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Hamilton Mechanical Equations with Four Almost Complex Structures on Riemannian Geometry

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Abstract: *The study concerns the Hamilton Mechanical Equations with Four Almost Complex Structures on Riemannian geometry. The four complex structures have been derived also important applications of Hamiltonian mechanical systems of the notions of Riemannian mentioned. Finally achieved that Almost complex structures have this systems in Mechanics and Physical Fields as well as in differential geometry.*

Key words*; differential geometry , almost complex structure, Hamiltonian Dynamics..*

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I. INTRODUCTION

Differential Geometry is studying Hamiltonian equations of Classical Mechanics. To show this, it is possible to find many articles in[4,5,6,7] and books [1,2.3] in the relevant fields.

If $H: T^*\mathcal{M} \longrightarrow R$ is a regular Hamiltonian function then there is a unique vector field Z_H on cotangent bundle $T^*\mathcal{M}$ such that dynamical equations

$$i_{Z_H}\phi = dH \tag{1}$$

where ϕ is the symplectic form and H stands for Hamiltonian function. The paths of the Hamiltonian vector field Z_H are the solutions of the Hamiltonian equations shown by

$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \quad , \quad \frac{dp_i}{dt} = \frac{\partial H}{\partial q^i} \tag{2}$$

where q^i and $(q^i, p_i), 1 \leq i \leq m$, are coordinates of $\mathcal{M}$ and $T^*\mathcal{M}$. The triple $(T^*\mathcal{M}, \phi, H)$, is called Hamiltonian system on the cotangent bundle $T^*\mathcal{M}$ with symplectic form ϕ. Let $T^*\mathcal{M}$ be symplectic manifold with closed symplectic form ϕ.

In this paper, we study dynamical systems with Three Almost Complex Structures . After Introduction in Section 1, we consider Historical Background paper basic . Section 2 deals with the study Almost Complex Structures. Section 3 is devoted to study Lagrangian Dynamics .Section 4 is devoted to study Hamiltonian Dynamics .

II. RIEMANNIAN MANIFOLDS

Definition Pseudo-Riemannian and Riemannian metric 2.1

1- A pseudo-Riemannian metric on a manifold $\mathcal{M}^\ell$ is a symmetric and nondegenerate covariant tensor field $\widehat{G}^{\otimes *}$ of second order. A manifold with
a pseudo-Riemannian metric is called a pseudo-Riemannian manifold.

2- A pseudo-Riemannian metric $\widehat{G}^{\otimes *}$ on $\mathcal{M}^\ell$ is said to be positive definite
if for all vector fields $\widehat{X}$ on $\mathcal{M}^\ell$ we have:

(1) $\widehat{G}^{\otimes *}_m(\widehat{X}_m, \dots, \widehat{X}_m) \geq 0 \quad \forall\, m \in \mathcal{M}^\ell$

(2) $\widehat{G}^{\otimes *}_m(\widehat{X}_m, \dots, \widehat{X}_m) = 0$ if and only $\widehat{X}_m = 0 \quad \forall\, m \in \mathcal{M}^\ell$

3 -A pseudo definite pseudo-Riemannian metric on a manifold is called a
Riemannian metric and a manifold with a Riemannian metric is called
a Riemannian manifold.[12]

Definition 2.2[13] The 2-form ω is Symplectic if ω is closed and ω_p is Symplectic for all $p \in M$.
If ω is Symplectic , then dim $T_p M = \dim M$ must be even.
Definition 2.3[14] A Symplectic manifold is a pair (M, ω) where M is a manifold and ω is a Symplectic form.
Definition 2.4 A symplectic manifold is a pair $(\mathcal{M}, \sigma)$such that $\mathcal{M}$ is a smooth manifold and σ is a closed non-degenerate differential 2-form on $\mathcal{M}$. This means that in each tangent space $T_p\mathcal{M}$, σ gives a non degenerate, bilinear, skew symmetric form $\sigma_p : T_p\mathcal{M} \times T_p\mathcal{M} \longrightarrow \mathbb{R}$ such that σ_p varies smoothly in p.

III. ALMOST COMPLEX STRUCTURES

Definition 3.1[6]

Let M be a smooth manifold. An almost complex structure J on M is a linear complex structure (that is, a linear map which squares to −1) on each tangent space of the manifold, which varies smoothly on the manifold. In other words, we have a smooth tensor field J of degree (1,1) such that $J^2 = -1$ when regarded as a vector bundle isomorphism $J : T\mathcal{M} \to T\mathcal{M}$ on the tangent bundle. A manifold equipped with an almost complex structure is called an almost complex manifold.

Integrable almost complex structures

Definition 3.2[8]

Every complex manifold is itself an almost complex manifold. In local holomorphic coordinates $Z = x_k + iy_k$ one can define the maps

$$J\left(\frac{\partial}{\partial x_k}\right) = \frac{\partial}{\partial y_k} \qquad , \quad J\left(\frac{\partial}{\partial y_k}\right) = -\frac{\partial}{\partial x_k}$$

Proposition 3.3

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}, \frac{\partial}{\partial x_7}, \frac{\partial}{\partial x_8}\right\}$$
$$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6, dx_7, dx_8\}$$

$$J\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} \quad , \quad J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1} \quad ,$$
$$J\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4} \quad , \quad J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}$$
$$J\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6} \quad , \quad J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$
$$J\left(\frac{\partial}{\partial x_7}\right) = \frac{\partial}{\partial x_8} \quad , \quad J\left(\frac{\partial}{\partial x_8}\right) = -\frac{\partial}{\partial x_7}$$

Let $z_1 = x_1 + ix_2$, $z_2 = x_3 + ix_4$, $z_3 = x_5 + ix_6$, $z_4 = x_7 + ix_8$

$$J^2\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} = J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1}$$
$$J^2\left(\frac{\partial}{\partial x_2}\right) = J\left(-\frac{\partial}{\partial x_1}\right) = -\frac{\partial}{\partial x_2}$$
$$J^2\left(\frac{\partial}{\partial x_3}\right) = J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}$$
$$J^2\left(\frac{\partial}{\partial x_4}\right) = J\left(-\frac{\partial}{\partial x_3}\right) = -\frac{\partial}{\partial x_4}$$
$$J^2\left(\frac{\partial}{\partial x_5}\right) = J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$
$$J^2\left(\frac{\partial}{\partial x_6}\right) = J\left(-\frac{\partial}{\partial x_5}\right) = -\frac{\partial}{\partial x_6}$$
$$J^2\left(\frac{\partial}{\partial x_7}\right) = J\left(-\frac{\partial}{\partial x_8}\right) = -\frac{\partial}{\partial x_7}$$
$$J^2\left(\frac{\partial}{\partial x_8}\right) = J\left(-\frac{\partial}{\partial x_7}\right) = -\frac{\partial}{\partial x_8} \qquad (3)$$

Proposition 3.4
The dual form J^* of the above J is as follows

$$J^{*2}(dx_1) = J^*(dx_2) = -dx_1$$
$$J^{*2}(dx_2) = J^*(-dx_1) = -dx_2$$
$$J^{*2}(dx_3) = J^*(dx_4) = -dx_3$$
$$J^{*2}(dx_4) = J^*(-dx_3) = -dx_4$$
$$J^{*2}(dx_5) = J^*(dx_6) = -dx_5$$
$$J^{*2}(dx_6) = J^*(dx_5) = -dx_6$$
$$J^{*2}(dx_7) = J^*(dx_8) = -dx_7$$
$$J^{*2}(dx_8) = J^*(-dx_7) = -dx_8 \qquad (4)$$

Theorem 3.5 [5] Let $\mathcal{M}$ be m-real dimensional configuration manifold .A tensor field J on $T^*\mathcal{M}$ is called an almost complex structure on $T^*\mathcal{M}$ if at every point p of $T^*\mathcal{M}$, J is endomorphism of the tangent space $T_p{}^*(\mathcal{M})$ such that $J^2 = -1$ are complex is $J^{*2} = J^* \circ J^* = -1$ is called structures are complex manifold

IV. HAMILTONIAN DYNAMICAL SYSTEMS

In this section, we shall obtain the version of Hamiltonian equations for classical mechanics structured with Four Almost Complex Structures on Symplectic Geometry

Definition 4.2[4]. A Hamiltonian system is a triple $(M;\omega;H)$, where $(\omega;H)$ is a Symplectic manifold and $H \in C^\infty(M)$ is a function, called the Hamiltonian function.

Suppose that an almost real structure, a Liouville form and 1-form on $T^*\mathcal{M}$ are shown by Φ^*, λ and ω, respectively. Then we have

$$\omega = \frac{1}{2}(x_1 dx_1 - x_2 dx_2 + x_3 dx_3 - x_4 dx_4 + x_5 dx_5 - x_6 dx_6 + x_7 dx_7 - x_8 dx_8) \qquad (5)$$

And

$$\lambda = \frac{1}{2}\big(x_1 J^*(dx_1) + x_2 J^*(dx_2) + x_3 J^*(dx_3) + x_4 J^*(dx_4) + x_5 J^*(dx_5) + x_6 J^*(dx_6) + x_7 J^*(dx_7) + x_8 J^*(dx_8)\big) \qquad (6)$$

We substitute equation (5) in equation (6) we get

$$\lambda = \Phi^*(\omega) = \frac{1}{2}[-x_1 dx_2 + x_2 dx_1 - x_3 dx_4 + x_4 dx_3 - x_5 dx_6 + x_6 dx_5 - x_7 dx_8 + x_8 dx_7] \qquad (7)$$

differential of λ

$$\phi = -d\lambda = = -d\frac{1}{2}[-x_1 dx_2 + x_2 dx_1 - x_3 dx_4 + x_4 dx_3 - x_5 dx_6 + x_6 dx_5 - x_7 dx_8 + x_8 dx_7]$$

It is known that if ϕ is a closed 2- form on $T^*\mathcal{M}$, then ϕ_H is also a symplectic structure on $T^*\mathcal{M}$.

$$\phi = dx_2 \wedge dx_1 + dx_4 \wedge dx_3 + dx_6 \wedge dx_5 + dx_8 \wedge dx_7 \qquad (8)$$

If Hamiltonian vector field X_H associated with Hamiltonian energy H is given by

$$X_H = X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}$$

Calculates a value X_H and ϕ

$$i_{X_H}\phi = \phi(X_H) = (dx_2 \wedge dx_1 + dx_4 \wedge dx_3 + dx_6 \wedge dx_5 + dx_8 \wedge dx_7)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6} + X^7\frac{\partial}{\partial x_7} + X^8\frac{\partial}{\partial x_8}\right) \qquad (9)$$

$$i_{X_H}\phi = -X^1 dx_2 + X^2 dx_1 - X^3 dx_4 + X^4 dx_3 - X^5 dx_6 + X^6 dx_5 - X^7 dx_8 + X^8 dx_7 \qquad (9)$$

So we find that

$$X^1 = \frac{\partial}{\partial x_2}, \quad X^2 = -\frac{\partial}{\partial x_1}, X^3 = \frac{\partial}{\partial x_4}, X^4 = -\frac{\partial}{\partial x_3}, X^5 = \frac{\partial}{\partial x_6}, X^6 = -\frac{\partial}{\partial x_5}, X^7 = \frac{\partial}{\partial x_8}, X^8 = -\frac{\partial}{\partial x_7}$$

Moreover, the differential of Hamiltonian energy is written as follows:

$$dH = -\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} + \frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} - \frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} + \frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} - \frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} + \frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} - \frac{\partial H}{\partial x_8}\frac{\partial}{\partial x_7} + \frac{\partial H}{\partial x_7}\frac{\partial}{\partial x_8} \qquad (10)$$

Suppose that a curve

$$\alpha: \mathrm{I} \subset \mathrm{R} \rightarrow \mathrm{T}^*\mathcal{M} = R^{2n}$$

is an integral curve of the Hamiltonian vector field X_H, i.e.,

$$X_H(\alpha(t)) = \frac{d\alpha(t)}{dt} \quad , \quad t \in I.$$

In the local coordinates, if it is considered to be

$$\alpha(t) = \left(x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t), x_7(t), x_8(t)\right)$$

we obtain

$$\frac{d\alpha(t)}{dt} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6} + \frac{dx_7}{dt}\frac{\partial}{\partial x_7} + \frac{dx_8}{dt}\frac{\partial}{\partial x_8} \quad (11)$$

Taking the equation(10) = the equation (11)

$$-\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} + \frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} - \frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} + \frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} - \frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} + \frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} - \frac{\partial H}{\partial x_8}\frac{\partial}{\partial x_7} + \frac{\partial H}{\partial x_7}\frac{\partial}{\partial x_8} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6} \frac{dx_7}{dt}\frac{\partial}{\partial x_7} + \frac{dx_8}{dt}\frac{\partial}{\partial x_8} \quad (12)$$

By comparing the two sides of the equation we get the

$$-\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} \Rightarrow -\frac{\partial H}{\partial x_2} = \frac{dx_1}{dt}$$
$$\frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} = \frac{dx_2}{dt}\frac{\partial}{\partial x_2} \Rightarrow \frac{\partial H}{\partial x_1} = \frac{dx_2}{dt}$$
$$-\frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} = \frac{dx_3}{dt}\frac{\partial}{\partial x_3} \Rightarrow -\frac{\partial H}{\partial x_4} = \frac{dx_3}{dt}$$
$$\frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} = \frac{dx_4}{dt}\frac{\partial}{\partial x_4} \Rightarrow \frac{\partial H}{\partial x_3} = \frac{dx_4}{dt}$$
$$-\frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} = \frac{dx_5}{dt}\frac{\partial}{\partial x_5} \Rightarrow -\frac{\partial H}{\partial x_6} = \frac{dx_5}{dt}$$
$$\frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} = \frac{dx_6}{dt}\frac{\partial}{\partial x_6} \Rightarrow \frac{\partial H}{\partial x_5} = \frac{dx_6}{dt}$$
$$-\frac{\partial H}{\partial x_8}\frac{\partial}{\partial x_7} = \frac{dx_7}{dt}\frac{\partial}{\partial x_7} \Rightarrow -\frac{\partial H}{\partial x_8} = \frac{dx_7}{dt}$$
$$\frac{\partial H}{\partial x_7}\frac{\partial}{\partial x_8} = \frac{dx_8}{dt}\frac{\partial}{\partial x_8} \Rightarrow \frac{\partial H}{\partial x_7} = \frac{dx_8}{dt}$$

Thus Hamilton's equations are

$$-\frac{\partial H}{\partial x_2} = \frac{dx_1}{dt} \quad , \quad \frac{\partial H}{\partial x_1} = \frac{dx_2}{dt}$$
$$-\frac{\partial H}{\partial x_4} = \frac{dx_3}{dt} \quad , \quad \frac{\partial H}{\partial x_3} = \frac{dx_4}{dt}$$
$$-\frac{\partial H}{\partial x_6} = \frac{dx_5}{dt} \quad , \quad \frac{\partial H}{\partial x_5} = \frac{dx_6}{dt}$$
$$-\frac{\partial H}{\partial x_8} = \frac{dx_7}{dt} \quad , \quad \frac{\partial H}{\partial x_7} = \frac{dx_8}{dt} \quad (13)$$

Hence the triple $(\mathcal{M}, \phi, X_H)$ is shown to be a Hamiltonian mechanical system which are deduced by means of an almost real structure j^* and using of basis $\left\{\frac{\partial}{\partial x_i} : i = 1,2,3,4,5,6,7,8\right\}$ on the distributions $T^*\mathcal{M}$

V. CONCLUSIONS

Thus, the equations of Hamiltonian equations (13) with Four Almost Complex Structures. In classical mechanics, a dynamic movement Hamilton equations is found. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds.

REFERENCES

[1] M. De Leon, P.R. Rodrigues, Generalized Classical Mechanics and Fields Theory, North-Holland Math. St.,112, Elsevier Sc. Pub. Com. Inc., Amsterdam, 1985.
[2] M. De Leon, P.R. Rodrigues , Methods of Differential Geometry in Analytical Mechanics ,North-Holland Math. St.,152, Elsevier Sc. Pub. Com. Inc., Amsterdam, 1989.
[3] R. Miron, D. Hrimiuc, H.Shimada, S. V. Sabau, The Geometry of Hamilton and Lagrange Spaces, Kluwer Academic Publishers, 2001.
[4] Tekkoyun, M., "A Note On Constrained Complex Hamiltonian Mechanics" Differential Geometry-Dynamical Systems (DGDS), Vol.8, No.1 , (2006) 262-267.
[5] Tekkoyun M., Cabar G., Complex Lagrangians and Hamiltonians, Journal of Arts and Sciences, C¸ ankaya ¨ Univ., Fen-Ed.Fak., Issue 8/December 2007
[6] Mehmet Tekkoyun, Complex Lagrangian mechanics with constraints, arXiv:0902.4128v1 [math.DG] 24 Feb 2009.
[7] Oguzhan Celik, Zeki Kasap, Mechanical Equations with Two Almost
[8] Complex Structures on Symplectic Geometry, April 28, 2016.
[9] D. Mc Duff , Singularities and Positivity of Intersections of J-Holomorphic
[10] Curves Symplectic 4-Manifolds, Holomorphic Curves in Symplectic Geometry, Birkhauser, 1994; pp. 191-215.
[11] R. Miron, D. Hrimiuc, H. Shimada and S.V. Sabau, The Geometry of
[12] Hamilton and Lagrange Spaces, Kluwer Academic Publishers, New York, 2002.
[13] Zeki Kasap, Conformal E Lagrange Mechanical Equations on Contact 5- Manifolds, international Journal of Innovative Mathematical Research ,Volume 3-Lssue 5 May 2015,pp41-48. http//en. Wikipedia.org / wiki / almost complex structure.
[14] K. Mackenzie: Lie groupoids and Lie algebroids in differential geometry, Cambridge University Press, 1987.
[15] P.J. Higgins, K. Mackenzie: Algebraic constructions in the category of Lie algebroids ,J. Algebra, 129 (1990), 194-230.
[16] Edward Burkard -From Classical Mechanics to Symplectic Geometry-4 November 2013 .

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