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Numerical Simulation for the Folded Core Sandwich Plates under Torsion by Homogenization Method

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ABSTRACT:*In this paper, an analytic homogenization model for the torsion problem of folded core sandwich plates is presented. It is very difficult to determine the torsion rigidity of these kinds of 3D structures even numerically because of the boundary condition effects. Based on the gridwork homogenization model of Timoshenko, plate torsion is divided into two beam torsions. The plate torsion curvature is separated into two beam torsion rates and the beam torsion rigidities in these two directions are introduced to describe the torsion behavior of the orthotropic plates. The proposed analytical homogenization model allows replacing the 3D folded core sandwich by a 2D homogenized plate. This model is validated by comparing the results of Abaqus-3D and H-2D homogenization model, confirming the accuracy and effectiveness of the proposed model. This torsion homogenization model can be used not only for corrugated cardboard packaging structures, but also for naval and aeronautic composite structures.*

Keywords: *Analytic homogenization, torsion rigidity, orthotropic sandwich plates, folded core*

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I. INTRODUCTION

Sandwich structures have been used over a long time applications where the weight of the member is critical, such as packaging, civil, naval, automotive and aerospace industries due to their low mass to stiffness ratio and high impact absorption capacity [1]. As designers in the transportation industry strive to reduce fuel consumption and improve safety, composite sandwich structures that provide improved stiffness-to-weight ratio, are becoming an attractive alternative to metals for mass transport applications. A reduction in structural weight of one large component usually triggers positive synergistic effects for other parts of the vehicle. Therefore, using composite sandwich structures not only reduces weight, thereby improving fuel economy and increasing payload capacity, but also enables the design of aerodynamic, stable vehicles with a low center of gravity [2]. Some instances of their applications in daily life are corrugated cardboard panels used for packaging, folded core sandwiches used as structural floor and roof panels, metal corrugated roofs, hulks, automotive chassis and bumpers. In nature, where mechanical design required to be optimized, sandwich structures are used such as the human skull, which is made up of two layers of dense compact bone separated by a “core” of lower density material [3].

Folded core sandwich is one of the materials most used to make partitions or roofs in construction or automobile. The manufacturing process gives three characteristic directions: the machine direction (MD), the cross direction (CD), and the thickness direction (ZD). Folded core sandwich plate are produced by a converting process in which three or more layers are laminated together. The flat layers are called liners and the folded cores are referred to as flutes (Fig. 1). The numerical modelling of this kind of three-dimensional (3D) structures is too tedious and CPU time consuming, it will be more efficient to use 2D homogenized equivalent plates. Analytic homogenization is an efficient and accurate method, but it is often limited to simple cases. The numerical homogenization approach is commonly used for calculating effective rigidities, but this tedious procedure should be redone for every new section structure and the boundary conditions on a representative volume element are often difficult to define [4]. There are many homogenization models obtained by analytical methods [4-6], numerical methods [8, 9] and experimental methods [6, 7]. Many studies have done for tensile, bending, in-plane and out-of-plane shear problems, but research on torsion remains a major challenge.

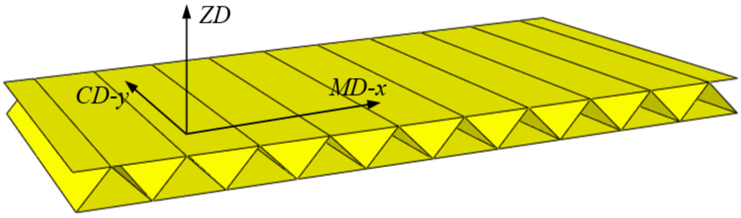

Fig. 1. The model of folded core sandwich plate

Timoshenko and Woinowski-Krieger [10] have proposed an expression for bending and torsion rigidities for various specific cases, such as reinforced concrete slabs, plywood, corrugated sheets and slabs, plywood, corrugated sheets, and plates reinforced by stiffeners. Ugural [11] has also come up with some similar formulas for the rigidities of several plates. An interesting application found in the book of Timoshenko and Woinowski-Krieger [10] is the torsion model for a gridwork system that allows leading the equivalent homogeneous plate. Timoshenko and Woinowski-Krieger proposed separating the plate torsion problem into two beam-like torsion problems. A torsion test and an analytical formula for measuring and calculating the torsional rigidity of a corrugated cardboard were proposed by Pommier and Poustis [12]; however, Carlson et al. [6] has shown that the torsional rigidity is highly dependent on the size of the cardboard, in that a larger sheet is more rigid than that given by the formula. In other studies, the torsion stiffness was calculated by integrating through the sheet thickness according to the theory of laminate plates. However, it should be noted that this theory is only valid for continuous thickness, for sandwich panels with many hollow cavities such as folded core plates, it should be considered them as intermittent 3D structures, and therefore theory of laminate plates should be adjusted [5].

In this paper, based on the theory of anisotropic plates of Timoshenko and Woinowski-Krieger and their homogenization model on gridwork system [10],the plate torsion is decomposed into two beam torsions in two directions: the plate torsion curvature is separated into two orthogonal beam torsion rates and the beam torsion stiffnesses in both directions are used to describe the torsion behavior of the orthotropic plates. These beam torsion rigidities can be easily determined analytically by beam theory or by numerical methods using finite elements. The results obtained by the H-2D homogenization model are compared with the results obtained by the 3D Abaqus model showing the accuracy and effectiveness of the proposed model.

II. TORSION THEORY OF ORTHOTROPIC PLATES AND ANALYTIC HOMOGENIZATION MODEL

In the classic theories of plate, the law of torsion behavior is written as follows:

$$-M_{xy} = M_{yx} = D_{33}\kappa_{xy} \tag{1}$$

whereD_{33} is the torsional stiffness per unit width of the plate, $\kappa_{xy}$is the torsion curvature:

$$\kappa_{xy} = -2w,_{xy} \tag{2}$$

$$\kappa_{xy} = \beta_x,_y + \beta_y,_x \tag{3}$$

where$w$ is the transverse displacement of a point A on the mid-surface of the plate, β_x is the angle of rotation from z to x of the normal of the plate at A, β_y is the angle of rotation from$z$to y of the normal [13]. It should be noted that we use (2) for thin plates according to theory of Kirchhoff, and for thick plates, we use (3) by theory of Mindlin. According to these definitions, we have the following relations:

$$\beta_x = \theta_y \quad ; \quad \beta_y = -\theta_x \tag{4}$$

whereθ_x and θ_y are the angles of rotation around x and y respectively.

The classic theories of plate return to assuming a single torsion curvature ($-2w,_{xy}$ or $\beta_x,_y+\beta_y,_x$) and a single torsion stiffness (D_{33}) for torsion moments ($-M_{xy}=M_{yx}$) on the sections in x and y.

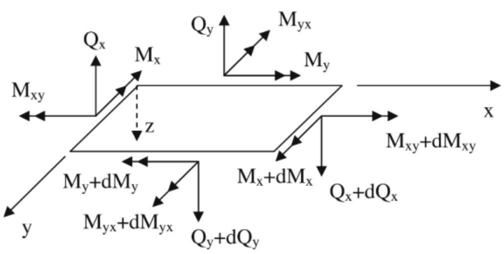

Fig. 2. The internal forces and moments in an elementary surface of a plate

Considering a plate subjected to a vertical distributed load q in the coordinate system in Fig. 2, we have the differential equation of equilibrium [10]:

$$M_{x},_{xx} + M_{yx},_{xy} + M_{y},_{yy} - M_{xy},_{xy} = -q \tag{5}$$

For an orthotropic plate, the three in-plane stresses and the corresponding bending-torsion moments are:

$$\begin{aligned} \sigma_x &= E_x^{'}\varepsilon_x + E^{''}\varepsilon_y = -z(E_x^{'}w,_{xx} + E^{''}w,_{yy}) \\ \sigma_y &= E_y^{'}\varepsilon_y + E^{''}\varepsilon_x = -z(E_y^{'}w,_{yy} + E^{''}w,_{xx}) \\ \tau_{xy} &= G\gamma_{xy} = -2Gzw,_{xy} \end{aligned} \tag{6}$$

with

$$E_x^{'} = \frac{E_x}{1-\nu_{xy}\nu_{yx}}; \quad E_y^{'} = \frac{E_y}{1-\nu_{xy}\nu_{yx}}; \quad E^{''} = \frac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} \quad ; \quad E_x\nu_{yx} = E_y\nu_{xy}$$

$$\begin{aligned} M_x &= \int_{-h/2}^{h/2} \sigma_x z dz = -D_x w,_{xx} - D_1 w,_{yy} \\ M_y &= \int_{-h/2}^{h/2} \sigma_y z dz = -D_y w,_{yy} - D_1 w,_{xx} \\ M_{xy} &= -\int_{-h/2}^{h/2} \tau_{xy} z dz = 2D_{xy} w,_{xy} \end{aligned} \tag{7}$$

with $D_x = \dfrac{E_x^{'}h^3}{12}$; $D_y = \dfrac{E_y^{'}h^3}{12}$; $D_1 = \dfrac{E^{''}h^3}{12}$; $D_{xy} = \dfrac{Gh^3}{12}$

where the Young modulus E_x and E_y, the shear modulus G, and the Poisson's coefficient ν_{xy} must be determined experimentally.

Substituting equation (7) into the differential equation of equilibrium (5), we obtain the following equation for an orthotropic plate:

$$D_x w,_{xxxx} + 2(D_1 + 2D_{xy})w,_{xxyy} + D_y w,_{yyyy} = q \tag{8}$$

where $2D_1$ represents the Poisson effect of the bending on the torsion curvature and D_{xy} is the torsion stiffness of the considered plate.

We note that the distributions of strains and stresses are not necessarily linear through the thickness in the case of sandwich plates with cavities such as folded core sandwich plate. Therefore, we cannot use the integration in equation (7) to calculate stiffnesses as in the case of laminated plates.

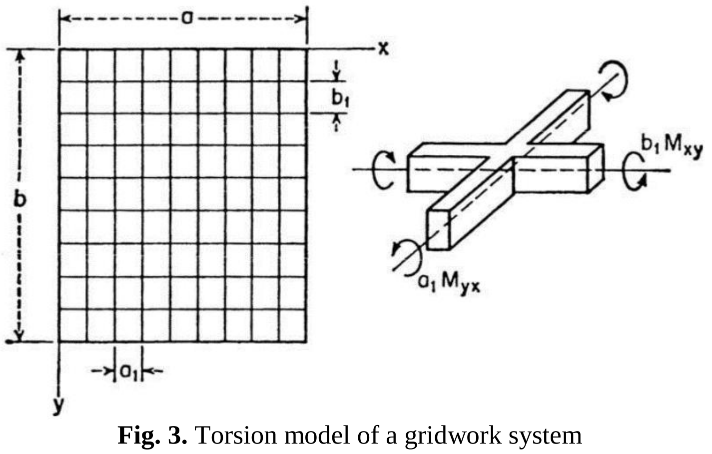

Fig. 3. Torsion model of a gridwork system

Timoshenko et al. [10] proposed a homogenization model for the torsion of a gridwork system in order to take into account the different torsion stiffnesses according to x and y (Fig. 3). The torsion stiffness of each beam was calculated according to the classical beam theory and then distributed uniformly over the intervals (a_1 or b_1), respectively:

$$M_{xy} = \frac{Gj_x}{b_1}\theta_x,_x = \frac{Gj_x}{b_1}w,_{yx}$$
$$M_{yx} = \frac{Gj_y}{a_1}\theta_y,_y = -\frac{Gj_y}{a_1}w,_{xy} \qquad (9)$$
$$w,_{yx} = w,_{xy}$$

where Gj_x is the torsion stiffness of each beam along x, Gj_y is the torsion stiffness of each beam along y, $\theta_x,_x$and $\theta_y,_y$, represent the torsion rates around x and y, -2$w,_{xy}$is the torsion curvature, $M_{xy}$and M_{yx} are the torsion moments per unit length, b_1 and a_1 are the intervals between two beams along x and y respectively.

It should be noted that: 1) beam torsion rates in both directions are identical ($\theta_x,_x$=$w,_{xy}$;$\theta_y,_y$=-$w,_{yx}$) and equal to one half of torsion curvature (-2$w,_{xy}$)of the equivalent plate; 2) the torsion stiffnesses of beams in two directions can be very different, but their coupling gives a unique torsion stiffness of the equivalent homogenized plate.

Substituting equation (9) into the differential equation of equilibrium (5), we obtain the following equation [10]:

$$\frac{E_x i_x}{b_1}w,_{xxxx} + \left(\frac{Gj_x}{b_1} + \frac{Gj_y}{a_1}\right)w,_{xxyy} + \frac{E_y i_y}{a_1}w,_{yyyy} = q \qquad (10)$$

where$E_x{\cdot}i_x$ is the bending rigidity of each beam along x, $E_y{\cdot}i_y$ is the one along y.

If we compare equation (10) with equation (8) (with D_1=0), the torsion stiffness of the equivalent homogenized plate can be obtained:

$$D_{xy} = \frac{1}{4}\left(\frac{Gj_x}{b_1} + \frac{Gj_y}{a_1}\right) = \frac{1}{4}\left(\frac{GJ_x}{b} + \frac{GJ_y}{a}\right) \qquad (11)$$

whereGJ_x and GJ_y are the total torsion rigidities of all beams along x and along y respectively.

We can deduce important conclusions: 1) Torsion stiffnesses in both directions (as beams without coupling) are different, which leads to two different internal torsion moments. 2) In the homogenized plate, these two torsion rigidities are coupled and give the only torsion rigidity: $-M_{xy} = M_{yx} = D_{33}\kappa_{xy}$. 3) The torsion stiffness of the plate is not a simple sum of the rigidities of two beam torsions, a factor of ¼ originated from the coupling effect. By using these conclusions, the complex calculation of the torsion stiffness of folded core sandwich plates becomes very simple.

When we impose a torsion angle θ_x (around x) on the gridwork (length L = a along x, width B = b along y), we obtain a torsion rate $\theta_x,_x$=$w,_{yx}$=θ_x/L around x and also an equal torsion rate $\theta_y,_y$=-$w,_{xy}$=-θ_x/L around y. Using the Clapeyron theorem [15], the work of the external moment is equal to the internal energy of deformation:

$$M_t\theta_x = \int_0^L Gj_x\left(\theta_x,_x\right)^2 dx + \int_0^B Gj_y\left(\theta_y,_y\right)^2 dy = \int_A \left(\frac{GJ_x}{B} + \frac{GJ_y}{L}\right)\left(w,_{xy}\right)^2 dA \qquad (12)$$

Inthe equivalent homogenized plate, the equation of energy can be written as follows:

$$M_t\theta_x = \int_A D_{xy}\kappa_{xy}^2\ dA = \int_A D_{xy}\left(-2w,_{xy}\right)^2 dA \qquad (13)$$

where (-2$w,_{xy}$) = κ_{xy} is the plate torsion curvature, which can be divided into two equal beam torsion rates ($w,_{yx}$=$\theta_x,_x$and$w,_{xy}$=-$\theta_y,_y$). By comparing equations (12) and (13), the same formula is obtained for calculating the torsion stiffness of the equivalent plate:

$$D_{xy} = \frac{1}{4}\left(\frac{GJ_x}{B} + \frac{GJ_y}{L}\right) \qquad (14)$$

We note that in the case of Mindlin's theory for thick plates, the torsion curvature in equation (13) is replaced by equation (3).

III. CACULATION OF THE TORSION RIGIDITIES FOR FOLDED CORE SANDWICH LATES

For folded core sandwich plates, numerical simulations with beam elements showed that it has two very different torsion rigidities on the *MD* and *CD* sections. For the *CD* section of folded core sandwich, the torsion around $y$is considered as the torsion of a beam having a closed thin-walled section composed of several cells. The finite element calculations showed that the *MD* section was an opened section having a very small beam torsion rigidity. Finally, an analytical solution was obtained for this very complicated torsion problem by neglecting the torsional stiffness *MD* [4]:

$$D_{xy} = \frac{1}{4}\left(\frac{GJ_{MD}}{B} + \frac{GJ_{CD}}{L}\right) \approx \frac{GJ_{CD}}{4L} \tag{15}$$

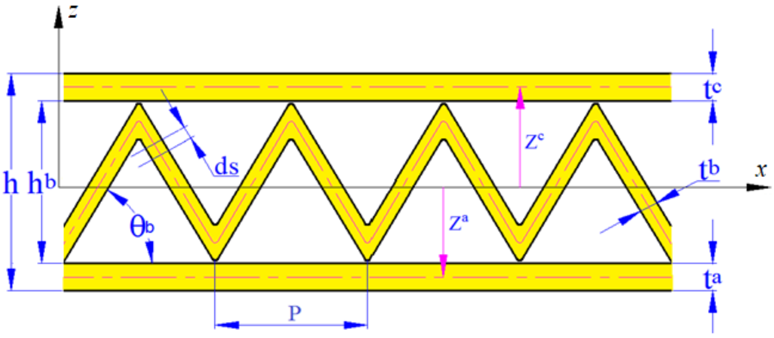

Fig. 4. Geometry of *CD* section of folded core sandwich plate

Based on the Bredt torsion theory for the structures having a thin wall section with several closed cells [16] and the finite element calculations, it has been shown that the flow of the shear stress is very low in the internal walls when the folded core sandwich is very wide (for example, 50 periods). Thus the section *CD* can be considered as a section with a single closed cell and we have an analytical formula for the torsion rigidity of the homogeneous plate [4]:

$$D_{xy} \approx \frac{GJ_{CD}}{4L} = \frac{4S^2}{4L\oint \frac{ds}{Gt}} = \frac{Lh^2}{\frac{L}{G^a t^a} + \frac{L}{G^c t^c} + \frac{2l}{G^b t^b}} \approx \frac{h^2 G^a t^a G^c t^c}{G^a t^a + G^c t^c} \quad \left(l \ll L\right) \tag{16}$$

where the superscripts *a*, *b*, and *c* correspond to the lower layer, the corrugated core layer and the upper layer respectively; *G* and *t* are shear modulus and thickness of layers respectively; *L* and *l* are the length of the plate and the length of one-half of a period of folded core (Fig. 4).

The length of one-half of a period of folded corecan be defined as following:

$$l = \sqrt{\left(h^b\right)^2 + \frac{P^2}{4}} \tag{17}$$

IV. NUMERICAL VALIDATION OF HOMOGENIZATION MODEL

To validate the proposed homogenization model (H-2D Model), we used a folded core sandwich plate of length L = 320 mm, width B = 280 mm, and *CD* section as shown in Fig. 4 (with$h$ = 4 mm, $t^a = t^c$ = 0.20 mm, t^b = 0.15 mmand$P$ = 8 mm). The material properties of each layer are given in Table 1. Hence, it can be easily calculated the torsion rigidity of the plate according to equation (16) as D_{33} = 788.96 Nmm. First, we discretize the three layers of the folded core with the S4R shell elements in Abaqus to obtain the Abaqus-3D model; then discretize the mid-surface of the folded core sandwich plate by the S4R shell elements associated with homogenization model (using the "user's subroutine UGENS" [17]) to obtain the H-2D Model. The comparison of the results allows demonstrating the effectiveness and accuracy of the proposed homogenization model.

Table 1. Parameters of the layers forming the folded core sandwich

	E_{11}(MPa)	E_{22}(MPa)	G_{12}(MPa)	ν_{12}
Layer a	2372.6	704.2	493.1	0.377
Layer b	1094.7	856.4	165.9	0.421
Layer c	2372.6	704.2	493.1	0.377

In two types of simulation (Abaqus-3D and H-2D Model), a rigid plate is attached to one side of the folded core sandwich to better apply torque. Calculations by the H-2D model are very fast, while the Abaqus-3D calculations take a lot of time. The comparison of the results obtained by the two models as well as the percentage error of these results is presented in Table 2. For torsion in *MD* and *CD* section, we find that Abaqus-3D simulation uses more than 11 times of the CPU time comparing to the H-2D model. The numerical results given by the two models are nearly identical. The torsion results in *MD* and *CD* sections are slightly different due to the boundary effects. Indeed, if we increase the size of the plate several times, the results will give the smaller error.

Table 2. Comparison between Abaqus-3D and H-2D Model for torsion in MD and CD section

M=1000 Nmm	MD Torsion		CD Torsion	
Torsion rigidity D_{33}=788.96 Nmm	Torsion angle θ_1 (rad)	CPU Time (s)	Torsion angle θ_2 (rad)	CPU Time (s)
Abaqus-3D	0.17379	52.8	0.1656	53.7
H-2D Model	0.17368	4.8	0.1623	4.9
Error (%)	0.06	11 times	1.97	11 times

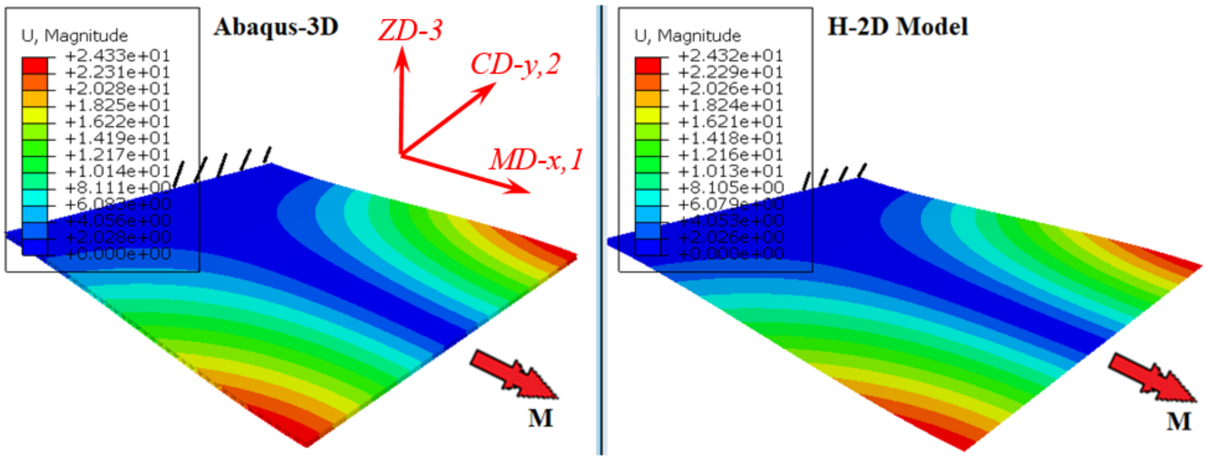

Fig. 5. Displacement and deformation of the folded core sandwich under torsion in MD section

Thedeformation shape and displacement values of the folded core sandwich plate obtained by the Abaqus-3D simulations and the H-2D homogenization model are shown in Fig. 5. We see that the Abaqus-3D model give the results are very close to the H-2D homogenization model. The comparison shows that the H-2D model proposed for folded core composite panels is quite accurate and effective.

V. CONCLUSION

In this paper, an analytical homogeneous model for folded core sandwich plates in torsion has been proposed. The comparison of the results obtained by the Abaqus-3D numerical simulations and the Abaqus-Ugens 2D model has demonstrated the accuracy and effectiveness of the proposed homogenization model for the folded core sandwich plates under torsion. The homogenization model allows to significantly reduce in the time required for building a geometry model, the finite element modeling time as well as the computational time for the folded core sandwich plates. This model can be easily applied to complex composite panels made of different materials, depending on the purpose of use in areas such as packaging, construction, navy, and aerospace.

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REFERENCES

[1] Iman Dayyani, Saeed Ziaei-Rad and Michael I Friswell, The mechanical behavior of composite corrugated core coated with elastomer for morphing skins, *Journal of Composite Materials*, 48, 2014, 1623–1636.
[2] A. Zinno, A. Prota, E. Di Maio, C.E. Bakis, Experimental characterization of phenolic-impregnated honeycomb sandwich structures for transportation vehicles, *Composite Structures*, 93, 2011, 2910–2924.
[3] Iman Dayyani, Saeed Ziaei-Rad, Hamid Salehi, Numerical and experimental investigations on mechanical behavior of composite corrugated core, *Applied Composite Materials*, 19, 2012, 705–721.
[4] B. Abbès, Y.Q. Guo, Analytic homogenization for torsion of orthotropic sandwich plates: Application to corrugated cardboard, *Composite Structures*, 92, 2010, 699–706.
[5] P.T.M. Duong, B. Abbès, Y.M. Li, A.D. Hammou, M. Makhlouf, Y.Q. Guo, An analytic homogenization model for shear-torsion coupling problems of double corrugated-core sandwich plates, *Journal of Composite Materials*, 47, 2013, 1327-1341.
[6] Carlsson LA, Nordstrand T, Westerlind B., On the elastic stiffness of corrugated core sandwich plate, *Journal Sandwich Structure Materials*, 3, 2001, 253–267.
[7] Aboura Z., Talbi N., Allaoui S., Benzeggagh ML., Elastic behaviour of corrugated cardboard: experiments and modelling, *Composite Structures*, 63, 2004, 53–62.

[8] Buannic N, Cartraud P, Quesnel T, Homogenization of corrugated core sandwich panels, *Composite Structures*, 59, 2003, 299–312.
[9] Biancolini M.E., Evaluation of equivalent stiffness properties of corrugated board, *Composite Structures*, 69, 2005, 322–328.
[10] Timoshenko S.P., Woinowski-Krieger S., *Theory of plates and shells* (2nd edition, McGraw-Hill International Editions, 1959).
[11] Urugal A.C.,*Stresses in plates and shells*(McGraw-Hill Book Company, 1981).
[12] Pommier J.C, Poustis J, Box compression strength, today McKee and tomorrow? *Tappi Journal*, 41(8), 1987,399.
[13] Berthelot J.M., *Matériaux composites - Comportementmécaniqueetanalyse des structures*(Deuxièmeédition Masson, 1996, 620 pages).
[14] Nordstrand T., Carlsson L.A., Allen H.G., Transverse shear stiffness of structural core sandwich, *Composite Structures*, 27,1994, 317-329.
[15] Love A.E.H., *A Treatise on the Mathematical Theory of Elasticity*(4th revised edition, Cambridge University Press, Cambridge, 1927).
[16] Rivello RM., *Theory and analysis of flight structures*(NewYork, NY: McGraw-Hill, 1969).
[17] ABAQUS User's Manual, Version 6.9, Simulia 2009.

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