American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-7, pp-310-314
www.ajer.org
Research Paper Open Access

Deduce The nth Derivative And Generalize Some Properties of Z-Transformations

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ABSTRACT: *The study presented the importance of z- transformation in our daily lives and briefly explains the relationship between z- transform, Laplace transform and Fourier transform and their use in signal processing techniques for continuous and discrete functions. Also introduced mathematical descriptive for differential equation and difference equation.*
Finally, during this study, researcher were able to drive general rule for differentiation of z-transformation and generalize some z-transform properties, such as linearity properties of z- transformation for n functions. This work does not go into Engineering applications.

Keyword: *z-transform, signal process, continuous ,discrete, properties, sequences.*

Date of Submission: 03 -07 - 2017 Date of acceptance: 31-07-2017

I. INTRODUCTION

In last century we witnessed the explosion of digital communication and media, the need for methods to analyze and process digital data is becoming more important than ever.JPEG images, MP3& MP4 songs, MPEG-2 videos and ZIP files etc. are all processed using digital processing techniques. Mathematical descriptive relationship of input -output of a system is formulated either in the time domain or in the frequency domain. Time-domain system analysis methods are based on differential equations, which describe the system output as a weighted combination of differential (i.e. the rates of change) of the system input and output signal. while frequency -domain methods mainly the Laplace transform [4].

In communication Engineering, two basic types of signals are encountered, continuous- time signals and discrete- time signals. Continuous time signals are defined for continuous values of the independent variable namely time and denoted by a function $\{f(t)\}$. Discrete time signals are defined only at discrete set of values of independent variable namely time and denoted by a sequences $\{f(n)\}$.

Z- transform, is an indispensable mathematical tool for design, analysis and monitoring of system. The z-transform is the discrete-time counter- part of Laplace transform and generalization of the Fourier transform of a sampled signal. Like Laplace transform, the z-transform allow insight into the transient behavior, the steady state behavior, and the stability of discrete-time system. So that z-transform is essential for the study of digital filters and systems [1,2]. On the other hand z-transform plays an important role in analysis of linear discrete time signals.

II. DEFINITIONS AND GENERAL CONCEPTS

2.1. Differential Equation: A differential equation is a mathematical equation that relates some function with its derivatives and their derivatives present their rates of change, which defines a relationship between the two. The input and output of a continuous system which are related by differential equation can be solved by Laplace transform techniques. However, the difference equation which applicable to sampled system are used z-transform techniques and forms the relation between its input and output [4].

2.2.Difference Equation: A difference equation is an equation which expresses a relation between an independent variable and the successive values of the dependent variable or successive difference of the dependent variable. This present a Linear Time Invariant (LTI) system and obeys all its usual properties[4].

2.3.Definition:[3].If $\{f(n)\}$ is a sequence defined for $n = 0, \pm 1, \pm 2 \dots \dots \dots$, then $\sum_{-\infty}^{\infty} f(n) z^{-n}$ is called the two side or bilateral z- transform of $\{f(n)\}$ and denoted by $z\{f(n)\}$ or $f^{-}(z)$ when z is a complex variable in general.

If $\{f(n)\}$ is a casual sequence, i.e. if $f(n) = 0$, for $n < 0$, then z-transform is called one side or unilateral z-transform of $\{f(n)\}$ and defined as

$$Z\{f(n)\} = f^{-}(z) = \sum_{0}^{\infty} f(n) z^{-n} \quad . \qquad (2.3.1)$$

Since z- transforms relates to sequences, we first review the notation associated with sequences. A finite sequence $\{x_k\}_0^n$ is an ordered set of $n + 1$, real or complex numbers, these numbers are ordered so that position in sequence is important. The position is defined by the position index k, where k is an integer. if the number of elements in the set is infinite then this leads to infinite sequence

$$\{x_k\}_0^{\infty} = \{x_0, x_1, x_2, \dots \dots \dots .\} \qquad (2.3.2)$$

When dealing with sampled function of time t, it is necessary to have a mean of allowing for $t < 0$. To do this we make the sequence of numbers to extend to infinity on both sides of initial position x_0, and write

$$\{x_k\}_{-\infty}^{\infty} = \{\dots \dots \dots \dots \dots, x_{-2}, x_{-1}, x_0, x_1, x_2, \dots \dots \dots .\} \qquad (2.3.3)$$

Sequences $\{x_k\}_{-\infty}^{\infty}$ for which $x_k = 0$ $(k < 0)$ are called causal sequence, by analogy with continuous -time causal function $f(t)H(t)$ defined as follows $f(t)H(t) = \begin{cases} 0 & (t < 0) \\ f(t) & (t \geq 0) \end{cases}$

(2.3.4)

while for some finite sequence it is possible to specify the sequence by listing all the elements of the set. Normally a sequence is specified by giving a formula for the general element x_k [3].

2.4.Definition: The z-transform of sequence $\{x_k\}_{-\infty}^{\infty}$ is defined in general as

$$\chi\{x_k\}_{-\infty}^{\infty} = X(z) = \sum_{k=-\infty}^{\infty} \frac{x_k}{z^k} \qquad (2.4.1)$$

whenever the sum exists. z is a complex variable, to be defined. The process of taking the z- transform of a sequence thus produces a function of a complex variable z , whose form depends upon the sequence itself. The symbol χ denotes the z-transform operator. When it operates on a sequences $\{x_k\}$ it transforms the letter into the function $X(z)$ as a z-transform pair , which sometimes written as $\{x_k\} \leftrightarrow X(z)$.

The sequences $\{x_k\}_{-\infty}^{\infty}$ are called causal, that is when $x_k = 0 \quad (k < 0)$, so the z-transform in (2.4.1) reduces to

$$\chi\{x_k\}_0^{\infty} = X(z) = \sum_{k=0}^{\infty} \frac{x_k}{z^k} \qquad (2.4.2)$$

2. 5. The Region of Converge (ROC)

Since the z-transform is a finite power series, it exists only for those values of the variable z , for which the series converges to a finite sum. The region of convergence (ROC) of $X(z)$ is the set of all the values of z for which $X(z)$ attains a finite computable value. To find the value of z for which the series converges, we use the ratio test or root test.[1].

The objectives of this work, to deduce a derivation rule for z-transform and generalize some properties, which could be implement in the Engineering field.

Corollary: Let us consider the following the sequence z- transformation $\{x_k\} = \{3^k\} \quad (k \geq 0)$

Proof: from equation (1.4.2) $\chi\{3^k\} = \sum_{k=0}^{\infty} \frac{3^k}{z^k} = \sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^k$

The last term is a geometric series, with common ratio $r = {}^{3}/_{z}$ between the consecutive terms. The series converges for $|z| > 3$

so $\sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^k = \lim_{k \to \infty} \frac{1-(3/z)^k}{1-3/z} = \frac{1}{1-3/z}$ which leads to

$\chi\{3^k\} = \frac{z}{z-3} \quad$ for $|z| > 3$

Hence $\{x_k\} = \{3^k\}$ and $X(z) = \frac{z}{z-3}$ are z- transform pair.

The function $X(z) = \frac{z}{z-3}$ is a generating function for the sequence $\{3^k\}$ in the senses that the coefficient of z^{-k} in the expansion of $X(z)$ in powers of $1/z$ generates the kth term of the sequence $\{3^k\}$ which can be formed as follow

$\frac{z}{z-3} = \frac{1}{1-3/z} = (1 - \frac{3}{z})^{-1}$ where $|z| > 3$

$(1 - \frac{3}{z})^{-1} = 1 \left(\frac{3}{z}\right)^0 + \left(\frac{3}{z}\right)^1 + \left(\frac{3}{z}\right)^2 \dots \dots \dots \dots \left(\frac{3}{z}\right)^k \dots \dots .$

Then 3^k is coefficient of z^{-k} as we expected.

To generalize the above result $\chi\{a^k\}$, the z-transformation of the sequences $\{a^k\}$, where a is a real or complex constant, is

$\chi\{a^k\} = \sum_{k=0}^{\infty}\left(\frac{a}{z}\right)^k = \frac{1}{1-a/z}$, such that $|z| > |a|$

Therefore $\chi\{a^k\} = \frac{z}{z-a}$, such that $|z| > |a|$

Example: Show that $\chi\left\{\left(-\frac{1}{a}\right)^k\right\} = \frac{az}{az+1}$ $\quad |z| > \frac{1}{a}$

Solution: Let us use $\chi\{a^k\} = \frac{z}{z-a} \Rightarrow \chi\left\{\left(-\frac{1}{a}\right)^k\right\} = \sum_{k=0}^{\infty}\left(\frac{-\frac{1}{a}}{z}\right)^k = \frac{z}{z-(-\frac{1}{a})} = \frac{az}{z+1}$; $|z| > \frac{1}{a}$, so that $\chi\left\{\left(-\frac{1}{a}\right)^k\right\} = \frac{az}{az+1}$; $\forall\ |z| > \frac{1}{a}$

III. GENERALIZATION OF Z- TRANSFORMATION FROM DIFFERENTIATION:

The differentiation in z-transform is very important, this leads us to study it in depth in order generate some properties with differentiation in general and especial cases.

Let the function $\chi\{a^k\} = \frac{z}{z-a}$.

By differentiating $\chi\{a^k\}$ with respect to a

$\frac{d\chi\{a^k\}}{da} = \chi\left\{\frac{d\{a^k\}}{da}\right\} = \frac{d}{da}\left(\frac{z}{z-a}\right)$; this give $\chi\{ka^{k-1}\} = \frac{z}{(z-a)^2}$; $\forall\ |z| > |a|$

in special case $a = 1$, we get, $\chi\{k\} = \frac{z}{(z-1)^2}$ where $|z| > |1|$

as mentioned in [3] and other scientific recourses.

Corollary: Let the function $\chi\{a^k\} = \frac{z}{z-a}$. Then the differentiation $\frac{d^n\chi\{a^k\}}{da^n} = \frac{n!z}{(z-a)^{n+1}}$

We can generalize the above method to obtain nth derivative, using mathematical induction.

The second derivative $\frac{d^2\chi\{a^k\}}{da^2} = \chi\left\{\frac{d^2\{a^k\}}{da^2}\right\} = \frac{d}{da}\left(\frac{z}{z-a}\right) = \frac{2z}{(z-a)^3} = \frac{2!z}{(z-a)^{2+1}}$

The third derivative $\frac{d^3\chi\{a^k\}}{da^3} = \chi\left\{\frac{d^3\{a^k\}}{da^3}\right\} = \frac{d}{da}\left(\frac{2z}{(z-a)^3}\right) = \frac{6z}{(z-a)^4} = \frac{3!z}{(z-a)^{3+1}}$

The fourth derivative $\frac{d^4\chi\{a^k\}}{da^4} = \chi\left\{\frac{d^4\{a^k\}}{da^4}\right\} = \frac{d}{da}\left(\frac{6z}{(z-a)^4}\right) = \frac{24z}{(z-a)^5} = \frac{4!z}{(z-a)^{4+1}}$

Hence for nth derivative $\frac{d^n\chi\{a^k\}}{da^n} = \chi\left\{\frac{d^n\{a^k\}}{da^n}\right\} = \frac{d}{da}\left(\frac{d^{n-1}\{a^k\}}{da^{n-1}}\right) = \frac{n!z}{(z-a)^{n+1}}$ where $|z| > a$

Theorem: Z-transformation of the sequence $\chi\{k\}$, take the form

$\chi\{k\} == \chi\{0, 1, 2, 3 \ldots\ldots\ldots\ldots\} = \sum_{k=0}^{\infty}\frac{k}{z^k} = \frac{z}{(z-1)^2}$.

We can assume that the function $\chi\{bk\}$ as scalar multiplication, such that b is any integer

Thus $\chi\{bk\}$ can take the following form for different value of b. Let $b = 2$, then

$\chi\{2k\} == \chi\{0, 2, 4, 6 \ldots\ldots\ldots\ldots\} = 2\sum_{k=0}^{\infty}\frac{k}{z^k} = \frac{2z}{(z-1)^2}$

Therefore $\chi\{2k\} = 2\chi\{k\} = \frac{2z}{(z-1)^2}$

$b = 3$, then $\chi\{3k\} == \chi\{0, 3, 6, 9 \ldots\ldots\ldots\ldots\} = 3\sum_{k=0}^{\infty}\frac{k}{z^k} = \frac{3z}{(z-1)^2}$

Therefore $\chi\{3k\} = 3\chi\{k\} = \frac{3z}{(z-1)^2}$

Hence

$\chi\{bk\} = b\sum_{k=0}^{\infty}\frac{k}{z^k} = \frac{bz}{(z-1)^2}$

Let us consider z-transform for the sequence $x_{k=\{a,a^2,a^3,a^4,\ldots\ldots\ldots a^k\}}$, such that $0 \le k \le N-1$and a is real number.

To find, the general term of sequence is $x_k = a^k$, so z-transform as

$\chi\{x_k\} = \sum_{k=0}^{N-1} a^k z^{-k} = \frac{1-(a/z)^N}{1-a/z} = \frac{1.z^N - a^N}{z^{N-1}z-a}$ where $z \ne a$

Corollary: If the sequence $x_k = ka^k$, such that $n \in N$. Then z- transform for the sequences is $\chi\{ka^k\} = \frac{az}{(z-a)^2}$; $|z| > |a|$.

.

Proof: $\chi\{x_k\} = \chi\{ka^k\} = \sum_{k=0}^{\infty} \frac{ka^k}{z^k} = \sum_{k=0}^{\infty} ka^k z^{-k}$........... (1)

$\chi\{ka^k\} = \left(\frac{a}{z}\right)^1 + 2\left(\frac{a}{z}\right)^2 + 3\left(\frac{a}{z}\right)^3 + 4\left(\frac{a}{z}\right)^4 + \cdots$ (2)

By multiplying the equation (**) by $\left(\frac{a}{z}\right)$ and subtract from (*), we get

$\left(\frac{a}{z}\right)\chi\{ka^k\} = \left(\frac{a}{z}\right)^2 + 2\left(\frac{a}{z}\right)^3 + 3\left(\frac{a}{z}\right)^4 + 4\left(\frac{a}{z}\right)^5 + \cdots$

$\chi\{ka^k\} - \left(\frac{a}{z}\right)\chi\{ka^k\} =$

$\left(\left(\frac{a}{z}\right)^1 + 2\left(\frac{a}{z}\right)^2 + 3\left(\frac{a}{z}\right)^3 + 4\left(\frac{a}{z}\right)^4 + \cdots\right) - \left(\left(\frac{a}{z}\right)^2 + 2\left(\frac{a}{z}\right)^3 + 3\left(\frac{a}{z}\right)^4 + 4\left(\frac{a}{z}\right)^5 +\right)$

$=\sum_{k=1}^{\infty} \frac{a^k}{z^k} = \sum_{k=1}^{\infty} \left(\frac{a}{z}\right)^k$, thus $\left(1-\frac{a}{z}\right)\chi\{ka^k\} = \frac{a/z}{1-a/z} = \frac{a}{z-a}$, where $|z| > |a|$

Therefore $\chi\{ka^k\} = \frac{az}{(z-a)^2}$; $|z| > |a|$

IV. Z- TRANSFORM PROPERTIES

Z- transform has applicable properties in engineering filed , our observation in this section, there are many properties need to generalize as mathematician view in any problem specially that concerns with applications.

4.1 The Linearity property: If $\{x_k\}$, $\{y_k\}$ are sequences having z transforms X(z) and Y(z)respectively such that α and β are any constants, real or complex, then

$\chi\{\alpha x_k + \beta y_k\} = \alpha\chi\{x_k\} + \beta\chi\{y_k\} = \alpha X(z) + \beta Y(z)$

Proof:

By using the definition $\chi\{x_k\}_0^{\infty} = X(z) = \sum_{k=0}^{\infty} \frac{x_k}{z^k}$

Then $\chi\{\alpha x_k + \beta y_k\} = \sum_{k=0}^{\infty} \frac{\alpha x_k + \beta y_k}{z^k} = \alpha \sum_{k=0}^{\infty} \frac{x_k}{z^k} + \beta \sum_{k=0}^{\infty} \frac{y_k}{z^k} = \alpha X(z) + \beta Y(z)$

The intersection region for existence of the z-transform, in the z plane, the linear sum will be the intersection of the regions of existence (the common region) of the individual z-transforms X(z) and Y(z) as proved in [3].

Thus we can generalize the above theorem to many functions, as follow

If $\{x_k\}$, $\{y_k\}$ … … … … … … $\{h_k\}$, …. are sequences having z -transforms X(z) ,Y(z) … … … …. and H(z) respectively and if α, β ....and σ are any constants, real or complex, then

$\chi\{\alpha x_k + \beta y_k + \cdots + \sigma h_k\} = \alpha\chi\{x_k\} + \beta\chi\{y_k\} + \cdots \sigma\chi\{h_k\} + \cdots = \alpha X(z) + \beta Y(z) + \cdots + \sigma H(z) + \cdots$

Proof:

By using the definition $\chi\{x_k\}_0^{\infty} = X(z) = \sum_{k=0}^{\infty} \frac{x_k}{z^k}$

$\chi\{\alpha x_k + \beta y_k + \cdots + \sigma h_k + \cdots\} = \sum_{k=0}^{\infty} \frac{\alpha x_k + \beta y_k + \cdots + \sigma h_k + \cdots}{z^k}$

$= \alpha \sum_{k=0}^{\infty} \frac{x_k}{z^k} + \beta \sum_{k=0}^{\infty} \frac{y_k}{z^k} + \cdots + \sigma \sum_{k=0}^{\infty} \frac{h_k}{z^k} + \cdots = \alpha X(z) + \beta Y(z) + \cdots \sigma H(z) + \cdots$

Here above the intersection region for existence of the z-transform, in the z plane, and linear sum will be the intersection of the regions of existence (the common region) of the individual z −transforms with X(z), Y(z), … … … … … …. and H(z).

Example: The continuous -time function $f(t) = \sin\omega t H(t)$, where ω a constant , is sampled in the idealized sense at interval T to generate the sequence $\sin k\omega T$.

Determine the z-transform of the sequence above.

Solution: The function, $\sin k\omega T = \frac{1}{2}(e^{ik\omega T} - e^{-ik\omega T})$, by using linearity property. We get, $\chi\{\sin k\omega T\} = \chi\{e^{ik\omega T} - e^{-ik\omega T}\} = \frac{1}{2}\chi\{e^{ik\omega T}\} - \frac{1}{2}\chi\{e^{-ik\omega T}\}$

Since $|e^{ik\omega T}| = |e^{-ik\omega T}| = 1$, this implies

$\chi\{\sin k\omega T\} = \frac{1}{2}\frac{z}{z-e^{i\omega T}} - \frac{1}{2}\frac{z}{z-e^{-i\omega T}}$, where $(|z| > 1)$

Which gives z- transform pair

$\chi\{\sin k\omega T\} = \frac{1}{2}\frac{z(z-e^{-i\omega T}) - z(z-e^{i\omega T})}{z^2 - z(e^{i\omega T} + e^{-i\omega T}) + 1} = \frac{z\sin(\omega T)}{z^2 - 2z\cos(\omega T) + 1}$, $(|z| > 1)$

4.2 Time delay(First shift property): The a shifted delay version of the sequence $\{x_k\}$ denoted by $\{y_k\}$, with $y_k = x_{k-k_0}$.

Here k_0 is the number of steps in the delay, such that $k_0 = 3$,then $y_k = x_{k-3}$, therefore $y_0 = x_{-3}$; $y_1 = x_{-2}$; $y_2 = x_{-1}$; $y_3 = x_0$ and $y_4 = x_1$

and so on .So that the sequence $\{y_k\}$ is simply the sequence $\{x_k\}$ moved backwards or delayed, by three steps. From the first definition of z-transform.

$$\chi\{y_k\} = \sum_{k=0}^{\infty} \frac{y_k}{z^k} = \sum_{k=0}^{\infty} \frac{x_{k-k_0}}{z^k} = \sum_{p=-k_0}^{\infty} \frac{x_p}{z^{p+k_0}}$$

Where we have written $p = k - k_0$. If $\{x_k\}$ is a causal sequence, so that

$x_p = 0 (p < 0)$, then $\chi\{y_k\} = \sum_{p=0}^{\infty} \frac{x_p}{z^{p+k_0}} = \frac{1}{z^{k_0}} \sum_{p=0}^{\infty} \frac{x_p}{z^p} = \frac{1}{z^{k_0}} X(z)$

It's clearly that $X(z)$ is the z-transform of $\{x_k\}$, so this lead as to the following result

$$\chi\{x_{k-n}\} = \frac{1}{z^n} \chi\{x_k\}$$

Example: Find $\chi\{x_{k-5}\}$, such that $\chi\{a^k\} = \frac{z}{z-a}$, where $(|z| > 5)$.

Solution: Here we have, $k_0 = 5 \Rightarrow \chi\{x_{k-5}\} = z^{-5}\chi\{a^k\} = z^{-5}\frac{z}{z-a} = \frac{1}{z^5 - az^4}, (|z| > 5)$.

4.3 Time advance (Second shift property): The shifted advance version of the sequence $\{x_k\}$ denoted by $\{y_k\}$, with

$y_k = x_{k+k_0}$. Here k_0 is the number of steps in the advance , such that $k_0 = 3$,then $y_k = x_{k+3}$, therefore $y_0 = x_3$; $y_1 = x_4$; $y_2 = x_5$; $y_3 = x_6$ and $y_4 = x_7$

Theorem: If $\{y_k\}$ is the single-step advanced version of the sequence $\{x_k\}$, then $\{y_k\}$is generated by $\chi\{x_{k+k_0}\} = z^{k_0}\chi\{x_k\} \quad \forall\, k \epsilon N$

Proof: By induction $\chi\{y_k\} = \sum_{k=0}^{\infty} \frac{y_k}{z^k} = \sum_{k=0}^{\infty} \frac{x_{k+1}}{z^k} = z \sum_{k=0}^{\infty} \frac{x_{k+1}}{z^{k+1}}$

and putting $p = k + 1$, it gives

$$\chi\{y_k\} = z \sum_{p=1}^{\infty} \frac{x_p}{z^p} = z\left(\sum_{p=0}^{\infty} \frac{x_p}{z^p} - x_0\right) = zX(z) - zx_0.$$

Where $X(z)$is z -transform of

For z-transform to time advance sequence $\{x_k\}$,therefore we can deduce the following result

$$\chi\{x_{k+1}\} = zX(z) - zx_0.$$

Similarly, we could extract a two -step advance sequence $\{x_{k+2}\}$

$$\chi\{x_{k+2}\} = z^2X(z) - z^2x_0 - zx_1.$$

Generally for k_0-step advance sequence $\{x_{k+k_0}\}$, z-transform can be formed as

$$\chi\{x_{k+k_0}\} = z^{k_0}X(z) - \sum_{j=0}^{k_0-1} x_j\, z^{k_0-j}$$

V. RESULT

In this work, we actually found the importance and meaningful of mathematics in our daily lives, it plays an important role as interpreter between nature and natural phenomena.

As an inevitable consequences of the study, we drive the general rule for differentiation of z-transformation and generalize some z-transform properties, such as linearity properties of z- transformation for n functions.

VI. CONCLUSION

In this study, the significance of z- transform has been introduced especially in signal processing. The relationship between mathematical tools (Fourier, Laplace and Z-transform) established.

The most general objectives of the study have been achieved.

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Abdulaziz B.M.HameD " Deduce The nth Derivative And Generalize Some Properties of Z-Transformations." American Journal of Engineering Research (AJER) 6.7 (2017): 310-314