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Dynamic Response Attenuation with Elastic Metamaterial Application

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ABSTRACT: *In mechanical engineeringMetamaterial is a relatively incipient area of research. The beginning of the studies on this subjectwas in electromagneticwaves field and, posteriorly, acoustics used similar principles to attenuate sound wavesDue to the similarity between mechanical and electromagnetic waves, the attempt to use elastic metamaterials to mitigate mechanical waves was successfully applied.This paper aims to present the basic unit cell of an elastic metamaterial and show the concept of negative effective mass when the effective mass is subjected to a dynamic force, namely a harmonic excitation.Finally, the unit cell presented will be part of a chain of effective masses and the response to a harmonic excitation in the beginning of the chain will be verified at the end of the chain, proving the existence of a bandgap.*

Keywords: *Metamaterial, Single Resonator, Mechanical Wave Mitigation.*

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I. INTRODUCTION

Metamaterials are broadly defined as typically man made materials specially designed to achieve certain unusual properties which are not found in nature. First applications of metamaterials were in field of electromagnetic waves and some features of these materials were negative permittivity and negative refraction index [1]. Daring proposals like invisibility cloaks are a focus of attention in the area of metamaterials. Thefreedom of design that metamaterials provide to redirect the electromagnetic waves wasused to propose a strategy to develop an invisibility cloak that completely excludes all electromagnetic fields [2].Other authors [3] discuss a new type of invisibility cloak that provides all camouflaged objects with the appearance of a flat conducting sheet, with the advantage that none of the cover parameters is unique and can be fabricated to be isotropic.

As known, mechanical waves have a mathematical similarity with electromagnetic waves and so researchers started the studies of elastic metamaterials whose main feature is the negative dynamic effective mass which allow blocking the mechanical wave propagation.

In this paper, it will be reviewed the topic unit cell which is the fundamental element ofan elastic metamaterial and the application of the unit cell in a chain of masses containing single resonators. Based on an infinite chain, the dispersion equation will be evaluated and its corresponding dispersion curve indicating the bandgap will be shown. Further, a numeric simulation using FEA (Finite Element Analysis) will be developed to verify the capacity of attenuation of the elastic metamaterial under bandgap frequencies.

II. UNIT CELL FOR A SINGLE RESONATOR

As mentioned, applications using metamaterials were first used in the field of electromagnetic waves, where researchers investigated negative properties as, for example, negative refractive index [4]. Due to the mathematical analogy of mechanical and electromagnetic waves, elastic metamaterials were recently explored. This new branch of metamaterials, known both as mechanical either elastic metamaterials, consists of adapted microstructures that present unusual mechanical properties, such as negative effective modulus of elasticity and / or negative effective mass density. A system containing negative mass properties consists of a chain of mass units, each mass being linked by a spring to an internal mass.

This resonator unit is shown in Fig. 1, where the outer cell unit has mass $m_1$and displacement u_1. The internal resonator has mass m_2 and displacement u_2. The stiffness of the spring is linear with stiffness coefficient k_2 and connects the external mass to the internal resonator.

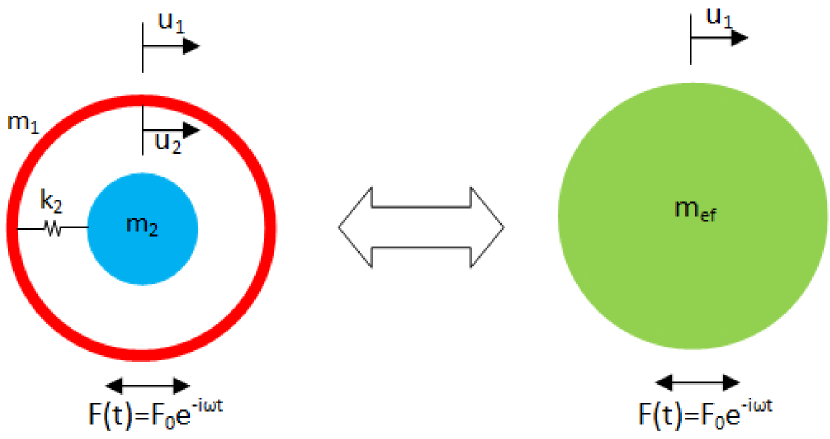

Fig 1: Single resonator and equivalent effective mass

The free-body diagram of each of the masses m_1 and m_2 provides following equations:

$$m_1\ddot{u}_1 = F + k_2(u_2 - u_1) \qquad 1$$

$$m_2\ddot{u}_2 = -k_2(u_2 - u_1) \qquad 2$$

Considering the mass displacements as harmonic wave behavior, as well as the applied force, according to (3) and (4):

$$u_\gamma(x,t) = \hat{u}_\gamma e^{-i\omega t} \qquad 3$$

$$F(t) = F_0 e^{-i\omega t} \qquad 4$$

Where γ = 1, 2, corresponding to the masses m_1 and m_2. (1) and (2) can be solved replacingin (3) and (4). Thus, the simplified relation is presented in (5):

$$0 = F_0 + \left(m_1 + \frac{\omega_2^2 m_2}{\omega_2^2 - \omega^2}\right)\omega^2 \hat{u}_1 \qquad 5$$

Where $\omega_2 = \sqrt{k_2/m_2}$is the natural frequency of the internal mass resonator m_2. Applying the equation of motion in the effective mass m_{eff}based on (4), we obtain the relation:

$$F_0 = -m_{eff}\omega^2 \hat{u}_1 \qquad 6$$

From (5) and (6), we conclude that the effective mass m_{eff} is given by:

$$m_{eff} = \left(m_1 + \frac{\omega_2^2 m_2}{\omega_2^2 - \omega^2}\right) \qquad 7$$

It is observed in (7) that the effective mass, m_{eff}, is dependent on the frequency ω. Assumingm_{st} as the static mass (m_1 + m_2), the normalized effective mass can be obtainedm_{eff}/m_{st} :

$$\frac{m_{ef}}{m_{st}} = 1 + \frac{\theta}{1+\theta}\left[\frac{\left(\omega/\omega_2\right)^2}{1 - (\omega/\omega_2)^2}\right] \qquad 8$$

Where θ is the ratio of the internal mass, m_2, and the external mass, m_1.Figure 2 shows how the ratio m_{ef}/m_{st} varies when$\omega/\omega_2$is considered in the abscissa axis. It can be seen that there is a narrow band gap band near the local resonance frequency of the internal mass m_2. Huang et. al. (2009) [5] showed that this negative effective mass corresponds to the band gap region of the dispersion curve when the wave propagation is considered.

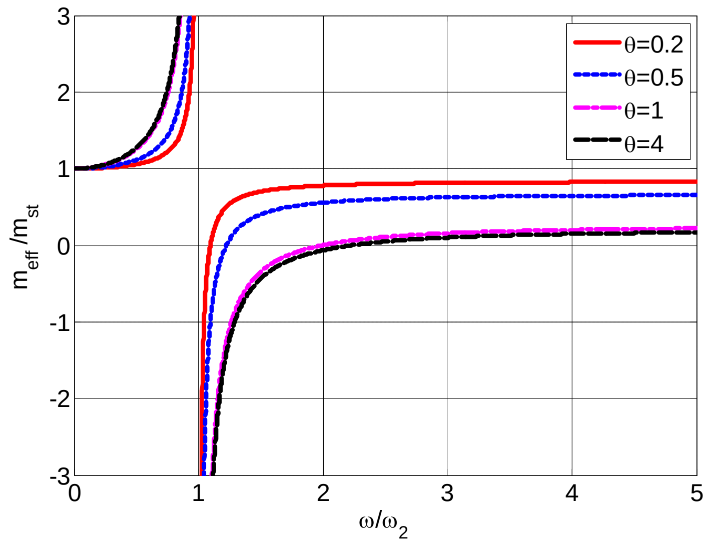

Fig 2: Normalized effective mass versus $\boldsymbol{\omega/\omega_2}$ ratio (single resonator)

III. APPLICATION OF THEUNIT CELL

3.1Effective Mass Chain

Consider the mass chain represented in Figure 3, where the initial masses (j = 1,2,3) of mass m are connected by springs of stiffnessk_1. The following masses$j$ = 3 to j = 9, besides being connected by springs of stiffnessk_1 each other, they are also connected to other masses, namelym_2, by springs of stiffnessk_2. Lastly, masses j = 10 to j = 13 are connected by the same springs of stiffnessk_1 of the beginning of the chain.The distance between the masses m is given by a.

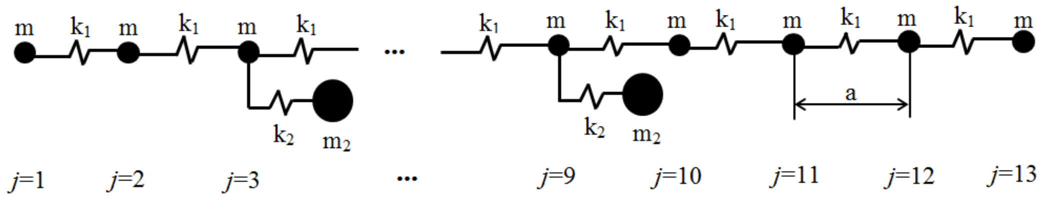

Fig 3:Mass Chain with Resonators

As shown in the sub-item 3.1.1-Simple Resonator, the masses j = 3 to j = 9 consist a single resonator. Thus, we can represent the mass chain of Figure 3.5, as shown in Figure 3.6, replacing the masses j = 3 to j = 9 by effective masses, $m_{eff.}$

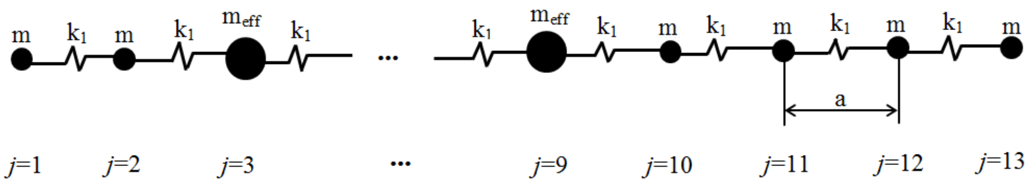

Fig 4:Chain with Effective Masses

Thus, for the masses j = 3 to j = 9, the equations of motion for the j-th unit are given by:

$$m_1^{(j)} \frac{d^2 u_1^{(j)}}{dt^2} + k_1 \left(2u_1^{(j)} - u_1^{(j-1)} - u_1^{(j+1)}\right) + k_2\left(u_1^{(j)} - u_2^{(j)}\right) = 0 \quad 9$$

$$m_2^{(j)} \frac{d^2 u_2^{(j)}}{dt^2} + k_2\left(u_2^{(j)} - u_1^{(j)}\right) = 0 \quad 10$$

Where $u_\gamma^{(j)}$ represents the displacements of the masses γ of the *j-th* cell. The harmonic wave solution for the *(j + n) -th* cell is expressed by the formula:

$$u_\gamma^{(j+n)} = \beta_\gamma e^{i(qx + nqa - \omega t)} \quad 11$$

Where β_γ is the complex wave amplitude, q is the wave number, ω is the angular frequency and $\gamma = 1$ and 2 corresponds to the masses m and m_2. The substitution of (11) in (9)and (10)yields two homogeneous equations for $\beta_1$and $\beta_2$fromwhich the dispersion equation is obtained as

$$m_1 m_2 \omega^4 - [(m_1 + m_2)k_2 + 2m_2 k_1(1 - \cos(qa))]\omega^2 + 2k_1 k_2(1 - \cos(qa)) = 0 \quad 12$$

Using the trigonometric relation $1 - cos(qa) = 2sen^2\left(\frac{qa}{2}\right)$, (12) can be simplified as shown in (13):

$$m_1\omega^4 - \left(4k_1 sen^2\left(\frac{qa}{2}\right) + \omega_2^2(m_2+m_1)\right)\omega^2 + \omega_2^2 4k_1 sen^2\left(\frac{qa}{2}\right) = 0 \quad 13$$

where$\omega_2^2 = k_2/m_2$.

3.2 Dispersion Curve

Taking into account the example presented by [6], where an experimental study is done with the data presented in Table 1, onecan plot the frequency (ω) as a function of the wave number (q), which represents the dispersion curve of (13).

The dispersion relations of a periodic structure can be evaluated by the Bloch theorem, where the wave vector varies along the boundaries of the irreducible Brillouin zone [7, 8]. The start and the end of the passband and bandgap coincides with the edges of the irreducible Brillouin zone.

Table1: DataUtilized

Data	Value
m_1	101.1 g
m_2	46.47 g
k_1	117 N/m
k_2	74 N/m
a	0.5 m

With the data presented in Table 1, $\omega_2 = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k_2}{m_2}} = 6.35$ Hz.

For the proposed one-dimensional system, these edges are at q = 0 and q = π / ain(13). Substituting these values into (13) yields ω = 0 and 7.67 Hz for q = 0 and ω = 5.75 Hz and 11.96 Hz for q = π / a.

Figure 5 shows the dispersion curve for (13). A bandgap frequency is observed between the red lines represented by the frequencies of 5.75 Hz and 7.67 Hz. In addition, it is observed that the cut-off frequency is 11.96 Hz, as calculated through the edges of the irreducible Brillouin zone.

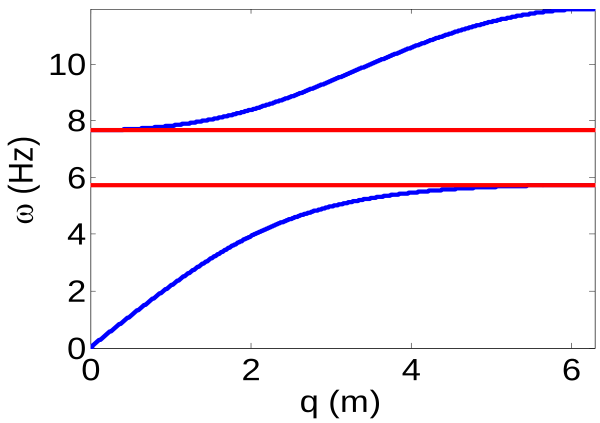

Fig 5:Dispersion Curve

3.3 Finite Element Analysis of the Effective MassChain

To verify the blocking property of the waves in the range represented in the dispersion curve of Fig.5, a Finite Element Analysis (FEA) simulation of the chain represented by Fig. 3 was developed, where the first mass $j = 1$) was harmonically excited with amplitude of 0.01 m making a sweep of frequencies from 0 to 12 Hz and the response obtained was the displacement of the last mass ($j = 13$).

The result can be observed in Fig.6, identifying the frequencies that limit the bandgap (5.75 Hz $<\omega<$7.67 Hz), where, in this range, the amplitude of the displacement is drastically reduced. Note that near the cutoff frequency (11.96 Hz), the displacement response is also reduced, but not so much as in the bandgap. This reduction can be better observed when the simulation is done only with the effective masses (Fig.7). It is even clearer that only in the bandgap range the amplitude of the displacement of the last mass is decreasingwhen the first mass is excited harmonically in the frequency range from 0 to 12 Hz.

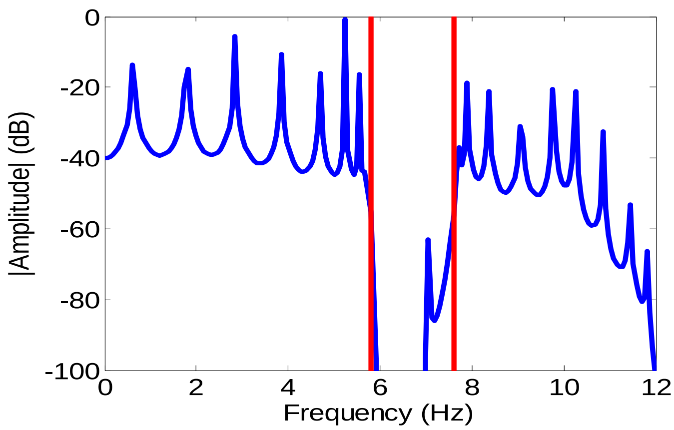

Fig 6:Amplitude module *vs.* frequency

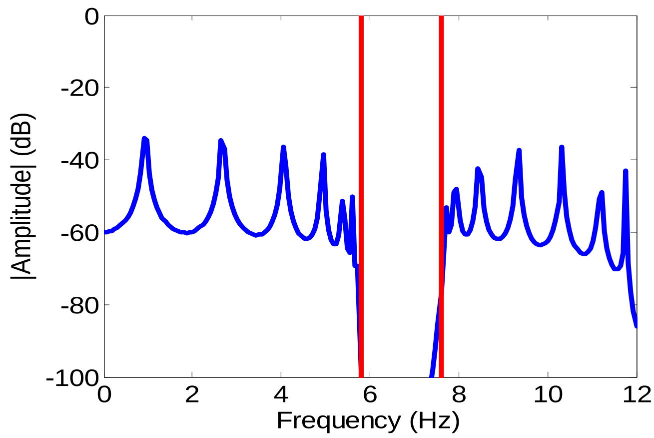

Fig 7:Amplitude module *vs.* frequency(m_{eff})

IV. CONCLUSION

The concept of unit cell for elastic metamaterial was reviewed and its negative effective mass for dynamic input was discussed.The unit cell component of a single resonator was applied to a chain of masses creating a chain of effective masses. Based on masses and stiffness of the chain, one could develop the dispersion equation of an idealized infinite chain subjected to a harmonic excitation. Plotting the dispersion equation, one could obtain the dispersion curve and identify the frequency where the wave propagation is blocked, namely bandgap, between 5.75 and 7.67 Hz. Simulation using FEA was developed sweeping the frequency from 0 Hz to the cut-off frequency (11.96 Hz) and it was possible to verify the attenuation of the amplitude response in the last mass of the chain in the bandgap interval.

So, it was proved that the elastic metamaterial has an effective blocking effect in the wave propagation when the frequency of propagation is in the bandgap defined by dispersion equation. Depending on masses and stiffness, it was possible to tune the appropriated interval to block the desired frequencies.

However, one difficulty to build an elastic metamaterials is geometric and material limitations. Both of these characteristics are fundamental to determine the masses and stiffness that will define the negative effective mass of the metamaterial and its respective bandgap.

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