain and feedback gain must be equal, i.e. feedback coefficient for the control actuator shall be equal to $K_{FB} = -k_\vartheta$ [5], [26].

To analyze stability of such a system, we shall analyze its characteristic equation. In accordance with above, we shall therefore assume that there are no measurements $\dot{y}$, that is $W_{CM}\,\dot{y} = 0$. So, the characteristic equation for the closed system subjected to the invariance conditions will be:

$$s^2\left(-K_{FB} - \frac{1}{K_{CA}}s\right) - W_{AC}C_{y\delta} = 0. \quad (14)$$

Let W_{AC} have a known form: $W_{AC} = \dfrac{k_\vartheta + k_{\dot{\vartheta}}s}{W'_{AC}(s)}$, where $W'_{AC}(s)$ is a polynomial from s, characterizing delay of stabilization controller. Then the characteristic equation shall be as follows:

$$s^2\left(k_\vartheta - \frac{1}{K_{CA}}s\right) - \frac{(k_\vartheta + k_{\dot{\vartheta}}s)C_{\vartheta\delta}}{W'_{CA}(s)} = 0$$

or

$$\frac{W'_{CA}(s)}{K_{CA}}s^3 - k_\vartheta W'_{AC}(s)s^2 + k_{\dot{\vartheta}}C_{\vartheta\delta}s + k_\vartheta C_{\vartheta\delta} = 0 \quad (15)$$

The above relation (15) demonstrates that the characteristic equation of the closed system does not meet stability requirements as there appear members $k_\vartheta s^2 W'_{AC}(s)$, with negative components. To compensate them it is necessary [26]:

1. To enter second derivative action into the control mode $k_{\ddot{\vartheta}}s^2$.

2. To enter equivalent delay block $W'_{CA}(s)$, into the feedback loop of the control actuator. The equivalent delay block shall be written as a polynomial which denominator is minimum the order of the polynomial $W'_{AC}(s)$.

After the above requirements have been satisfied, the characteristic equation shall be as follows:

$$\frac{W'_{CA}(s)W_{AC}(s)}{K_{CA}}s^3 + \left(k_{\ddot{\vartheta}}C_{\vartheta\delta}W'_{CA}(s) - k_{\vartheta}W'_{CA}(s)\right)s^2 + k_{\dot{\vartheta}}C_{\vartheta\delta}W'_{CA}(s)s + k_{\vartheta}C_{\vartheta\delta}W'_{CA}(s) = 0 \quad (16)$$

The necessary condition for the system stability as follows from (16) is that $k_{\ddot{\vartheta}}C_{\vartheta\delta} > k_{\vartheta}$, and the coefficients at corresponding members in a polynomial $W'_{CA}(s)$ should not be less than the coefficients of the polynomial $W'_{AC}(s)$.

The required stability conditions obtained are consistent with the known criteria in the invariant theory. According to them, the response rate of the loop, which guarantees invariance (here the actuator loop), must be not less than the response rate of the main loop [9].

If the above condition is met providing the order of polynomial denominator in the loop of control actuator equivalent delay $W'_{CA}(s)$ equal to the order of polynomial $W'_{AC}(s)$, the invariant stabilization system shall be "rough" because execution of invariant conditions results in the degeneration of leading members in the characteristic equation of the system, and thus a sufficient degree of stability can be achieved by selection of parameters in the control mode (Polynomial Block)[26].

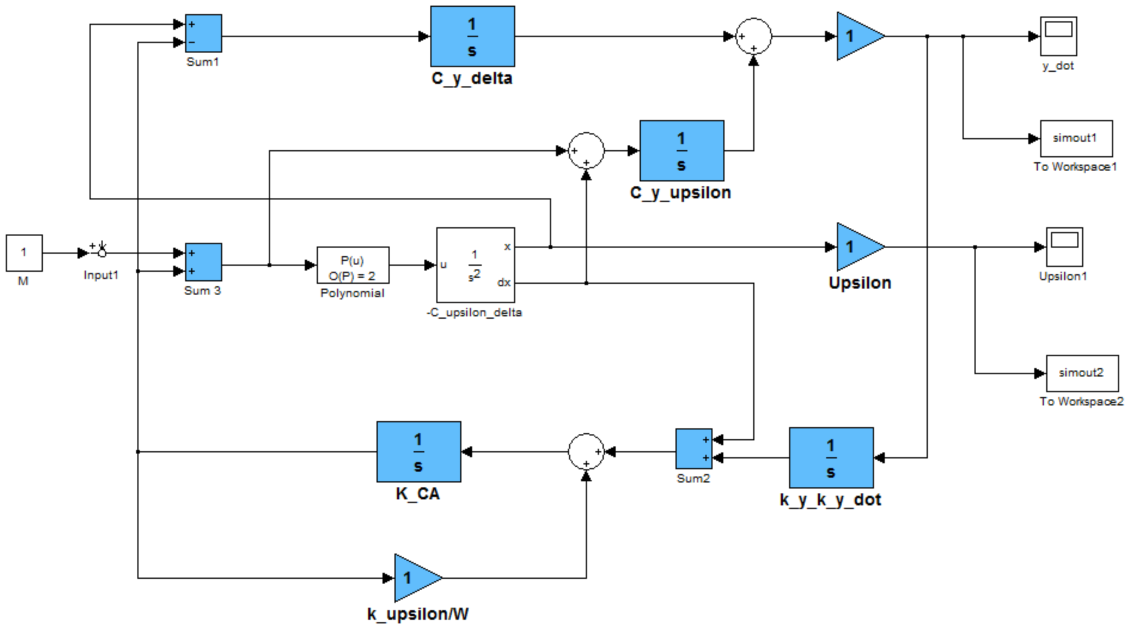

Fig. 4.Block diagram of the center of mass stabilization system invariant under disturbing moment

A block diagram of the center of mass stabilization system invariant under disturbing moment $M = const$ is shown in Fig. 4.

By analyzing the invariant stabilization system you can draw the following conclusions:

1. The stabilization system under consideration ensures invariance of the controlled variable $\dot{y}$ only under one influence, i.e. disturbing moment M. Since its influence has the greatest impact on the stabilization accuracy of the spacecraft center of mass, use of the proposed system plus normal center of mass stabilization channel which enables to introduce components proportional to velocity and the object drift coordinate into the control mode can significantly improve the control accuracy $\dot{y}$.
2. The peculiarities of the system considered include the need to introduce equivalent delay loop into feedback of the control actuator, which in its turn results in the complexity of the control actuator design and the need to use positive feedback.

4.5. Synthesis of a Stabilization System Partially Invariant under Destabilizing Force and Disturbing Moment

It is possible to synthesize an invariant stabilization system still other way. As mentioned earlier, absolute inequality to zero of the system's main determinant (8) is a necessary criterion for implementation of an open loop invariant system considering the absolute invariance conditions [17]:

$$\Delta_{ij} = \begin{vmatrix} a_{11} & ..a_{1,j-1} & a_{1,j+1} & ..a_{n1} \\ ... & ... & ... & ... \\ a_{i-1,1} & ..a_{i-1,j-1} & a_{i-1,j+1} & ..a_{i-1,n} \\ a_{i+1,1} & ..a_{i+1,j-1} & a_{i+1,j+1} & ..a_{i+1,n} \\ ... & ... & ... & ... \\ a_{n1} & ..a_{n,j-1} & a_{n,j+1} & ..a_{nn} \end{vmatrix} \equiv 0 \quad (17)$$

If this criterion is met, the invariant system can be synthesized. In practice, when such a system being synthesized, the absolute invariance conditions are only partially met with accuracy to the first few members of the absolute invariant expression (17) expanded into series. Therefore, such systems are invariant to the value of ε (partially invariant).

We used the same principle for research and synthesis in the previous paragraphs of this work: we checked the criterion for physical implementation of the invariant stabilization system described above under disturbing moment and destabilizing force. As a result, we revealed a possibility to synthesize an invariant system only under disturbing moment. In the synthesis process, the condition of absolute invariance (7) was replaced by a partial invariance condition (12). That is, the synthesis resulted in a partial invariant stabilization system under disturbing moment [26].

This work suggests a different approach to the synthesis of partially invariant systems, in which unlike the "traditional" approach described above, absolute invariance conditions are replaced with partial invariance conditions but considering physical implementation of the invariant system.

As noted above, disturbances influencing the spacecraft over the active phase are slowly changing, so it is necessary to fulfill the invariance conditions for drift velocity under disturbances themselves rather than under their derivatives [26]. Therefore, invariance conditions for the case under consideration can be obtained from the expressions (4) by accepting $s = 0$:

$$\begin{cases} k_{\dot{\vartheta}} C_{\vartheta\delta} = 0 \\ C_{y\delta} K_{FB} + k_{\dot{\vartheta}} C_{\vartheta\delta} = 0 \end{cases},$$

whence it follows that

$$K_{FB} = 0, k_{\dot{\vartheta}} = 0, \quad (18)$$

that is, the system must have no control over the angular deflection of the object, and the feedback of the control actuator must be open. It is easy to see that when the conditions (18) are met, the expression determining the open system (6) is not identically equal to zero, and therefore, a partially invariant system under disturbing moment and force can be implemented.

Suppose the partial invariance conditions have been obtained, the characteristic equation for the closed system shall be:

$$\frac{s}{K_{OD}} + W_{CS}(s)\frac{C_{\vartheta\delta}}{s^2} - W_{CM}(s)\frac{C_{y\vartheta}s^2 - C_{y\vartheta}C_{y\delta}}{s^3} = 0, \; W_{CS} = \frac{k_{\dot{\vartheta}}s}{W'_{AS}}, W_{CM} = \frac{k_{\dot{y}}}{W'_{AS}},$$

where W'_{AS} is a polynomial characterizing delay of stabilization controller.

By expanding the expressions for W_{CS} and W_{CS}, we shall get a characteristic equation of the system as follows:

$$\frac{W'_{AS}}{K_{OD}} s^4 + 0s^3 + \left(C_{\vartheta\delta}k_{\dot{\vartheta}} - C_{y\vartheta}k_{\dot{y}}\right)s^2 + 0s + C_{y\vartheta}C_{\vartheta\delta}k_{\dot{y}} = 0.$$

As follows from this characteristic equation, this stabilization system is unstable because there are zero coefficients in the 3rd and the 1st degrees S it is necessary to introduce second derivative actions from center of mass drift velocity into control action to ensure stability $k_{\ddot{y}}s$. In which case the characteristic equation takes the form:

$$\frac{W'_{AS}}{K_{OD}} s^4 + \left(C_{\vartheta\delta}k_{\ddot{\vartheta}} - C_{y\vartheta}k_{\ddot{y}}\right)s^3 + \left(C_{\vartheta\delta}k_{\dot{\vartheta}} - C_{y\vartheta}k_{\dot{y}}\right)s^2 + C_{y\vartheta}C_{\vartheta\delta}k_{\ddot{y}}s + C_{y\vartheta}C_{\vartheta\delta}k_{\dot{y}} = 0. \quad (19)$$

As follows from the equation (19), it is necessary that $C_{\vartheta\delta}k_{\ddot{\vartheta}} > C_{y\vartheta}k_{\ddot{y}}$ and $C_{\vartheta\delta}k_{\dot{\vartheta}} > C_{y\vartheta}k_{\dot{y}}$ to ensure stability of the system. As in the previous case (Paragraph *IV.4*), this system is "rough", because execution of invariance conditions $(K_{FB} = 0, k_{\vartheta} = 0)$ does not reduce the order of the characteristic equation, and therefore sufficient system stability can be achieved by selecting parameters in the control action [26].

It should be noted that the proposed center of mass stabilization system, which is close to an invariant one due to consistent disturbing influences M and F, has significant advantages over the previously synthesized system (in Paragraph *4.3*).

First, the system provides a simultaneous partial invariance of the controlled value under both disturbances: moment and force.

Second, in order the invariance conditions are met, it is necessary only to open feedback of the control actuator before the start of the active phase, while a positive feedback should be introduced for the system previously considered (Paragraph *4.4*). With some precision, it is necessary to ensure that the values K_{FB} and $k_{\vartheta} = 0$ coincide.

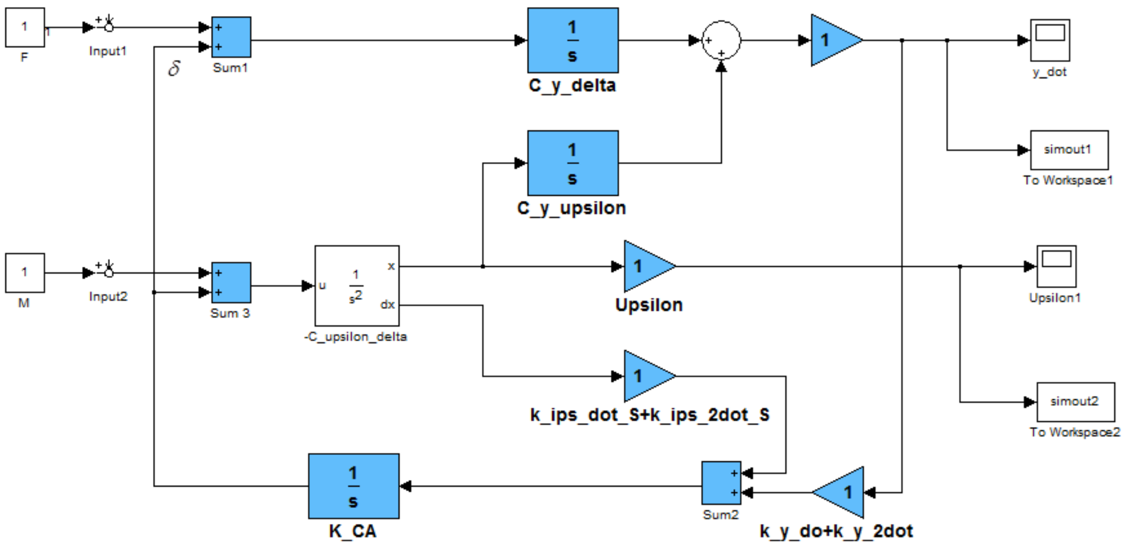

Fig. 5. Block diagram of a partially invariant center of mass stabilization system

Third, in order to provide stability and "roughness" of the previous system (Paragraph *4.4*) it is necessary to introduce an equivalent delay loop in the feedback of control actuator complementary to introduction of an additional second derivative action from the deflection angle $k_{\ddot{\vartheta}}s$, whereas the system under consideration does not require it.

Fourth, the proposed system does not require two distribution channels for two disturbances in the controlled object for its implementation.

In view of the benefits listed, we can assert that a stabilization system, which is close to an invariant one in moment and force, is more suitable for practical application. It should also be noted that, while the proposed stabilization system is not entirely invariant under influences M and F, yet as previously noted (Paragraph *4.2*) [5], [26], disturbing influences M and F in practice are slowly changing in time, and therefore application of a partial invariant stabilization system, as the author has shown above, has a significant effect in improving the accuracy of the stabilization of the center of mass tangential velocities.

Note: For the sake of brevity, we shall hereinafter imply a stabilization system, which is partially invariant under influencing disturbances under "invariant stabilization system".

As may be seen from the analysis of the characteristic equation of the system in question, the dynamic delay of the stabilization controller, in this case, unlike the system in Paragraph *IV.4*, does not have a fundamental impact on the structure and stability of the system, so the dynamic delay of the stabilization controller will not be taken into account hereinafter. Thus the block diagram for the stabilization system under consideration will be as follows (Fig. 5).

4.6. Additional Possibilities for Improving the Accuracy of Partially Invariant Stabilization Systems

As noted above ([5], [26], [28]), the accuracy of the center of mass stabilization system can be improved by using additional algorithms. The initial adjustment of the operating device (before the active phase) in the position, which was taken at the end of the previous active phase used in practice in a number of cases, is one of such algorithms.

Here, we use the fact that the main factor, i.e. disturbing moment determined by the thrust misalignment and the displacement of the spacecraft center of mass relative to the X- axis slowly changes over time, and preliminary adjustment of the operating device allows for immediate creation of a control moment partially compensating for this disturbance.

Mathematical modeling shows that the simultaneous use of invariant algorithms and the initial adjustment of the operating device give an increase in the accuracy of the drift velocity twice or more as compared to the model stabilization system using similar additional algorithms [5].

The additional algorithm specified has the following mathematical interpretation. Let the differential equation be used for a tangential velocity control error $\dot{y}(t)$

$$\left(a_0 s^n + a_1 s^{n-1} + \ldots + a_n\right)\dot{y}(t) = \left(b_0 s^m + b_1 s^{m-1} + \ldots + b_m\right) f(t) \quad (20)$$

where $f(t)$ is disturbing influence.

The equation solution (20) has two components: transitional $\dot{y}_t(t)$ and forced $\dot{y}_e(t)$, i.e. $\dot{y}(t) = \dot{y}_t(t) + \dot{y}_e(t)$ [15]. The forced component $\dot{y}_e(t)$ is an isolated solution to this equation, and due to the partial invariance of the system, the component will be close to zero for slowly changing disturbing influences $\left(f(t) \approx const\right)$ $\dot{y}_e(t) \to 0$.

The transitional component $\dot{y}_t(t)$ is a general solution of the equation (20) without the right side and is determined by the initial conditions of the transition process, so it is possible to reduce the transitional component to zero $\dot{y}_t(t) = 0$.

For this case, the values of the phase coordinate vector of the dynamic system at the time moment $t = 0$:

$$\overline{X}_0 = \left(\vartheta_0, \dot{\vartheta}_0, \delta_0, \dot{\delta}_0\right)^T$$

shall be initial conditions for transition process. In a typical system operation mode, if influence of the errors in spacecraft orientation and stabilization systems are neglected during the passive phase, all vector components $\overline{X}_0$ have zero values.

The final conditions of the transition process $\overline{X}_f$ can be determined based on the conditions of the steady mode $(s = 0)$. In this case the system (1) shall be as follows:

$$\begin{cases} -C_{y\vartheta}\vartheta - C_{y\delta}\delta = F \\ C_{\vartheta\delta}\delta = M \end{cases}$$

By expressing disturbances M and F through equivalent deviations of the steering control δ_M, δ_F $(M = C_{\vartheta\delta}\delta_M, F = C_{y\delta}\delta_F)$, and also taking into account the fact that $C_{y\vartheta} = C_{y\delta}$, for the spacecraft with rotating operating device, we shall write equations for the steady mode as follows:

$$\begin{cases} C_{y\vartheta}\vartheta(\vartheta + \delta + \delta_F) = 0 \\ -C_{\vartheta\delta}\delta + C_{\vartheta\delta}\delta_M = 0 \end{cases}$$

From the first equation, we obtain an expression for deflection of steering control δ_f in the steady mode:

$$\delta_f = \delta_M.$$

The angle value ϑ_f for the steady mode shall be obtained from the second system equation:

$$\vartheta_f = -\delta - \delta_F.$$

Because disturbing moment is the main disturbance to which a spacecraft is subject in the active phase, the value δ_f in the last expression can be ignored and we shall consider that $\vartheta_f \approx -\delta_M$.

Thus, the initial conditions vector of the transition process ensuring proximity of transition component to zero shall be as follows [26]:

$$\overline{X}_0 = \left(-\delta_M, 0, \delta_M, 0\right)^T. \quad (21)$$

Consequently, an additional algorithm for initial adjustment of the operating device based on the results of the previous adjustment is nothing more than the approximation of the component δ_0 of the initial conditions vector to the value of δ_f, which corresponds to the steady mode.

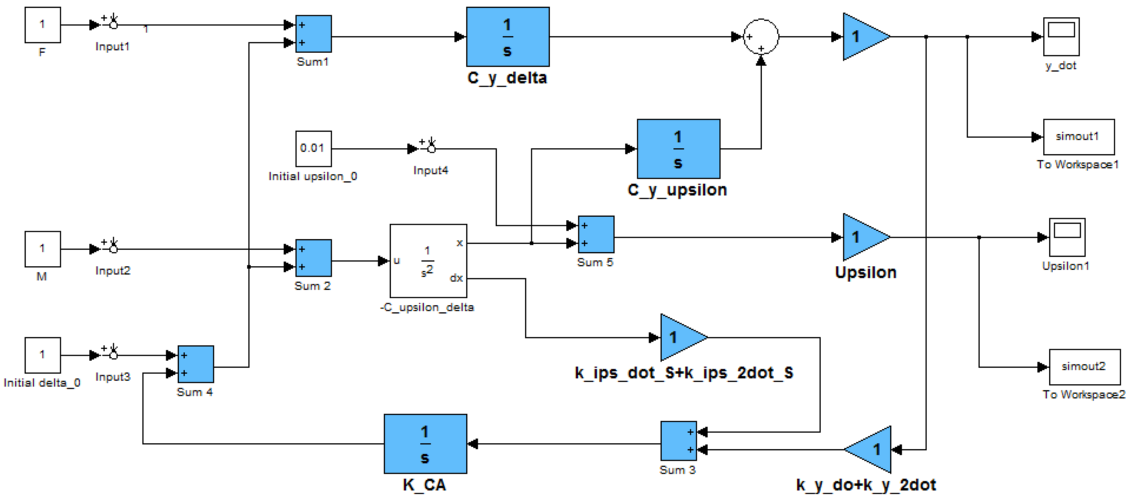

Fig. 6.Block diagram of a partially invariant center of mass stabilization system with self-configuration elements

Therefore, the observed additional algorithm contributes to reduction of the transitional component of the tangential velocity control process $\dot{y}(t)$. However, (21) demonstrates that in order start and end condition vectors could fully match, it is necessary that the spacecraft body deflection angle in the transverse plane be equal to the steering control deflection angle at the initial moment of time in the steady mode taken with the reverse sign. Accordingly, an additional algorithm with self-configuration elements should ensure that the operating device is adjusted (before the start of the active phase) in the position which was at the end of the previous active phase and must also rotate the spacecraft to an appropriate angle (Fig. 6-7) [29] to improve the stabilization accuracy in this case. Although the suggested self-configuration elements have the same drawbacks as those mentioned above (namely, initial adjustment errors, storage of a transient value of operating device deflection angle, complexity of the control system), still use of invariant algorithms is a great way to compensate for these drawbacks [26]. The transition process in the invariant stabilization system has significantly less decay time than in the model system, which significantly increases the probability for the achievement of the steady mode by the end of the active phase, and hence the probability that an operating device deviation value which is more close to the established one shall be stored. Because a forced component of the transition process is practically absent when invariant algorithms are used, tangential velocity control error shall depend, by contrast to the typical systems, only upon errors made during initial adjustment of operating device and the spacecraft body rather than upon disturbances, if the suggested self-configuration elements are used.

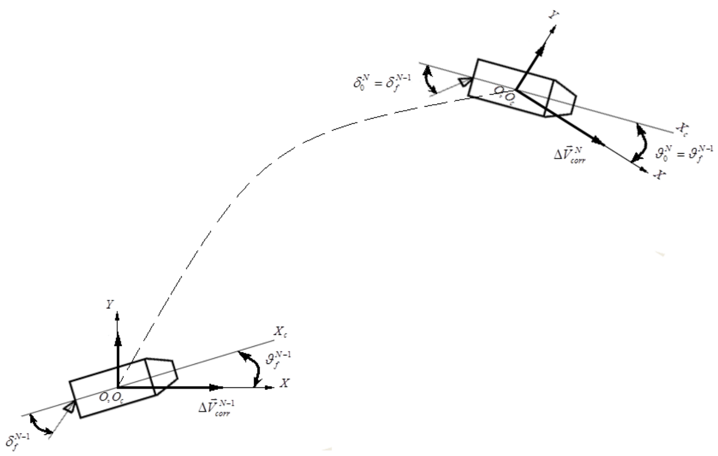

Fig. 7.Initial adjustment of the operating device and the spacecraft body following the previous adjustment

The mathematical modeling shows that simultaneous use of initial adjustment of operating device and the body of a spacecraft with invariant stabilization algorithms improves the accuracy of regulation approximately by one order as compared with a typical stabilization system using the same configuration elements [28, 29].

Transition process diagrams for the invariant and standard stabilization systems are shown in Fig. 8-9 and correspond to maximum value case m_F^H, m_M^H.

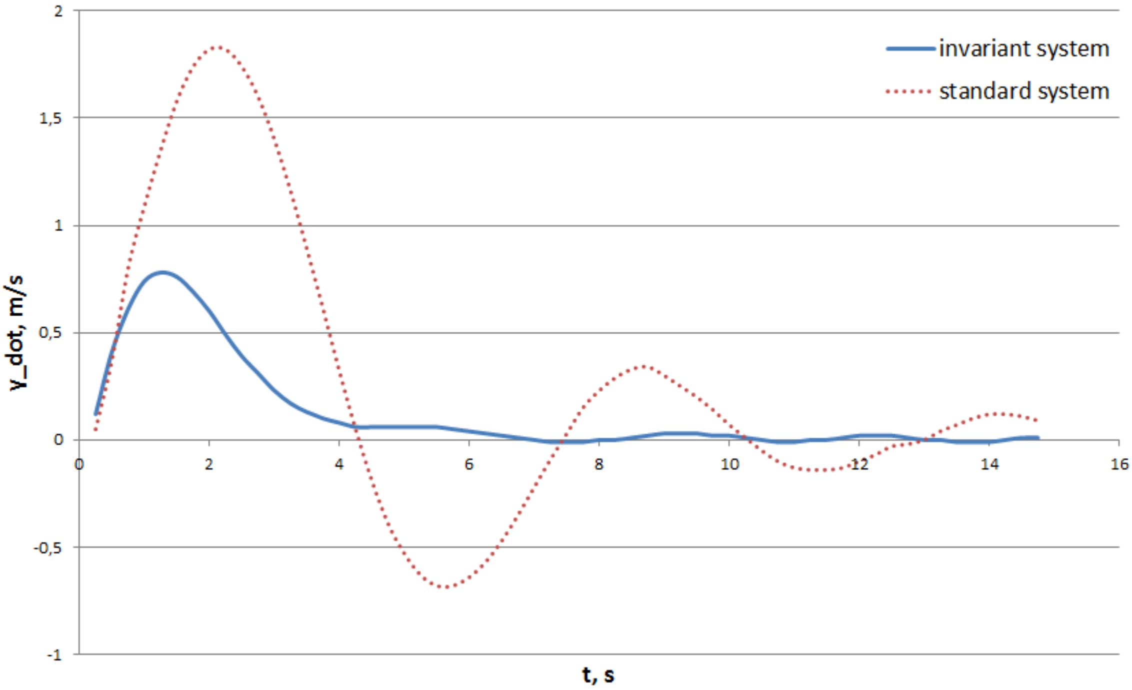

Fig. 8.Spacecraft drift velocity transition processes in the normal plane in the invariant and standard stabilization systems $(m_F^H = 0.3\deg; m_M^H = 3.5\deg)$

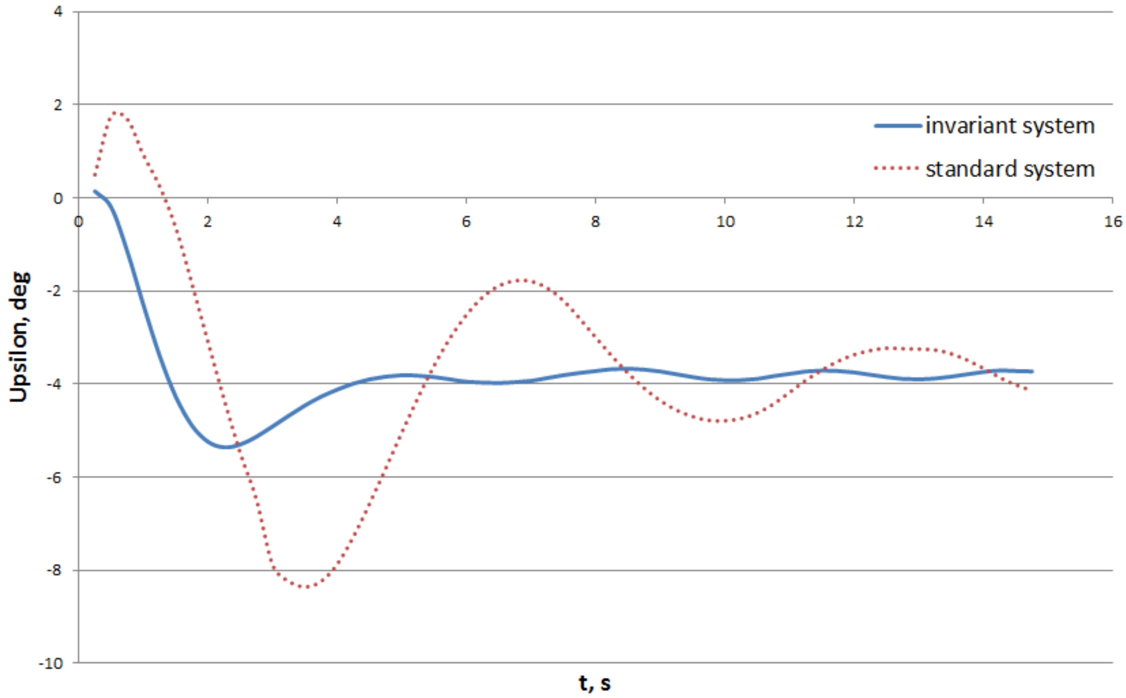

Fig. 9.Spacecraft X-axis angular deviation transition processes in the normal plane in the invariant and standard stabilization systems $(m_F^H = 0.3\deg; m_M^H = 3.5\deg)$

As the mathematical modeling shows (Fig. 8-9), application of the invariant algorithm in this case improves the accuracy of center of mass roll stabilization twice or three times. Transition processes in the invariant stabilization system have significantly less attenuation time than in the standard system. Random disturbances caused by fluctuating PS operating conditions during normal operation, as well as random AVS measurement errors in have no significant impact on the stabilization accuracy.

V. CONCLUSION

1. In practice, a partially invariant under disturbing moment stabilization system is the easiest one to implement. In order the invariance conditions could be met, there must be a positive feedback of the control actuator with a gain which is equal to the gain according to angular deflection of the object in the angular stabilization channel. Stability of the system is ensured by introduction of an additional second derivative action from the object's deflection angle into the action as well as by introduction of an equivalent delay loop in the feedback of control actuator in order to compensate for the dynamic delay of the stabilization controller.
2. It is also possible to synthesize a stabilization system, which shall be partially invariant under two disturbances simultaneously, i.e. moment and the force. Open feedback of the control actuator and exclusion of control according to object's deflection angle from the angular stabilization channel are the invariance conditions in this case Stability is achieved through the introduction of second derivative actions from object's deflection angle and the center of mass drift coordinate into the control action. Such a stabilization system has obvious advantages over a system, which is invariant under disturbing moment, and therefore more suitable for practical implementation.
3. A partially invariant fewer than two disturbances (moment and force) stabilization system provides a significant increase in the accuracy of the center of masstangential stabilization velocities as compared to known stabilization systems.

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