

Reactive Energy Generated By High Voltage Lines. A Calculation Methodology

Alphonse Omboua

Doctor of Applied Sciences from the University of Liège; Professor at the University of Congo Brazzaville.

Abstract: Problems related to capacitive effects on high voltage lines remain subject to several assumptions and continue to be poorly circumscribed. The calculation of the capacitance matrices, the predetermination of the reactive powers generated by the capacitors between the various conductors of the line, are not yet unanimous and continue to pose problems of methodology. Of course, the reactive energies generated by high-voltage power lines remain topical issues of concern. The relationships between the discrete capacitances (C_{ij}) relating to the line geometry and the equivalent capacitance (C) are to be established. This article presents a methodology for calculating the reactive energies generated by a high voltage line, after determining the capacitances between the different conductors, it gives a clarification on the location of the reactive energies generated by the various capacitors.

Keys Words: Capacitive effect, Reactive energy, High voltage line

I. INTRODUCTION

Impacts related to the reactive energy generated by high-voltage power lines continue to occupy the debates on the transmission of electricity. The capacitors relating to the geometries of the lines generate reactive energy whose localization deserves a clarification. These capacitors are numerous:

- The capacitors between the different conductors of the line between them;
- Between the different conductors of the line and the plan of the ground;
- Between the active conductors and the guard cables

Because of these linear capacitances, the high voltage line generates reactive energy in considerable quantity. This capacitive effect is often detrimental in the operation of high voltage networks, because of the too much reactive energy it generates in the lines: this is one of the causes of overvoltages in power networks. [1]

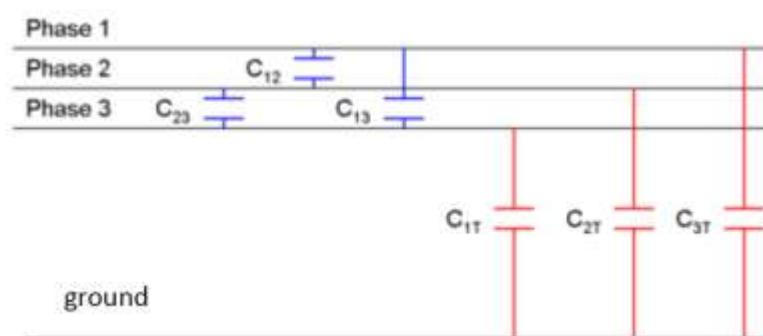


Fig.1 Capacitors in a three-phase system

This paper presents the calculation of the discrete capacitances (C_{ij}), according to the geometry of a line in order to deduce the equivalent capacity (C) to be taken into account in the equivalent T or π diagrams of a high voltage line. Several calculation methods are proposed in the literature (matrix of potential coefficients, matrix of capacities, [2] etc.) but there is always a problem of clarity and logic.

II. THE CAPACITORS AND THE CAPACITIVE EFFECT

In general, a capacitor is the assembly formed by two facing surfaces and which are in electrostatic influence; these surfaces are called reinforcements and are generally close to one another, separated by an

insulator called a dielectric. Let a capacitance capacitor (C) subjected to an alternating voltage $u(t)$ and crossed by an intensity $i(t)$ and $(+q)$, $(-q)$ the electrical charges on each armature.

We have $q=Cu$ with $q = \int(i)dt$ (1)

In complex notation, we have $\bar{I} = I_m e^{j\omega t}$ and (2)

$$\bar{q} = \int I_m e^{j\omega t} dt = \frac{I_m}{j\omega} e^{j\omega t} \quad (3)$$

$$\text{So } \bar{U} = \frac{\bar{q}}{C} = \frac{I_m}{jC\omega} e^{j\omega t} \quad (4)$$

$$= \frac{I_m}{C\omega} (-j) e^{j\omega t} = \frac{I_m}{C\omega} e^{-j\pi/2} \cdot e^{j\omega t} \quad (5)$$

$$\bar{U} = \frac{I_m}{C\omega} e^{j(\omega t - \frac{\pi}{2})} \quad (6)$$

We see that the voltage $u(t)$ is behind $(-\pi/2)$ with respect to the intensity $i(t)$.

In the case of a pure coil, this phase shift would be $(+\frac{\pi}{2})$.

Phase shift	φ	Interpretation
Capacitor	$-\pi/2$	The capacitor generates reactive energy
Coil	$+\pi/2$	The coil consumes reactive energy

It is then understood that the coil consumes reactive energy while the capacitor itself generates reactive energy in the circuit which is in parallel with it. The capacitor is therefore a reactive energy generator and the capacitive effect is therefore related to a capacitor equivalent energized.

A capacitor (C), subjected to the voltage U and crossed by a current of current I, generates reactive energy Q_0 :

$$Q_0 = XI^2 = \frac{1}{C\omega} \cdot I^2 \text{ with } I = \frac{U}{X} = C\omega U, \text{ so } Q_0 = \frac{1}{C\omega} \cdot (C\omega)^2 \cdot U^2 = C\omega U^2 \quad (7)$$

III. POWER GENERATED BY CAPACITORS

To better understand the situation of power lines, we consider the following diagram of a circuit stopper. Here, $L\omega = 1/C\omega$.

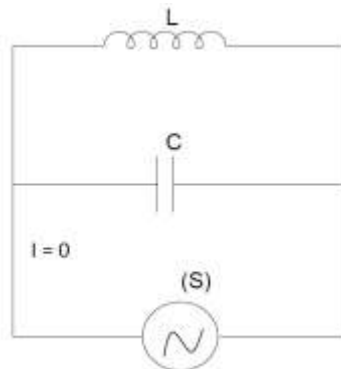


Fig.2. Example of a circuit stopper

In this case where $1/C\omega = L\omega$, the capacitor becomes an energy source for the coil because the main current that comes from the source (S) is zero [$i(t) = 0$] [3].

In this circuit, the reactive energy generated by the capacitor of capacitance C is entirely consumed by the inductance coil L, and it appears however that the source (S) is at rest. This example makes us understand that the capacitor sends its energy on the impedance which is in parallel with it; this allows us to deduce that the various capacitors formed by the conductors of the lines generate undesirable reactive energy on the windings of the transformers at the starting (A_1) and arrival (A_2) stations, ie $Q/2$ in A_1 and $Q/2$ in A_2 . In the case of high voltage lines, the capacitor existing between two respective conductors (k) and (j) for example generates the reactive energy Q_C at the terminals of the impedances Z_{kj} which would be placed between these two conductors at the stations of departure (A_1) and of arrival (A_2).

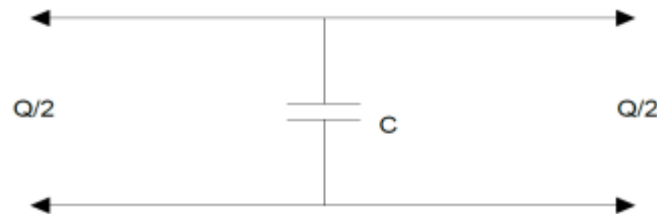


Fig 3 : Capacitive effect between two conductors of a line

If (j) and (k) are two phase conductors, for example, the impedance Z_{kj} is nothing but the impedance of the winding of a transformer of the starting station A_1 or of the arrival station A_2 and the voltage U_{jk} is equal U : the voltage of the line (The impedance of the line is neglected before that of the winding of the transformers in the stations).The energy generated by the capacitance capacitor C_{jk} is: $Q_{jk} = C_{jk} \cdot \omega \cdot (U_{jk})^2$, half of which will be reflected on the windings of each of the transformers of the stations A_1 and A_2 . Thus, the reactive energy generated by the respective capacitors of the line is such that:

$$Q_{12} = C_{12} \omega (U)^2, \quad Q_{13} = C_{13} \omega (U)^2, \quad Q_{23} = C_{23} \omega (U)^2 \quad (8)$$

At the starting station A_1 , this energy of the capacitors of the line "attacks" the secondary windings of the transformer while at the arrival station A_2 , this reactive energy "attacks" the primary windings of the transformer of the station.

IV. LOCALIZATION OF THE REACTIVE ENERGIES GENERATED BY THE VARIOUS CAPACITORS OF A POWER LINE

In the configuration of the high voltage lines, the various capacitors constituted by the conductors of the line appear as sources of reactive energy supplying respectively impedances which are in parallel with them. Our interest is focused on the impedances that exist between the three phase conductors of the line, as shown in the figure below:

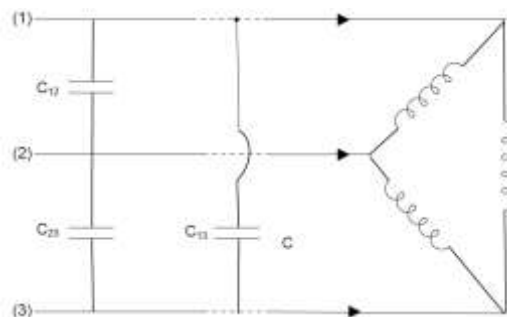


Fig 4 : Example of transformer windings in triangle

The reactive energies generated by the various capacitors between the phase conductors and the ground are located between the phases and the ground and therefore do not present any major impacts to the electrical network; the same applies to the capacitor energies between the phases and the guard cables which are connected to ground through the pylons. Only the energies generated by the capacitors (C_{12} , C_{13} , C_{23}) between active conductors interest us because these energies arrive on the windings of the transformers of the starting stations A_1 and the arrival stations A_2 of the line. This energy which arrives at the stations A_1 and A_2 is $Q/2$ with $Q = Q_{12} + Q_{13} + Q_{23} = (C_{12} + C_{13} + C_{23}) \omega (U)^2 \quad (9)$

In the case of networks with low reactive energy consumption, this energy generated by the capacitive effect can generate overvoltages because the capacitors C_{12} , C_{13} and C_{23} send this energy directly to the windings of the transformers of the stations A_1 and A_2 .

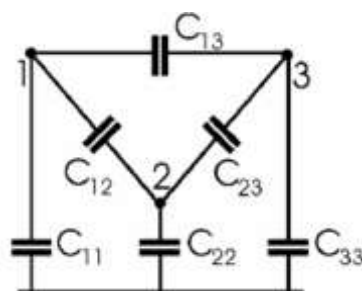


Fig 5 Capacitors in the case of a single line

V. EQUIVALENT CAPACITOR OF A LINE

For the purposes of modeling a power line in (T) or in (π), [3]the value of the capacitance C to be taken into account is that which would produce the same capacitive effect as the capacitors of respective capacitances C₁₂, C₁₃, C₂₃, such as: (C₁₂ + C₁₃+C₂₃) ω (U)² =3 C ω (U)²(10)

Either by phase C = (C₁₂ + C₁₃+ C₂₃)/3 with Y= Cω(11)

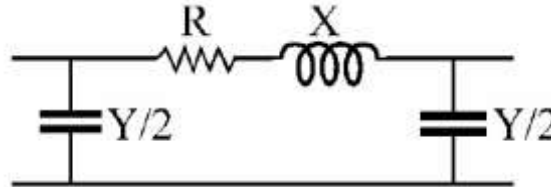


Fig.6. A model in π of a power line

VI. CALCULATION OF CAPACITORS BETWEEN PARALLEL CONDUCTORS- CASE OF POWER LINE CONDUCTORS

For the determination of capacitances between parallel conductors, electrostatic problems must be solved by applying the direct calculation methods. Consider two rectilinear conductors A and B of respective radii r₁ and r₂, which are parallel to one another and whose lengths are very great with respect to their distance D and whose cross-section is very small compared to their spacing and perpendicular to the plane of the figure.

For the calculation of capacitances of capacitors, the conductors can be assimilated as carriers of electric charges uniformly distributed with the charge density λ = q in coulombs per meter; we must assume that one has a charge density (+q) and the other (-q).

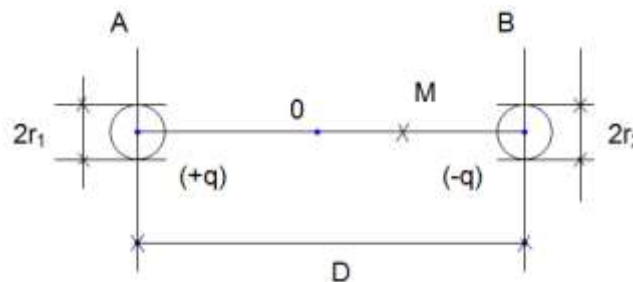


Fig7.Two conductors of a power line

The electric field created at a distance (d) by a linear electrical charge of density λ is E = λ / 2πε₀d (see electrostatic course). Let $\vec{OM} = x$

The electrostatic field created by conductor A at point M is $\vec{E}_A M = \frac{q}{2\pi\epsilon_0(\frac{D}{2}+x)} \vec{i}$ and that created by conductor B

is $\vec{E}_B M = \frac{q}{2\pi\epsilon_0(\frac{D}{2}-x)} \vec{i}$ and $\vec{E}_M = \vec{E}_A M + \vec{E}_B M$

$$\vec{E}_M = \frac{q}{2\pi\epsilon_0} \left(\frac{1}{(\frac{D}{2}+x)} + \frac{1}{(\frac{D}{2}-x)} \right) \vec{i} \tag{12}$$

The potential difference $V_{AB} = V_A - V_B$ is then such that

$$V_A - V_B = \int_A^B E_M dx \text{ and } V_A - V_B = \frac{q}{2\pi\epsilon_0} \int_A^B \left(\frac{1}{(\frac{D}{2}+x)} + \frac{1}{(\frac{D}{2}-x)} \right) dx \tag{13}$$

$$= \frac{q}{2\pi\epsilon_0} \left[\log \left(\frac{D}{2} + x \right) - \log \left(\frac{D}{2} - x \right) \right]_A^B = \frac{q}{2\pi\epsilon_0} \left[\log \left(\frac{\frac{D}{2}+x}{\frac{D}{2}-x} \right) \right]_A^B \tag{14}$$

At point A	$x_A = -\frac{D}{2} + r_1$
At point B	$x_B = \frac{D}{2} - r_2$

$$V_A - V_B = \frac{q}{2\pi\epsilon_0} \left(\log \left(\frac{\frac{D}{2} + \frac{D}{2} - r_2}{\frac{D}{2} - \frac{D}{2} + r_2} \right) - \log \left(\frac{\frac{D}{2} - \frac{D}{2} + r_1}{\frac{D}{2} + \frac{D}{2} - r_1} \right) \right) \quad (15)$$

$$= \frac{q}{2\pi\epsilon_0} \left[\log \left(\frac{D-r_2}{r_2} \right) - \log \left(\frac{r_1}{D-r_1} \right) \right] \quad (16)$$

$$= \frac{q}{2\pi\epsilon_0} \log \left(\frac{D-r_2}{r_2} \times \frac{D-r_1}{r_1} \right) \quad (17)$$

$$V_A - V_B = \frac{q}{2\pi\epsilon_0} \log \left(\frac{D(1-\frac{r_2}{D})}{r_1 r_2} \right) \quad (18)$$

$$= \frac{q}{2\pi\epsilon_0} \log \frac{D^2}{r_1 r_2} \left(1 - \frac{r_1}{D} \right) \left(1 - \frac{r_2}{D} \right) \quad (19)$$

$$= \frac{q}{2\pi\epsilon_0} \log \frac{D^2}{r_1 r_2} \left(1 - \frac{r_1}{D} - \frac{r_2}{D} + \frac{r_1 r_2}{D^2} \right) \quad (20)$$

here : $r_1 \ll D$ and $r_2 \ll D$ so $1 - \frac{r_1}{D} - \frac{r_2}{D} + \frac{r_1 r_2}{D^2} \sim 1$ and so

$$V_A - V_B \approx \frac{q}{2\pi\epsilon_0} \log \left(\frac{D^2}{r_1 r_2} \right) \text{ then } V_{AB} \approx \frac{q}{2\pi\epsilon_0} \log \left(\frac{D^2}{r_1 r_2} \right) \quad (21)$$

Considering A and B as the armatures of a capacitor, one can write $q = CV_{AB}$

$$\text{so } C = \frac{q}{V_{AB}} = \frac{2\pi\epsilon_0}{\log \left(\frac{D^2}{r_1 r_2} \right)} \quad (22)$$

The capacitance of the capacitor between two parallel, non-identical conductors n^0_1 and n^0_2 of radii $r_1 \neq r_2$, separated from the distance D is then:

$$C_{12} = \frac{\pi\epsilon_0}{\log \left(\frac{D}{\sqrt{r_1 r_2}} \right)} \quad (23)$$

If the conductors are identical, we have $r_1 = r_2 = r_3 = r$ (case of the phase conductors of a line), then

$$C = \frac{\pi\epsilon_0}{\log \left(\frac{D}{r} \right)} \text{ in Farads /m} \quad (24)$$

D: spacing between the two conductors and (r) the radius of the conductor

VII. CALCULATING THE CAPACITOR BETWEEN A PARALLEL CYLINDRICAL CONDUCTOR TO A PLAN - CASE OF ELECTRIC LINE CONDUCTORS WITH THE SOIL PLAN

In the case of two parallel cylindrical conductors (FIG. 8), the mediator plan (P) is an equipotential surface.

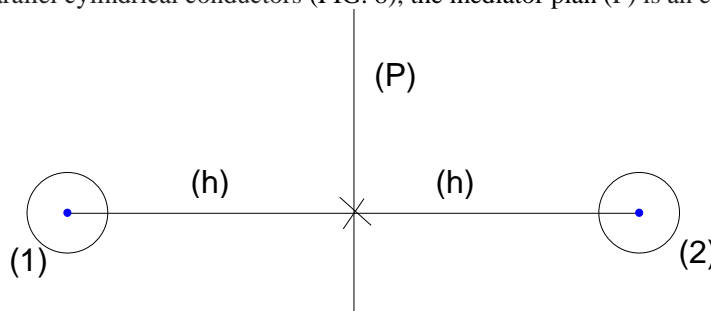


Fig.8 The mediator plan between two conductors

The situation is that of two capacitors in series: the first capacitor is formed by the conductor n^0_1 and the mediator plane (P) and the second capacitor constituted by the conductor n^0_2 and the mediator plan (P).

Let C_{1P} and C_{2P} respectively be the capacitances of the capacitors formed by the mediator plan and the conductors n^0_1 and n^0_2 . Considering the case of the two identical conductors, we have $C_{1P} = C_{2P} = C_0$

We write the relation of two capacitors in series of equivalent capacitance C .

Let $C = \frac{\pi\epsilon_0}{\log(\frac{D}{r})}$ With : $\frac{1}{C} = \frac{1}{C_{1P}} + \frac{1}{C_{2P}} = \frac{2}{C_0}$ so $C_0 = 2C = \frac{2\pi\epsilon_0}{\log(\frac{D}{r})}$ is the capacitance of the capacitor formed by a cylindrical conductor of radius (r) parallel to a plan at the distance $h = D / 2$. [4]. So $C_{1P} = C_{2P} = C_0 = \frac{2\pi\epsilon_0}{\log(\frac{2h}{r})}$ in Farad/m

$$C_0 = \frac{2\pi\epsilon_0}{\log\left(\frac{2h}{r}\right)} \quad \text{In Farads/m} \quad (25)$$

In the case of power lines, (h) is the distance between the conductor and a horizontal plan (in this case, the ground plan) and (r) is the radius of the conductor.

$\epsilon_0 = 8,841941 \cdot 10^{-12}$ F/m, $\epsilon_r \approx 1$, the relative permittivity of the air

VIII. APPLICATIONS

Most of the high voltage lines in African countries are in 220 kV. They have the following configuration [5]:

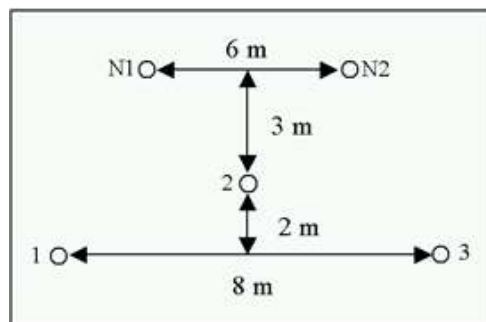


Fig 9: Configuring of 220 kV HV line

n^0_1 , n^0_2 and n^0_3 are the phase conductors, N_1 is the first guard cable and N_2 is the second guard cable that incorporates optical fiber. The n^0_1 and n^0_3 phases are located fourteen (14) meters from the ground; other dimensions are shown in the figure above (Fig 9). The cross-section of the phase conductors is 570 mm^2 , giving an average radius of $r = 13.46 \text{ mm}$ and the calculations give: $C_{12} = C_{23} = 10,7 \text{ nF/km}$ and $C_{13} = 9,7 \text{ nF/km}$ therefore $C = 10,36 \text{ nF/km}$ and the reactive energy generated by this line is: $Q = Q_{12} + Q_{13} + Q_{23} = (C_{12} + C_{13} + C_{23}) \omega (U)^2 = 3 C \omega (U)^2 = 0,472 \text{ MVAR/km}$

In general, 80% of this reactive power must be compensated by the reactances for powering up the system.

IX. CONCLUSIONS

From this study, we developed a methodology for determining the different capacitances constituting the geometry of a high voltage line:

- Capacitances between the various active conductors of the line (C_{12} , C_{13} , C_{23});
- Capacitances between phase conductors and guard cables;
- Capacitances between the phase conductors and the soil plan (C_{1S} , C_{2S} , C_{3S});
- Capacitances between the guard cables and the soil plan (C_{GS})

This results in a calculation of the reactive energy generated by a high voltage line and the equivalent capacitance C to be taken into account in the modeling schemes of the power lines.

REFERENCES

- [1]. Jignesh.Parmar- Electrical notes and articles- March 21,2011 - <https://electricalnotes.wordpress.com/2011/03/21/importance-of-reactive-power-for-system>
- [2]. Calculation of the R,L,C characteristics of a three-phase junction - Transport and Distribution of Electrical Energy - University of Liège - Practical work manual - p1.3 + p1.20 - <http://www.tdee.ulg.ac.be/userfiles/file/1.pdf>
- [3]. Gilbert Gastebois: gilbert.gastebois.pagespersoorange.fr/java/rlc/bouchon/theorie_bouchon.htm
- [4]. EE 740 – Transmission Lines -Spring 2013 <http://www.egr.unlv.edu/~eebag/TRANSMISSION%20LINES.pdf>
- [5]. Ph.D. Alphonse Omboua –2002- University of Liege - Belgium . p35. <http://www.tdee.ulg.ac.be/userfiles/file/these-omboua.pdf>