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A Nomogram Of Performances In Endurance Running Based On Logarithmic Model Of Péronnet-Thibault

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Abstract: *In their study on the modeling of the contributions of aerobic and anaerobic metabolism in function of the duration of the race, Péronnet and Thibaut proposed the slope of the relationship between the natural logarithm of running duration and the fractional utilization (EI) of maximal aerobic speed (MAS) as an index of endurance capability for running exercises whose durations are longer than 7 min. However, this logarithmic equation does not enable to predict the performances on other distances. In the present study, three nomograms of the performances in running have been designed from the logarithmic model proposed by Péronnet and Thibault: a first nomogram for the distances that are the most often ran (1500, 3000, 5000 and 10000 m) in international competitions, a second nomograms corresponding to running performances beyond 7 min as in the model of Péronnet and Thibault and a third nomogram including a large range of distance (from 1000 to 20000m). From the performances over two distances, these nomograms enable to estimate 1) the performances over other distances; 2)MAS; 3) the index of endurance capability EI.*

Keywords: Modelling, Endurance, Elite runners, Endurance index

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I. INTRODUCTION

In 1989, Péronnet and Thibault [1] proposed a model that took into account the contributions of aerobic and anaerobic metabolism to total energy output in function of the duration of the race. A runner is only capable of sustaining his maximal aerobic power for a finite period of time. In agreement with previous studies, Péronnet and Thibault assumed that maximal aerobic power can be sustained over 7 min (420 s). In this interesting study, Péronnet and Thibault also proposed the slope of the relationship between the natural logarithm of running duration and the fractional utilization of maximal aerobic speed (MAS) as an index of endurance capability.

$$100\ S/MAS = C - EI^*\ln(t_{lim}/420) \qquad \text{Equation 1}$$

where C was a constant close to 100 and slope EI an endurance index. This endurance index was significantly related (r = 0.853) to ventilatory threshold, expressed as a percentage of MAS, in a group of marathon runners [2]. The lower the absolute value of EI, the higher the endurance capacity is assumed to be. For example, endurance indexes computed from personal best performances were equal to 8.14 for Ryun, an elite middle-distance runner and 4.07, for Derek Clayton, an elite long distance runner. Equation 1 can be modified as following:

$$S = MAS^*[C - EI\ \ln(t_{lim}/420)]/100$$
$$S = MAS - E\ \ln(t_{lim}/420) \qquad \text{Equation 2}$$

$$\text{where } E = MAS^*EI/100 \qquad \text{Equation 3}$$

In the model of Péronnet-Thibault, two values of S and t_{lim} are sufficient to compute Equation 1.
$S_1 = MAS - E\ \ln(t_{lim1}/420)$
$S_2 = MAS - E\ \ln(t_{lim2}/420)$
$S_1 - S_2 = E\ \ln(t_{lim2}/420) - E\ \ln(t_{lim1}/420) = E\ \ln(t_{lim2}) - E\ \ln(t_{lim1})$

$E = (S_1 – S_2)/[\ln(t_{lim2}) – \ln(t_{lim1})]$
$MAS = S_2 + E \ln(t_{lim2}/420) = S_1 + E \ln(t_{lim1}/420)$

The values of E and MAS can also be estimated (2) by computing the linear regression between S and the natural logarithm of $t_{lim}/420$:
$S = a + b \ln(t_{lim}/420)$
where MAS and E are equal to a and b, respectively.

In the Péronnet-Thibault model, the value of a distance (D_{lim}) corresponding to t_{lim} is equal to:

$D_{lim} = f(t_{lim}) = t_{lim}*S = t_{lim}*[MAS – E \ln(t_{lim}/420)]$

However, It is not possible to predict the value of t_{lim} corresponding to a given distance from the inverse function $f^{-1}(D_{lim})$ that is unknown. The estimation of E, MAS, EI are very easy for people who know how to use the logarithms and the computational programs. For people who are not mathematician, the use of nomograms (or abaques) will be easier and more useful. Indeed, a nomogram, is a two-dimensional diagram designed to graphically compute a function without using a mathematical formula. In the present paper three nomograms are presented:

- a first nomogram for the distances that are the most often performed in international competitions (1500, 3000, 5000 and 10000 m);
- a second nomograms corresponding to running performances beyond 7 min as in the model of Péronnet and Thibault;
- a third nomogram including a large range of distances (from 1000 to 20000m).

The three nomograms presented in the present study enable not only the estimations of EI and MAS from the performances over two distances but also the predictions of performances over other distances.

II. METHODS

The three nomograms were designed with the graphics software SigmaPlot (Systat Software Inc. San Jose, USA) and the vector graphic editor CoreDRAW.

2.1 Design of the first nomogram (Fig. 1)

The first column of the table of data in SigmaPlot corresponded to time (from 210 to 7200 s) with 1 second increments. The second column corresponded to the same range of time but with 5 second increments. Column 3 corresponded to time with 30 second increments. Columns 4, 5 and 6 corresponded to the running speeds ($m.s^{-1}$) over 1500 m that were equal to 1500 divided by the values of time in columns 1, 2 and 3, respectively. Similarly, the speeds corresponding to 3000, 5000 and 10000 m were printed in the next columns.

The SigmaPlot graphs corresponding to the relationships between S and t_{lim} for each distance were drawn with a linear scale for ordinate (running speed) but with a logarithmic scale for abscissa (t_{lim}). For each distance (1500, 3000, 5000 and 10000 m), a line plot and two scatter plots were drawn. The line plot corresponding to a given distance was a curve (Fig. 1) and was computed from the time and velocity data corresponding to 1 second increments. For the 1500 m distance, a scatter plot of the points corresponding to the values of S at each 30-second interval was drawn from the data in column 5. The symbols of this scatters plot were little squares (2.5 x 2.5 mm) and were superimposed to the line plot. A second scatter plot of the points corresponding to the values of S at each 5-second interval was done from the data in column 6. The symbols of this second scatter plot were smaller squares (1 x 1 mm). The same kinds of plots were drawn for the other distances. The SigmaPlot graph at the origin of the first nomogram was constituted of the combinations of the line plots and scatter plots corresponding to the different distances. Thereafter, this graph was transferred to the software Corel Draw. Each square symbol of the scatter plots was transformed in a rectangle by decreasing its height and increasing its length. These rectangles corresponded to the small horizontal graduations on the nomogram (Fig. 1). The ordinates of the graphs drawn by SigmaPlot concerned the running speeds for the different distances. As t_{lim} depends on speeds, the values of speed for a given distance in the Sigmaplot graphs were "translated" in the values of t_{lim} for each distance in the CorelDRAW graphics (t_{lim} = distance/speed). For each distance, the highest graduation of the curve corresponded to a time lower than the world record. For example, the graduation at the top of the curve corresponding to 10000 m was equal to 22 min and was decremented with 15 seconds for each graduation, which corresponded to 3 minor graduations and 1 major graduation for each minute (Fig 1). For 5000 m, the best time was equal to 11 min and was decremented by 10 seconds, which corresponded to 5 minor graduations and 1 major graduation for each minute. For 1500 and 3000 m, the best times were equal to 3min 20s and 6min 35s and were decremented by 5 seconds, which corresponded to 5 minor graduations and 1 major graduation for each 30-second interval.

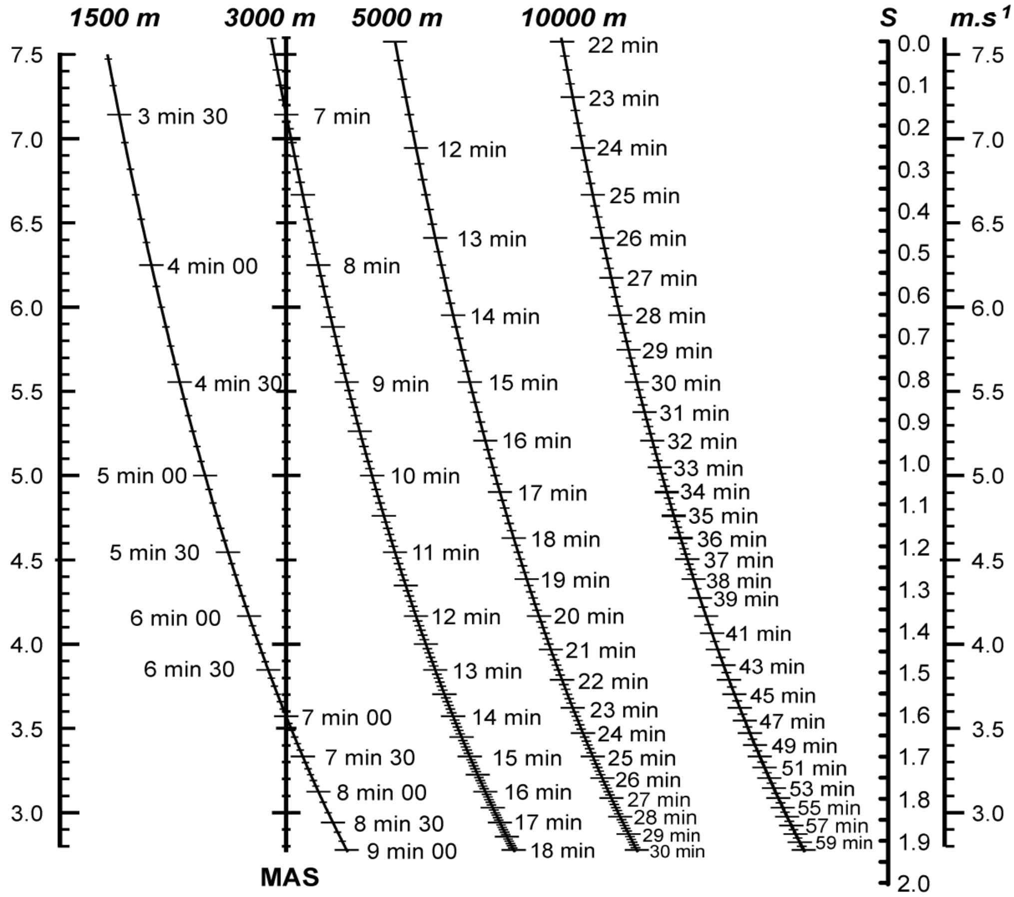

Fig. 1: nomogram of performances corresponding to 1500, 3000, 5000 and 10000 m, according to the model of Péronnet andThibault [1].

A vertical line corresponding to t_{lim} equal to 420 s (7 min) and two scales (from 2.8 to 7.5 m.s $^{-1}$) were added for the estimation of MAS. A scale named "Slope" (from 0.0 to 2.0) was added on the right of the graph for the estimation of slope E. The estimations of MAS and E are demonstrated (Fig. 2) with the example of a theoretical runner whose best performances would be 4 min for 1500 m and 15 min for 5000 m. First, the points corresponding to performances over the two distances (red dots) are plotted on the nomogram and a line joining these points (individual performance line, red line on Fig. 2) is drawn. Secondly, the value of MAS is determined by drawing a horizontal line (MAS line, blue line on Fig. 2) passing through in the intersection (blue dot) of the red line and the vertical line corresponding to 7 min. The blue and red lines intersect the Slope scale at E_1 and E_2 and the value of E is equal to the difference between E_2 and E_1 (green arrow). In the example presented on Fig. 2, MAS and E are estimated as equal to 5.96 m.s^{-1} and 0.52 (1.17 – 0.65), respectively. Endurance index EI is equal to 100 E/MAS. Therefore, EI (3) is equal to 8.72 (100*0.52/5.96). The running performances over 3000 and 10000 m correspond to red empty circles.

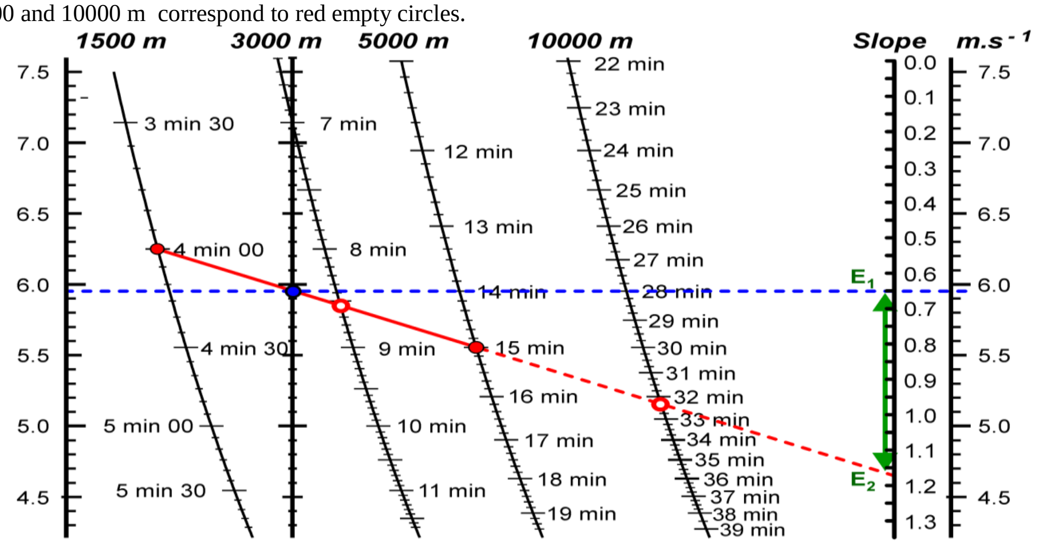

Fig 2: explanation of the estimations of MAS and E with the nomogram

2.2 Designs of the second and third nomograms

The second nomogram (Fig 3.) corresponds to a large range of running distances corresponding to t_{lim} longer than 7 min. It is not possible to write the values of t_{lim} corresponding to the graduations for many curves in the same nomogram, especially when the distances are close (for example 3000 m and 2 miles). Therefore, the horizontal graduations and labels were replaced with vertical lines corresponding to different values of t_{lim}. The third nomogram (Fig 3) corresponds to a larger range of running distances (from 1000 to 10000 m). The intervals between vertical lines (I) were equal to 5 s for t_{lim} between 3 and 7 min, 10 s for t_{lim} between 7 and 14 min, 20 s for t_{lim} between 14 and 28 min, 30 s for t_{lim} between 28 and 55 min, 1min for t_{lim} between 55 and 97 min and 2 min for t_{lim} between 97 and 133 min. Every six lines, the vertical lines (from 3 to 133 min) were labelled and their thicknesses were doubled. Two scales (from 2 to 8 $m.s^{-1}$) were added for the estimation of MAS. A scale (S) from 0.0 to 2.0 was added on the right of the graph for the estimation of slope E as in the first nomogram. The methods of estimation of MAS and E are the same as demonstrated in Fig. 2.

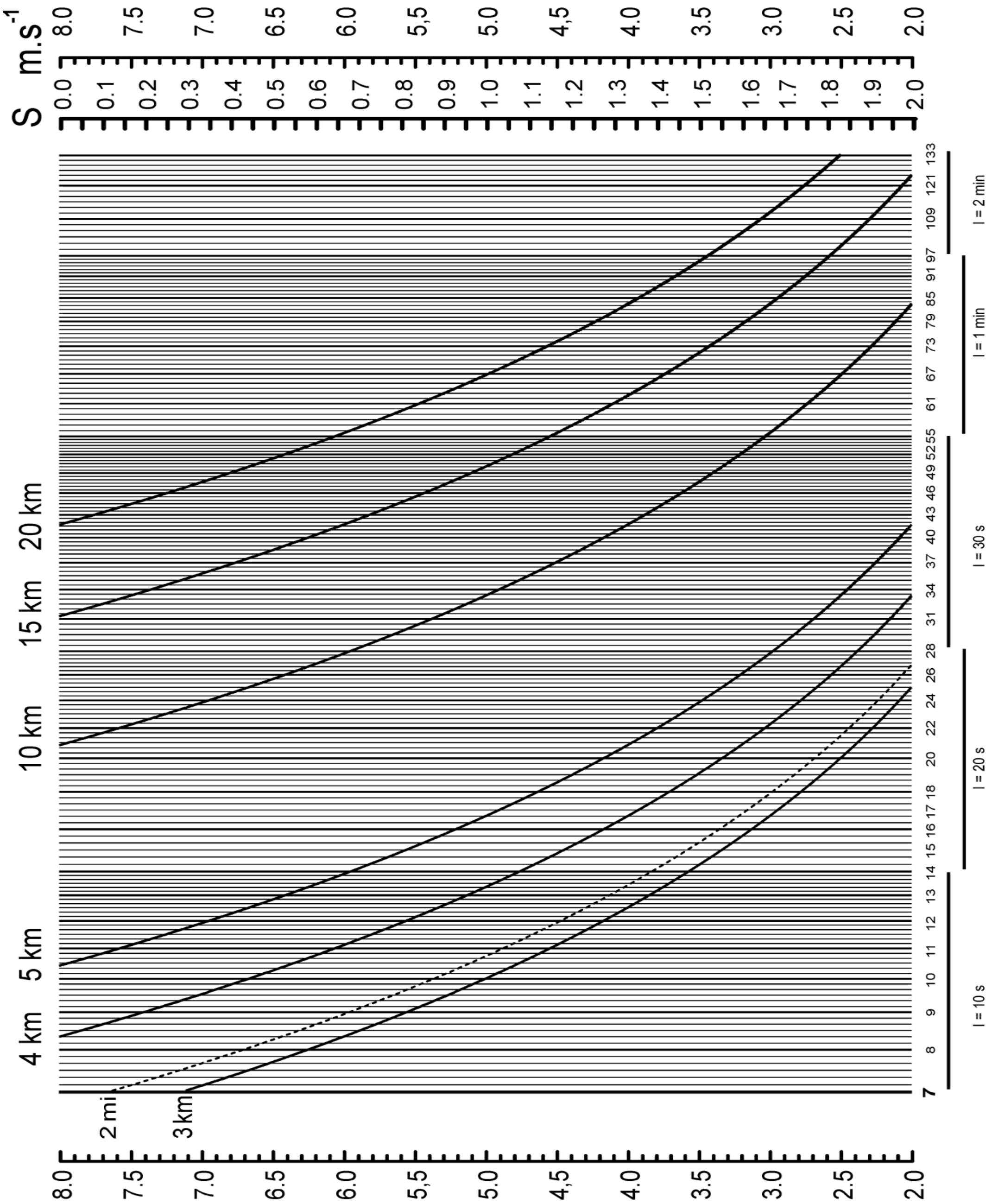

Fig. 3: nomogram of running performances (Péronnet-Thibault model) for exercise durations longer than 7 min. I = intervals between vertical lines (from 5 s to 2 min). The dashed curve corresponds to the performances over 2 miles.

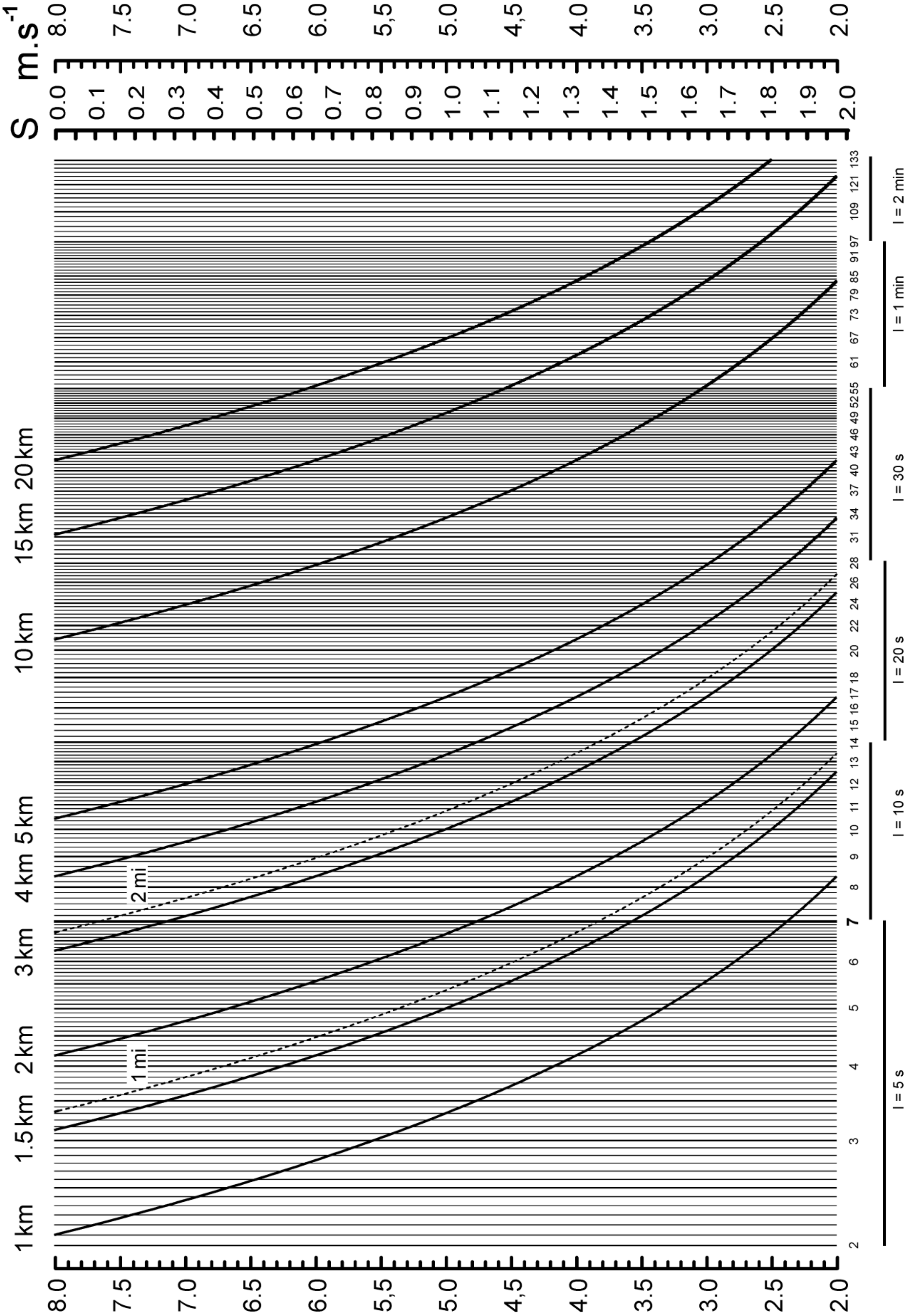

Fig. 4: nomogram of running performances (Péronnet-Thibault model) for distances rangeing from 1000 m to 20000 m. I = intervals between vertical lines (from 5 s to 2 min).The dashed curves correspond to the performances over 1 and 2 miles.

2.3 Validation of the model of Péronnet-Thibault

The validity of the Péronnet-Thibault model was tested by using the performances of elite endurance runners who were the best runners in the world (several Olympic medals and/or world records for each runner) and ran over all the different distances of the first nomogram. These data can be obtained in many sites of internet (Wikipedia…). The use of the performance of world elite runners in the validation of the Péronnet-Thibault model is interesting because it is assumed that the running data corresponded to the maximal performance for each distance. The best performances of world elite runners generally corresponded to the results of many competitions against other elite runners. Therefore, the motivation was probably optimal during these races. The best times of these elite endurance runners and the computed values of MAS and EI are presented in table 1.

	1500	*3000*	*5000*	*10000*	*MAS*	*EI*
Nurmi	233	500	868	1806	6.13	7.20
Zatopek	233	488	837	1734	6.22	5.35
Virén	222	463	796	1658	6.52	5.54
Aouita	209	449	778	1646	6.77	7.83
Gebrselassie	214	445	759	1583	6.78	5.05

Table 1: best times of elite endurance runners and their individual values of MAS and EI used in the validation of the model of Péronnet-Thibault.

III. RESULTS AND DISCUSSION

The validity of the first nomogram is demonstrated by the results of figure 2. Indeed, the estimations of MAS (5.96 $m.s^{-1}$) and E (0.52) are very close to the results of the computed regression between S and ln(tlim/420) that is:

$$S = 5.956 - 0.525 \ln(t_{lim}/420) = MAS - E \ln(t_{lim}/420)$$

The validity of the model of Péronnet-Thibault is also confirmed by the plot of the best performances of the elite endurance runners in the third nomogram (Fig. 5): the individual data corresponding to the different distances are aligned for each runner and the estimated values of MAS and EI (Table 3) are not significantly different ($P > 0.05$) from the values computed from the regression between S and $\ln(t_{lim}/420)$ in Table1 for EI (6.05 ± 1.24 vs 6.19 ± 1.24) as well as MAS (6.480 ± 0.316 vs 6.484 ± 0.302 $m.s^{-1}$).

Runners	E_1	E_2	E	MAS	EI
Nurmi	0.61	1.05	0.44	6.12	7.12
Zatopek	0.595	0.91	0.315	6.20	5.08
Virén	0.49	0.825	0.335	6,5	5.15
Aouita	0.395	0.915	0.520	6.79	7.66
Gebrsellassie	0.395	0.75	0.355	6.79	5.23

Table 2: Individual values of MAS and EI estimated from Fig.3

However, the present data do not prove that the Péronnet-Thibault and the nomograms are valid for the performances on track in subjects who are not elite endurance runners. Indeed, the logarithmic model proposed by Péronnet and Thibault [1] is not the only model than can theoretically describe with accuracy the performances of elite endurance runners [3, 4]. Indeed the power-law model proposed by Kennelly in 1906 can also describe the running performances of elite endurance runners [4]. But, in runners who are not elite endurance runners, the performances cannot theoretically be described by both models of Péronnet-Thibault and Kennelly [4]. Therefore, it would be interesting to compare the validity of Kennelly and Péronnet-Thibault model in non-elite endurance runners.

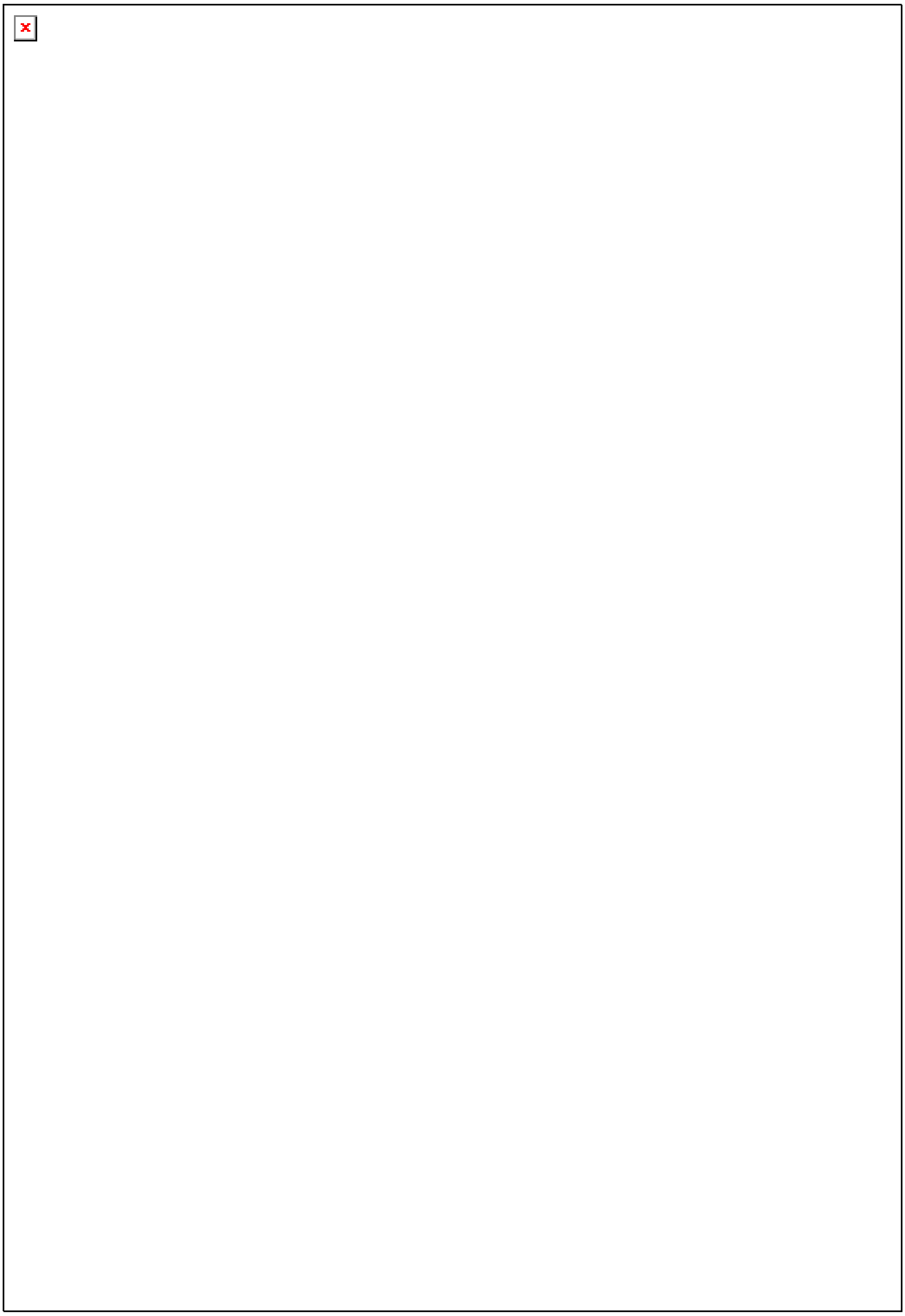

Figure 5: performances of elite endurance runners. I = intervals between vertical lines (from 5 s to 2 min). Black dots : Gebrselassie. Orange dots: Aouita. Blue dots: Virén. Green dots: Zatopek.White dots: Nurmi.

IV. CONCLUSIONS

The results of the present study indicate that the present nomograms based on Péronnet-Thibault model can describe the individual performances of elite runners on a track (1500, 3000, 5000 and 10000) with a high accuracy and enable the estimations of MAS, E and EI. However, the validity of the Péronnet-Thibault model and the interest of the present nomograms must be verified in subjects who are not endurance runners.

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