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Mathematical modelling of Maintenance Scheduling Based on Condition Monitoring

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ABSTRACT: *This study presents the application of condition monitoring in maintenance scheduling. The problems of machine inspection cost, condition monitoring instruments cost and anticipated failure cost in a repairable system were investigated. A Mathematical model was developed. Input data were obtained from an industry and combinatorial optimization procedure was used to solve the problem. Results obtained from the study provided a matrix for decision making that minimizes the total expected cost in the system for a one year planning horizon. The inspection interval of three months ie* $T_1 = T_2 = T_3 = T_4 = 3T_0$ *,with a total expected cost of ₦68,514,069.00 per annum and average monthly cost of ₦5,709,505.75 minimizes the CBM cost in the system.*

Keywords: *Mathematical modeling, CBM, maintenance, maintenance scheduling*

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I. INTRODUCTION

Condition based maintenance (CBM) is an equipment maintenance procedure based on detecting the condition of the equipment in order to evaluate whether it will fail during some future period and then acting appropriately to avoid the consequence of that failure (Bengston, 2004). It is maintenance action furthered on actual condition derived from tests. Maintenance is not carried out until there is an obvious need which will increase the availability of the equipment, as well as lower the maintenance cost. The acquired data could be used to determine whether the system is running at a normal operating condition. If the limits of the preset values are exceeded, the reason behind it can be adduced and prediction made for future equipment breakdown and failure. The available information is used to plan maintenance actions (Bengston, 2004). The system/component could be monitored continuously, in which case, the monitoring equipment is fixed on the system and connected to the computer for real time monitoring.

Asset inspection is an important approach to acquiring information for CBM decision-making. An inspection can incur additional costs. Some inspection methods even insist on the shutdown of the asset .Therefore inspection should be well conducted to reduce cost and enhance asset availability. Equipment inspection can be performed continuously or only on discrete time points. In practice continuous asset monitoring is often technically or economically impossible, therefore most CBM methods adopt discrete inspections. Ben-Daya and Duffuaa (1997) presented a condition based maintenance inspection model for a group of machines with the objective of determining the optimum maintenance cost. Huynh *et al.*(2013) developed an inspection maintenance model for a system subjected to deterioration.

Marseguerra *et al.* (2002) considered a continuously monitored multi-component system and used a Genetic Algorithm to determine the optimal CBM policy. Li and Pham (2005) presented a generalized CBM model subject to multiple competing failure processes based on degradation paths and accumulated shock damage. Cai, *et al*. (2012) applied proportional covariate model (PCM) to assess the wear characteristics of cutting tools on a machine. Saranga and Knezevic (2001) developed a mathematical model for reliability prediction of condition based maintenance systems in which the material is deteriorating as a Markov process.. Kallen and Nootwijk (2006) presented a decision model for the determining the optimal inspection interval of an item with sequential discrete states. Yam *et al.* (2001) presented an artificial neural network (ANN) to predict the deterioration of a gear box for appropriate decision support system. Ozor (2014) presented artificial neural network based maintenance system for industries Sinha *et al.* (2002) applied a neural networks model to predict the probability of failure of underground pipeline system. Chen and Trivedi (2005) suggested a semi-Markov decision process for the maintenance policy optimization of condition based preventive maintenance problems, and presented a method for a joint inspection rate and maintenance policy. Yang, *et al.* (2008) made use of Markov chain to describe the various states of each machine with the objective function that depicts the difference between the benefits of producing parts and cost of maintenance operations in the system.

The objective of this paper is to investigate how condition based maintenance can be applied in an industrial setting. The paper is concerned with how inspection can be used in CBM environment to minimize the total cost of maintenance. A mathematical model is presented with a unique method of solution and also validated

II. METHODOLOGY

In this study a mathematical model was developed and data were obtained from a flow station of a major oil company in Nigeria, which its identity is protected in this work. This data include: cost of monitoring equipment, cost of inspection, repair cost, increase cost of running equipment above threshold limit and down time of the equipment among others. There was also discussion with managers, supervisors, engineers and maintenance personnel on the implementation of condition based maintenance in the organization. Combinatorial optimization was developed and used as a method of solution. The result was validated using the model of Ben Daya and Duffuaa (1997)

III. MATHEMATICAL MODELLING

The mathematical model presented below consists of three parts: setup cost, failure cost and down time cost that may occur due to condition monitoring

The objective of the model is to determine the inspection time T_i for machine *i* as a multiple of the basic cycle so as to minimize the expected cost per unit time.

The following assumptions were made in the development of this model:

(i) The life of the machine is a random variable with probability density function *f*(t), where (t) is the life running time.

(ii). The repair times are negligible and repair brings the machine back to an in-control-state.

(iii). A cycle schedule is repeated every year, T

(iv) A constant inspection interval and its multiple to the basic cycle is assumed.

(a) Setup Cost

This cost consists of two parts: the cost of monitoring equipment to conduct CM and the cost of labour.

This cost (C_{bc}) consists of the cost incurred at every basic cycle T_o

(i)
$$C_{bc} = \frac{P - SV}{n_m} = A_v \quad (1)$$

A_v is the depreciation cost of the measuring equipment. It is spread over time. A straight-line depreciation method is assumed, P is the acquisition cost of the condition monitoring instrument, d_t is amortization factor, SV is the salvage value, n_m , is the planned number of years before replacement.

(ii) The total cost of inspection for the machine is given as

$$\sum C_{ml} = \sum_{i=1}^{N} \frac{T}{T_i} a_i \quad (2)$$

$$\sum C_{mi} = \sum_{i=1}^{N} \frac{T}{T_i} C_L t_{inspectt_{i,j}} \quad (3)$$

Where

$$a_i = C_L t_{inspect_{i,j}} \quad (4)$$

C_L is the labour rate of the inspection personnel and $t_{inspect_{i,j}}$ of machine *i*., N is the number of machines

(b) The downtime cost due to CM is given as:

$$C_{idi,j+1} = \sum_{i=1}^{N}\sum_{j=o}^{n-1} P_L t_{dmi} \tag{5}$$

$$j = 0,1,2....n-1$$

Where C_{tdi+j} is the down time cost due to condition monitoring of machine i in the interval j. P_L is the production/service loss per unit time and td_{mi} is the shut down time for CM inspection.

(c) The Failure Cost

The total failure cost for N machines in a given system in a particular horizon is expressed as

$$\sum C_{mi} = \sum_{i=1}^{N}\sum_{j=0}^{n_{i_i}} C_{j,j+1} \tag{6}$$

Where

$$C_{i,j+!} = \frac{r_i(1-e^{-\lambda_i t_{i,j+!}}) + s_i(t_{i,j+!}) + s_i t_{i,j+1} e^{-\lambda_i t_{i,j+1}} - \frac{s_i}{\lambda_i}(1-e^{-\lambda_i t_{i,J+1}})}{e^{-\lambda_i t_{i,j+!}}}$$

$$j = 0,1,2....n-1 \tag{7}$$

For machine *i* between $t_{i0} = 0$ and $t_{i,j+1}$ the cost of failure at each $C_{i,j+1}$ is calculated

An exponential distribution is assumed.

$$f_i(t) = \lambda_i e^{-\lambda_i t} \tag{8}$$

Where λ_i is the failure rate of the CM component in machine i. The failure rate is given as

$$\lambda_i = \frac{1}{MTBF_{ni}} \tag{9}$$

The mean time to failure is given as

$$MBTF_{ni} = \frac{U_T}{n_{fi}} \tag{10}$$

The total expected cost per cycle (TEC) of length T is obtained by summing together Equations and 1, 2.5 and 7

$$TEC = A_v + \sum_{i=1}^{N}\frac{T}{T_i}a_i + \sum_{i=1}^{N}\sum_{j=o}^{n-1_i} C_{idi,j+1} + \sum_{i=1}^{N}\sum_{j=0}^{n-1_i} C_{i,j+1} \tag{11}$$

The average cost per month (CPT).

$$CPT = \frac{A_v}{T} + \sum_{i=1}^{N}\frac{a_i}{T_i} + \sum_{i=1}^{N}\sum_{j=0}^{n_i-1}\frac{C_{idi,j+1}}{T} + \sum_{i=1}^{N}\sum_{j=0}^{n-1_i}\frac{C_{i,j+1}}{T} \tag{12}$$

Where A_v, $a_i, C_{idi,j+1}$ and $C_{i,j+1}$ are given in equations: Where A_v, a_i, $C_{idi,j+1}$ and $C_{i,j+1}$ are given in equations 1,2,5 and 6.

Ben-Daya and Duffuaa's (1997) model is used in the validation. The expected total cost (ETC) is given as:

$$ETC = \frac{A}{T_0} + \sum_{i=1}^{N}\frac{T}{T_i}a_i + \sum_{i=1}^{N}\sum_{j=0}^{n-1_i} C_{i,j+1} \tag{13}$$

Where

$$C_{i,j+!} = s_i t_{i,j+1} - \alpha_i s_i (1 - e^{-\lambda t_{i,j+1}}) \tag{`14}$$

$$\alpha_i = \frac{1}{\lambda_i} - r_i^p \tag{15}$$

$$r_i^p = \frac{r_i}{s_i} \tag{16}$$

From equation equations 14 and 15

$$C_{i,j+1} = r_i + s_i t_{i,j+1} - \frac{s_i}{\lambda_i}(1 - e^{-\lambda t_{i,j+1}}) - r_i e^{-\lambda t_{i,j+1}} \quad (17)$$

$10\ \%\ r_i \leq s_i \leq 50\ \%\ r_i$ (Ben-Daya and Duffuaa (1997)

A combinatorial optimization technique is used as a method of solution.
The formulation is:
Minimize the CBM cost (TEC)

$$TEC = A_v + \sum_{i=1}^{N} \frac{T}{T_i} a_i + \sum_{i=1}^{N} \sum_{j=o}^{n-1_i} C_{idi,j+1} + \sum_{i=1}^{N} \sum_{j=0}^{n-1_i} C_{i,j+1} \quad (18)$$

Subject to

$T_i = K\ T_o$ **,** K is an integer (19)

Where A_v, a_i, $C_{idi,j+1}$ and $C_{i,j+1}$ are given in equations 1,2,5 and 7

IV. RESULTS AND DISCUSSION

The basic information of the system is given in the data below:

T = 1 year

T_0 = 1 month- basic cycle

T_i = 1 month- inspection interval for all the 4 machines

N = 4 (Machines- three pumps and a generator)

n_i = 12

P = ₦30, 240, 0000

n_m = 5 years (depreciation assumption)

A_v = ₦ 6,048,000

C_L = ₦ 315000 per machine

$t_{inspecti_i}$ = 1hour for the 4 machines

a_i = ₦ 315000 for all the 4 machines

r_1 = ₦ 2,693,333, r_2 = ₦ 2,600,000, r_3 = ₦ 3,373,333, r_4 = ₦ 2,040,000

s_1 =₦ 673,333 *(25% r_1)* , s_2 =₦ 650,000 *(25% r_2)*, s_3 =₦ 843,330 *(25% r_3)*, s_4 = ₦ *510,000 (25% r_4)*

U_{T1} = 8712 hrs, U_{T2} = 8736 hrs, U_{T3} = 8736 hrs, U_{T4} = 8690 hrs

n_{f1} =1, n_{f2} = 1, n_{f3} = 1, n_{f4} = 2

$MBTF_1$ = 8712 hrs, $MBTF_2$ = 8736 hrs, $MBTF_3$ = 8736 hrs, $MBTF_4$ =4345 hrs

λ_1 = 0.00011/hr (1/yr.) , λ_2 = 0.00011/hr (1/yr.), λ_3 = 0.00011/hr (1/yr.)

λ_4 = 0.00023/hr (2/yr.)

For 1 month interval, $n_i = 12$ For 2 months interval, $n_i = 6$, For 3 months interval $n_i = 4$

The flow station has four operational machines, three pumps and a generator. The failure rate of the machines are :, $\lambda_1 = 1/year$, $\lambda_2 = 1/year$, $\lambda_3 = 1/year$ and $\lambda_4 = 2/year$ .The cost of inspection of each machine is ₦315000. The average maintenance cost of each machine per month is r_i and the increased maintenance cost s_i and other information are presented in the data above.

The failure cost for one month inspection interval as presented in table 1 increases progressively in the order of 20%, 18%,10%, 35%, 15%, 14%, 14%, 14%, 14%, 14%, 14% representing an average increase of 17 % per month. The failure cost in the month 6th, 7th, 8th , 9th,10th and 11th month remain fairly constant. However in two months inspection interval there is a sharp increase in the failure cost: 28% and 26%, 37%, 26% 26%

amounting to an average of 29% . This trend is repeated for inspection interval of three months, which increases by 36%, 35% and 44% respectively with an average of 38%.

The expected failure costs of the various combinations lie between the boundary of that of one month, two months and three months respectively. This is reflected in the total expected cost of the various combinations as shown in table 5.

The failure cost of machines 1, 2, 3 and 4 in the one month inspection interval are: ₦8,568,422.988 ₦11,994,473.890 ₦20,006,168.376 ₦45,051,258.317 representing 10%, 14%,23% and 53% of the total cost failure cost in the system.

For an inspection interval of one month, the total expected cost of maintenance is ₦106,788,384.46 while it is ₦81,234,998.55 and ₦68,514,069.00 for two months and three months inspection interval respectively. If more maintenance actions occur upon inspection on monthly basis then the total expected cost (TEC) per year will be higher than that of two months and three months respectively. The total failure cost in the system are ₦85,620,323.575, ₦65,627,013.273 and ₦57,426,068.596 representing 80%, 83% and 84% of the total expected cost (TEC) for one, two and three months inspection interval respectively(Summing failure costs in Tables 1,3 and 4.).The increase in inspection interval leads to the reduction of failure cost of the machine. However such practice requires caution. There is a reduction in total expected cost (TEC) by 31% by using two months inspection interval and 56 % for 3 months inspection interval compared to one month. Table 5 provides other options available to the company to reduce the cost of CBM in the system. These options were obtained from combinatorial optimization.

The set up cost of condition monitoring equipment is a constant depreciated cost of ₦6,048,000 per year or ₦ 504,000 per month especially as it is instrument based. Inspection contributes ₦315000 per month for a machine to the maintenance cost. This cost is should be controlled to minimize the total cost of maintenance.

The optimum basic inspection interval for the system need to be properly selected based on the machine type, experience, cost and operational characteristics to ensure that the machine is not allowed to run excessively above the threshold value or out-of-control state, which will result in unforeseen downtime before next inspection. The failure costs at any interval of time are very close to each other and it is beneficial to increase the inspection interval to an optimum time interval so as to reduce the total expected cost and yet reduce failures of machines in the system.

The data were applied on Ben-Daya and Duffuaas' model. The condition monitoring instrument cost in their work was defined as a routine cost occurred whenever an inspection were to be performed in the system. The condition monitoring instrument cost in this work model is however defined as a constant depreciated cost spread over five years and is assumed to be equivalent to the depreciated instrument cost in a monthly inspection interval basis when used in Ben-Daya and Duffuaas' model (1997).

The model presented in this work considers the effect of the complement of CDF on the failure cost. The compliment of the CDF is the reliability of the system within the stipulated time interval. The failure cost obtained from this research work is ₦85,620,323.575 compared to ₦67,697,377.933 in table 2 using Ben-Daya and Duffuaas' model (1997), representing a difference of ₦17,922,945.64 or 21%. The model presented in this work is more realistic for a multi-component unit over a long planning horizon of one year.

Table I shows Failure cost for one month inspection interval while Table 2 shows the Failure cost using Ben Daya and Duffuaa's model

Table :1. Expected Failure Cost for 1 Month Inspection Interval

M/C	Failure Cost/Month(s)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	236532.55	313622.72	392996.66	474892.87	559574.04	647329.49	738477.77	833369.59	932391.01	1035966.89	1144564.76	1258704.64
2	388579.59	481631.38	578916.16	680875.98	787997.76	900817.73	1019926.35	1145973.66	1279675.22	1421818.51	1573270.10	1743991.45
3	695439.81	838410.74	989294.95	1148918.83	1318192.62	1498118.79	1689801.16	1894454.95	2113417.80	2348161.89	2600307.21	2871649.64
4	654488.10	824791.28	1021322.09	2022808.57	2448916.37	2953224.83	3551481.47	4262573.61	5109149.29	6118360.12	7322749.87	8761392.71
$\sum_{1}^{4} C_{i,j}$	1975040.05	2458456.12	2982529.86	3327496.25	5114680.79	5999490.84	6999686.75	8136371.81	9434633.32	10924307.41	12640891.4	14626739.44

Table: 2. The expected failure cost results from Ben-Daya and Duffuaa's model (`1997)

M/C	Failure Cost/Month(s)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	217620.52	471859.51	710257.10	934079.91	1144493.26	1342569.32	1529294.50	1705576.39	1872249.96	2030083.46	2179783.70	2322007.63
2	162263.66	407692.44	637828.78	853895.40	1057017.24	1248229.32	1428483.87	1598657.02	1759554.82	1911918.88	2056431.52	2193726.94
3	153449.31	471877.36	770464.46	1050796.99	1314334.52	1562419.89	1796288.58	2017077.28	2225831.82	2423514.38	2611010.24	2789142.20
4	283391.69	590989.57	857890.20	1038547.32	1241837.56	1420443.63	1578154.98	1718179.32	1843231.93	1955611.22	2057262.77	2149837.74
$\sum_{1}^{4} C_{i,j}$	816725.18	1942418.88	2976440.54	3877319.62	4757682.58	5573662.16	6332221.93	7039490.01	7700868.53	8321127.94	8904488.23	9454714.51

Table 3 shows the failure cost for two months inspection interval

Table: 3. Expected Failure Cost for 2 Months Inspection Interval

M/c	Months/Failure Cost					
	2	4	6	8	10	12
1	498923.96	705322.80	933162.17	1187121.41	1472871.91	1797278.92
2	797626.98	1073829.54	1388249.76	1749228.50	2166875.00	2653427.62
3	1381910.15	1824782.63	2336481.60	2932028.406	3529628.21	4451322.49
4	1354884.60	2022285.79	2953255.96	6276568.35	9041560.66	12997846.04
$\sum_{1}^{4} C_{i,j}$	4,033,345.69	5,626,760.76	7,611,149.49	12,144,946.45	16,310,935.78	21,899,875,07

Table 4 shows the failure cost for three months inspection interval.

Table: 4. Expected Failure Cost for 3 Months Inspection Interval

Machine	Months/Failure Cost			
	3	6	9	12
1	789875.51	1196133.00	1687403.47	2292785.15
2	1229787.04	1800840.89	2517875.59	3430857.45
3	2067733.41	2989381.66	4165012.02	5681557.48
4	2120689.54	3760294.93	6522957.71	15172883.75
$\sum_{1}^{4} C_{i,j}$	6,208,085.5	9,746,650.48	14,893,248.79	26,578,083.83

The total expected cost per year and average cost per month for various combinations of basic cycles are shown in Table 5

Table: 5. TEC Cost of 1, 2 and 3 Month(s) Inspection Interval and Various Combinations

S/N	Combinations	TEC Cost ₦	CPT Cost ₦
1	$T_1 = T_2 = T_3 = T_4 = T_0$ (By the Company)	106,788,384.46	8,899,032.04
2	$T_1 = T_2 = T_3 = T_4 = 2T_0$	81,234,998.55	6,769,583.21
3	$T_1 = T_2 = T_3 = T_4 = 3T_0$	68,514,069.00	5,709,505.75
4	$T_1 = 2T_0 \; T_2 = T_3 = T_4 = T_0$	102,924,623.744	8,577,051.975
5	$T_2 = 2T_0 \; T_1 = T_3 = T_4 = T_0$	102,733,129.072	8,561,084.083
6	$T_3 = 2T_0 \; T_1 = T_2 = T_4 = T_0$	101,448,350.454	8,454,029.2
7	$T_4 = 2T_0 , T_1 = T_2 = T_3 = T_0$	94,494,048.650	7,874,040.054
8	$T_1 = T_2 = 2T_0 \; T_3 = T_4 = T_0$	98,869,368.360	8,239,114.03
9	$T_1 = T_3 = 2T_0 , T_2 = T_4 = T_0$	97,584,589.742	8,132,049.145
10	$T_1 = T_4 = 2T_0 , T_2 = T_3 = T_0$	90,630,287.938	7,552,523.994
11	$T_2 = T_3 = 2T_0 , T_1 = T_4 = T_0$	97,393,095.070	8,116,091.256
12	$T_2 = T_4 = 2T_0 , T_1 = T_3 = T_0$	90,438,793.267	7,536,566.105
13	$T_3 = T_4 = 2T_0 , T_1 = T_2 = T_0$	89,154,014.648	7,429,501.22
14	$T_1 = T_2 = T_3 = 2T_0 , T_4 = T_0$	93,529,334.358	7,794,111.196
15	$T_1 = T_2 = T_4 = 2T_0 , T_3 = T_0$	86,575,032.555	7,214,836.04
16	$T_2 = T_3 = T_4 = 2T_0 , T_1 = T_0$	85,098,759.265	7,091,563.272
17	$T_3 = T_1 = T_4 = 2T_0 , T_2 = T_0$	85,290,253.936	7,107,521.161
18	$T_1 = 3T_0 , T_2 = T_3 = T_4 = T_0$	101,666,143.471	8,437,771.367
19	$T_2 = 3T_0 , T_1 = T_3 = T_4 = T_0$	101,253,256.413	8,437,771.367
20	$T_3 = 3T_0 , T_1 = T_2 = T_3 = T_0$	99,165,885.535	8,263,823.794
21	$T_4 = 3T_0 , T_1 = T_2 = T_3 = T_0$	86,793,936.945	7,232,828.078
22	$T_1 = T_2 = 3T_0 , T_3 = T_4 = T_0$	96,131,015.428	8,010,917.952
23	$T_1 = T_3 = 3T_0 , T_2 = T_4 = T_0$	94,043,644.550	7,836,970.379
24	$T_1 = T_4 = 3T_0 , T_2 = T_3 = T_0$	81,671,695.960	6,805,974.663
25	$T_2 = T_4 = 3T_0 , T_1 = T_3 = T_0$	81,258,808.902	6,771,567.408
26	$T_2 = T_3 = 3T_0 , T_1 = T_4 = T_0$	93,630,757.492	7,802,563.124
27	$T_3 = T_4 = 3T_0 , T_1 = T_2 = T_0$	79,171,438.024	6,597,619.835
28	$T_1 = T_2 = T_3 = 3T_0 , T_4 = T_0$	88,508,516.507	7,375,709.708
29	$T_1 = T_2 = T_4 = 3T_0 , T_3 = T_0$	76,136,567.917	6,344,713.993
30	$T_2 = T_3 = T_4 = 3T_0 , T_1 = T_0$	73,636,309.981	6,136,359.165
31	$T_3 = T_1 = T_4 = 3T_0 , T_2 = T_0$	74,049,197.039	6,170,766.419
32	$T_1 = 2T_0 , T_2 = T_3 = T_4 = 3T_0$	69,772,549.269	5,814,379.105
33	$T_2 = 2T_0 , T_1 = T_2 = T_4 = 3T_0$	70,796,533.915	5,899,711.159

34	$T_3 = 2T_0, T_1 = T_3 = T_4 = 3T_0$	69,993,941.656	5,832,828.471
35	$T_4 = 2T_0, T_1 = T_2 = T_3 = 3T_0$	76,214,180.701	6,351,181.725
36	$T_1 = T_2 = 2T_0, T_3 = T_4 = 3T_0$	71,252,421.929	5,937,701.827
37	$T_1 = T_3 = 2T_0, T_2 = T_4 = 3T_0$	72,055,014.188	6,004,584.515
38	$T_1 = T_4 = 2T_0, T_2 = T_3 = 3T_0$	77,472,660.974	6,456,055.081
39	$T_2 = T_3 = 2T_0, T_1 = T_4 = 3T_0$	72,276,406.574	6,023,033.881
40	$T_2 = T_4 = 2T_0, T_1 = T_3 = 3T_0$	77,694,053.361	6,474,504.447
41	$T_3 = T_4 = 2T_0, T_1 = T_2 = 3T_0$	78,496,645.620	6,541,387.135
42	$T_1 = T_2 = T_3 = 2T_0, T_4 = 3T_0$	73,534,886.847	6,127,907.237
43	$T_1 = T_2 = T_4 = 2T_0, T_3 = 3T_0$	78,952,533.634	6,579,377.803
44	$T_2 = T_3 = T_4 = 2T_0, T_1 = 3T_0$	79,976,518.279	6,664,709.856
45	$T_3 = T_1 = T_4 = 2T_0, T_2 = 3T_0$	79,755,125,893	6,646,260.491
46	$T_1 = T_0, T_2 = T_3 = 2T_0, T_4 = 3T_0$	77,398,647.559	6,449,887.296
47	$T_1 = T_0, T_2 = T_4 = 2T_0, T_3 = 3T_0$	82,816,294.346	6,901,357.862
48	$T_1 = T_0, T_3 = T_4 = 2T_0, T_1 = 3T_0$	83,618,886.605	6,968,240.55
49	$T_2 = T_0, T_3 = T_4 = 2T_0, T_1 = 3T_0$	84,031,773.663	7,002,647.805
50	$T_2 = T_0, T_2 = T_4 = 2T_0, T_3 = 3T_0$	83,007,789.017	6,917,315.751
51	$T_3 = T_0, T_1 = T_2 = 2T_0, T_4 = 3T_0$	78,874,920.850	6,572,910.071
52	$T_3 = T_0, T_2 = T_4 = 2T_0, T_1 = 3T_0$	85,316,552.182	7,109,712.682
53	$T_4 = T_0, T_1 = T_2 = 2T_0, T_3 = 3T_0$	91,246,869.439	7,603,905.786
54	$T_4 = T_0, T_2 = T_3 = 2T_0, T_1 = 3T_0$	92,270,854.085	7,689,237.84

V. CONCLUSIONS

The Mathematical model presented in this study is effective to determine the total expected maintenance cost using CBM inspection for the machines in the system The failure rate of the machine, inspection cost, and the cost of condition monitoring equipment have great contributions on the maintenance cost of the system. The failure cost has the most significant effect on the total expected maintenance cost of the system. Shorter inspection interval increases the total expected maintenance cost of the system. A combinatorial optimization solution procedure used is suitable in solving this type of problem and has provided a template for effective maintenance cost decision making. The numbers of machines affect the matrix of maintenance decision template. This approach is highly recommended for CBM management of the system and similar systems.

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