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Strain Based Finite Element for Analysis of Cylindrical Shell Dam

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ABSTRACT: *A new cylindrical rectangular finite element is developed in this paper. The element has six nodal degrees of freedom at each of the four corner nodes. The displacement fields of the element satisfy the exact requirements of rigid body modes of motion. Shallow shell formulation is used and the element is based on an independent strain assumption insofar as it is allowed by the compatibility equations. A cylindrical shell problem for which a previous solution exists is first analyzed using the new element to test the efficieny of the element. The element is then used in the analysis of cylindrical shell dam structure .Results obtained by the present element are compared with those a vailable in the previous solution .These comparisions show that an efficient and accurate result can be obtained by using the present element . The distribution of various components of stresses is obtained and the effect of the central angle of the stiffness of the cylindrical shell dam is investigated also to give the designer an insight for the behavior of such structures*

Keywords: *Strain-based, cylindrical rectangular, Finite element.*

I. INTRODUCTION

Considerable attention has been given to applying the finite element method in the analysis to curved structures. The early work on the subject was presented by Grafton and Strome (1963) who developed conical segments for the analysis of shells of revolution. Jones and Strome (1966) modified the method and used curved meridional elements which were found to lead to considerably improved results for the stresses. Further research led to the development of curved rectangular and cylindrical elements (Brebbia*et al*, 1967; Bogner *et al*, 1967; Cantin *et al*, 1968 and Sabir *et al*, 1972). Several shell elements have been developed to analyze shell structures of different shape ,except the last one , all the previous elements are developed using the displacement formulationresulting in an improvement of the accuracy of the results. However, this improvement is achieved at the expense of more computer time as well as storag assemble the overall structure matrix. At the same time a new approach of elements was developed at Cardiff University ,Referred to as

The strain based approach .. This approach is based on determining the exact terms representing all the rigid body modes together with the displacement functions representing the straining of the element by assuming independent strain functions insofar as it is allowed by the compatibility equations. This approach has successfully employed in the development ofdifferent curved shell elements such as cylindrical , conical ,spherical ,hyperbolic elements (Ashwell *et al*, 1971, 1972; Sabir *et al*, 1975, 1982, 1983, 1987,1988 ,1996; El-Erris, 1987,1994, 1997; Bouzegari ,1988; Djoudi et al ,2003 ,2004, Hamadi et al 2007 and Mousa, 1994, 1998, 2015). . The results obtained from some examples show that these elements show the effiency and the performance of the strain based approach compared to the well known displacement formulation (Hamadi et al 2014) . The strain-based approach is employed in the present paper to develop a new rectangular cylindrical element having six degrees of freedom at each corner node, five of which are essential external degrees of freedom and the additional sixth is associated with the in-plne rotation of shell.The new element is first tested by applying it to the analysis of a clamped barrel vault for which a previous solution exists. The work is then extended to the analysis of a cylindricalshell dam structure .The distribution of the various components of stresses is obtained .The effect of the central angle of stiffness of the dam is investigated to give designers an insight into the behavior of such structures.

II. THEORETICAL CONSIDERATION FOR DEVELOPMENT OF DISPLACEMENT FUNCTIONS FOR THE NEW RECTANGULAR CYLINDRICAL SHEL ELEMENT

1.1. *Theoretical considerations*

A rectangular shallow cylindrical shell element and the associated curvilinear coordinates are shown in Fig. 1.

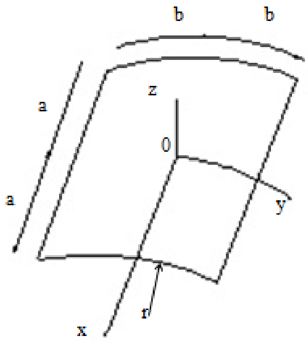


Fig. 1. Coordinate axes for rectangular cylindrical element

For the shown system of curvilinear coordinates, the simplified strain displacement relationship for the cylindrical shell elements can be written as:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{r}\ ,\ \varepsilon_y = \frac{\partial v}{\partial y},\ \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$
$$k_x = \frac{\partial^2 w}{\partial x^2}\ ,\ k_y = \frac{\partial^2 w}{\partial x^2},\ k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \qquad (1)$$

Where u, v and w = displacements in the x, y and z axes; $\varepsilon_x, \varepsilon_y$ and ε_{xy} = in-plane direct and shearing strains; k_x, k_y, and k_{xy} = the changes in direct and twisting curvatures; and r = the principal radius of curvature.
The above six components of strain can be considered independent, as they are a function of the three displacements u, v & w and must satisfy three additional compatibility equations. These compatibility equations are derived by eliminating u, v and w from Eq.(1), hence, they are:

$$\partial^2 \varepsilon_x / \partial y^2 + \partial^2 \varepsilon_y / \partial x^2 - \partial^2 \varepsilon_{xy} / \partial x \partial y + k_y / r = 0$$
$$\partial^2 k_{xy} / \partial x - 2\partial k_x / \partial y = 0 \qquad (2)$$
$$\partial k_{xy} / \partial y - 2\partial ky / \partial x = 0$$

In order to keep the element as simple as possible and to avoid the difficulties associated with internal non-geometric degrees of freedom, the developed element should possess six degrees of freedom at each of the four corner nodes: u, v, w, θ_x, θ_y and ϕ. Thus, the shape functions for a rectangular element should contain twenty four independent constants.
To obtain the displacement fields due to rigid body movements, all the six strains given by Equation 1 are equated to zero and the resulting partial differential equations are integrated to yield:

$$u = -a_1 \frac{x}{r} - a_2\left(-\frac{x^2}{2r}\right) + a_3 \frac{xy}{r} + a_4 + a_6 y$$
$$v = -a_1 \frac{y}{r} - a_3\left(\frac{x^2}{2r}\right) + a_5 - a_6 x \qquad (3)$$
$$w = -a_1 + a_2 x + a_3 y$$

These displacement fields are due to the six components of the rigid body displacements and are represented in terms of the constants a_1 to a_6. If the element has six degrees of freedom for each of the four corner nodes, the displacement fields should be represented by twenty four independent constants. Having used six constants for the representation of the rigid body modes, the remaining eighteen constants are available for expressing the displacements due to the strains within the element. These constants can be apportioned among the strains in several ways. For the present element, the following way is proposed:

$$\begin{aligned}
\varepsilon_x &= a_7 + a_8 y + a_{21} y^2 + 2a_{22} x y^3 - \frac{1}{r}\left[a_{16}\frac{y^2}{2} + a_{17}\frac{xy^2}{2} + a_{18}\frac{y^3}{6} + a_{19}\frac{xy^3}{6}\right]\\
\varepsilon_y &= a_9 + a_{10} x - a_{21} y^2 - 2a_{22} x y^3\\
\varepsilon_{xy} &= a_{11} + 2a_{22} x + 2a_{24} xy\\
k_x &= a_{12} + a_{13} x + a_{14} y + a_{15} xy\\
k_y &= a_{16} + a_{17} x + a_{18} y + a_{19} xy\\
k_{xy} &= a_{20} + [2a_{14} x + a_{15} x^2 + 2a_{17} y + a_{19} y^2]
\end{aligned} \qquad (4)$$

Equation 4 is derived by first assuming the un-bracketed terms and adding the terms between brackets to satisfy the compatibility condition “Eq.(2)”. It is then equated to the corresponding expressions in terms of *u*, *v*&*w* from Eq.(1) and the resulting equations are integrated to obtain:

$$\begin{aligned}
u &= a_7 x + a_8 xy - a_{10}\frac{y^2}{2} + a_{11}\frac{y}{2} + a_{12}\frac{x^3}{6r} + a_{13}\frac{x^4}{24r} + a_{14}\frac{x^3 y}{6r} + a_{15}\frac{x^4 y}{24r} + a_{20}\left(\frac{x^2 y}{4r}\right) a_{21} xy^2 + a_{22} x^2 y^3 + a_{24} y^2\\
v &= -a_8\frac{x^2}{2} + a_9 y + a_{10} xy + a_{11}\frac{x}{2} - a_{14}\frac{x^4}{24r} - a_{15}\frac{x^5}{120r} + a_{20}\left(-\frac{x^3}{12r}\right) - a_{21} x^2 y - a_{22} x^3 y^2 + a_{23} x^2\\
w &= -a_{12}\frac{x^2}{2} - a_{13}\frac{x^3}{6} - a_{14}\frac{x^2 y}{2} - a_{15}\frac{x^3 y}{6} - a_{16}\frac{y^2}{2} - a_{17}\frac{xy^2}{2} - a_{18}\frac{y^3}{6} - a_{19}\frac{xy^3}{6} - a_{20}\frac{xy}{2}
\end{aligned} \qquad (5)$$

The complete displacement functions for the element are obtained by adding the corresponding expressions for *u, v &w* from Equations 3 and 5. The translational degrees of freedom for the element are *u, v, w*. The three rotations about the *x, y & z* axes are given by:

$$\begin{aligned}
\theta_x &= \frac{\partial w}{\partial y} = a_3 - a_{14}\frac{x^2}{2} - a_{15}\frac{x^3}{6} - a_{16} y - a_{17} xy - a_{18}\frac{y^2}{2} - a_{19}\frac{xy^2}{2} - a_{20}\frac{x}{2}\\
\theta_y &= -\frac{\partial w}{\partial x} = -a_2 + a_{12} x + a_{13}\frac{x^2}{2} + a_{14} xy + a_{15}\frac{x^2 y}{2} + a_{17}\frac{y^2}{2} + a_{19}\frac{y^3}{6} + a_{20}\frac{y}{2}\\
\phi &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = a_6 - a_8 x + a_{10} y - a_{14}\frac{x^3}{6r} - a_{15}\frac{x^4}{24r} - a_{20}\left(\frac{x^2}{4r}\right) - 2a_{21} xy - 3a_{22} x^2 y^2 - a_{23} x - a_{24} y
\end{aligned} \qquad (6)$$

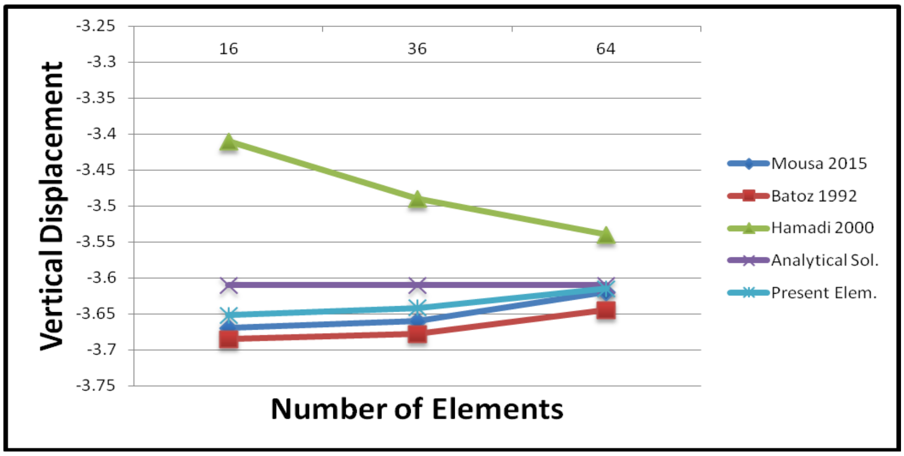
The Stiffness matrix [*K*] for the shell element is then calculated in the usual manner, i.e.,

$$[K] = [C^{-1}]^T \left\{\iiint [B^T] D B\, dv\right\} [C^{-1}] \qquad (7)$$

where *B* and *D* = the strain and elasticity matrices, respectively; and*C* = the matrix relating the nodal displacements to the constant a_1 to a_{24}.*B* can be calculated from equations (1), (3), (4) and D is given by substituting the matrices *B* and *D* into equation (5). The integration within the bracketed terms of equation (5) are carried out explicitly and the rest are computed to obtain the stiffness matrix [*K*].

III. PATCH CONVERGENCE TEST

This test is to be considered which is frequently used to test the performance of the shell elements is that of Scordelis-Lo Roof having the geometry as shown in Fig. (2). The shell has the following dimensions and material properties: thickness, t= 0.03 m, R= 3m, L = 6m, $\propto$= 40°, modulus of elasticity, E= 3 Pa, Poisson's ratio, μ= 0.0, density, ρ = 0.625 Pa. The straight edges are free while the curved edges are supported on rigid diagrams along their plan considering the symmetry of the problem only one quarter of the roof is analyzed.



The results obtained by the new present element for the vertical displacement at the midpoint B of the free edge and the center C of the roof are compared to other kinds of shells elements (Batoz *et al*, 1992 and Hamadi *et al*, 2000). The analytical solution of this problem is based on the shallow shell theory is given by Scordelis and Lo (1969). Convergence curves in Fig.3) show that the convergence of the present element faster convergence than the other. Then the new present element would be more efficient to use it in the analysis of proposed cylindrical shell under loads.

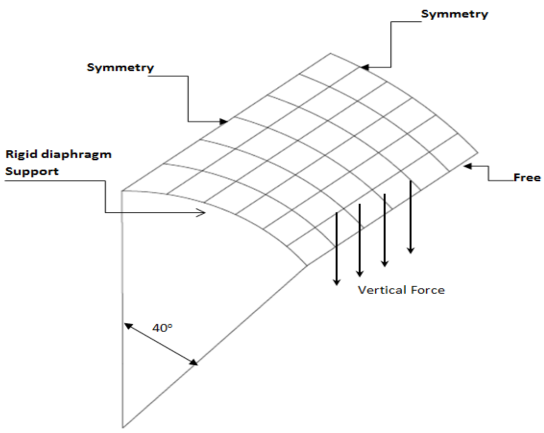


Fig.2 Geometry and finite element mesh of roof

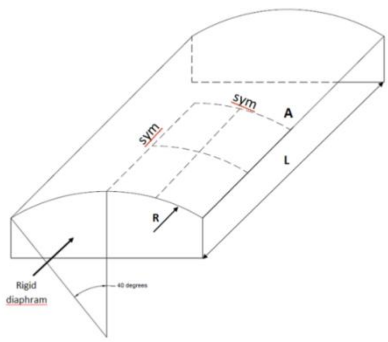


Fig.3 Convergence curve for the deflection ,w ,at point A

2. Consistant Load Vector

The simplest method to establish an equivalent set of nodal forces is the lumping process. An alternative and more accurate approach for dealing with distributed loads is the use of a consistent load vector which is derived by equating the work done by the distributed load through the displacement of the element to the work done by the nodal generalized loads through the nodal displacements. If a rectangular shell element is acted upon by a distributed load *q*per unit area in the direction of *w*, the work done by this load is given by:

$$P_1 = \int_{-a}^{a}\int_{-b}^{b} qw\,dx\,dy \qquad (8)$$

where a and *b* = the projected half length of the sides of the rectangular element in the *x* and *y* directions, respectively. If w is taken to be represented by:

$$\{w\}=[N^T]\{a\}=[N^T][C^{-1}]\{d\} \qquad (9)$$

where$[N^T]$ for the present element is given by Eqs.(3),(5):

$$\left[N^T\right]=[-1,x,y,0,0,0,0,0,0,0,0,0,-\frac{x^2}{2},-\frac{x^3}{6},-\frac{x^2y}{2},-\frac{x^3y}{6}\;-\frac{y^2}{2}-\frac{xy^2}{2}-\frac{y^3}{6}-\frac{xy^3}{6}-\frac{xy}{2}]$$

$\{a\}$ = the vector of the independent constants; $[C^{-1}]$= the inverse f the transformation matrix; and {d} = a vector of the nodal degrees of freedom. The work done by the consistent nodal generalized force through the nodal displacements {d} is given by:

$$P_2=\{F\}^T\{d\} \qquad (10)$$

Hence, from Eqs.(8), (9) and (10), the nodal forces are obtained, i.e.

$$\{F\}=\int_{-a}^{a}\int_{-b}^{b} q\left|N^T\right|\left[C^{-1}\right]dxdy=\{a\}[N]^{-1}dxdy \qquad (11)$$

When the loading is due to a linearly varying hydrostatic pressure, (Fig.4), $\{F\}$ for the rectangular element will be given by:

$$\{F\}=\gamma h\int_{-a}^{a}\int_{-b}^{b}(1+\frac{x}{h})\left|N^T\right|\left[C^{-1}\right]dxdy \qquad (12)$$

Where the 2*a* and 2*b* = the length of the element sides in the *x* and *y* directions respectively;□□= the water density; and *h*= the depth from the top of the water level to the centric of the considered element.

Eq. 12 gives the nodal forces for a single element; and the nodal forces for the whole structure are obtained by assembling the elements' nodal forces.

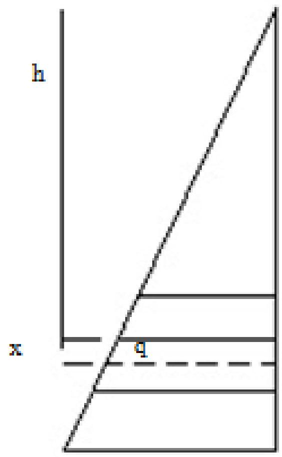


Fig.4. Pressure Distribution in the Vertical Direction

IV. PROBLEM CONSIDERED

2.1. ***Cylindrical shell dam structure***

The problem considered is that of a cylindrical shell dam where he boundary nodes are supported by a rigid valley, the dam subtending a central angle of 106 degrees, 3m thick , 30m high and has a radius of 43.25m, young's modulus and Poisson's ratio are 20×10^{8} kglm^{z} and o.15 respectively (Fig. 3)

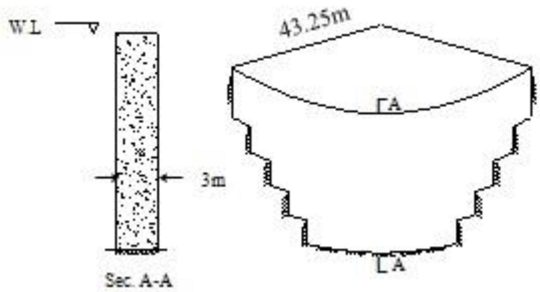
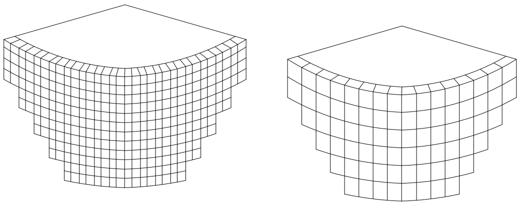


Fig.5. General dimensions of the cylindrical dam

Due to symmetry only one half of the dam is analyzed. Two meshes as shown in Fig.4 were considered.



(a) Fine (b) Moderate

Fig.6. Finite element meshes

The results for the radial deflection at the central line of the dam (crown line) for both meshes are shown in Fig.7. This figure also contains the results of the finite element solution obtained in reference [Bouzegari 1988] where a rectangular element with five degrees of freedom at each corner node is considered.

These results show that the two meshes give similar results and they are in very good agreement with these of finite element solution obtained in reference[Bouzegari 1988].

The direct stress resultants N_x& $N_y$in the *x* and *y* directions respectively and the bending momentsstress resultants M_x& M_y about *y* and *x* axes respectively are compared along the crown line.The results are based on the fine and moderat meshes are presented in Figs.8-11 respectively.

It is clearly seen that the two meshes gave similar results and they are in very good agreement with those obtained in reference [Bouzegari 1988].

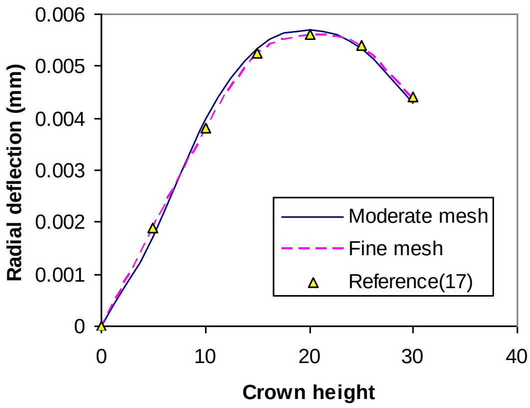


Fig.7. Redial deflection at the crown line

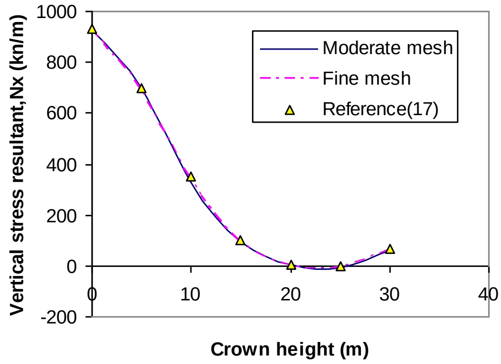


Fig.8. Direct stress resultant (Nx) at the crown line

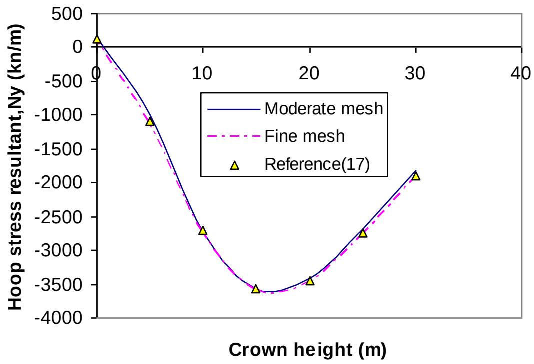


Fig.9. Hoop stress resultant (Ny) of the crown line

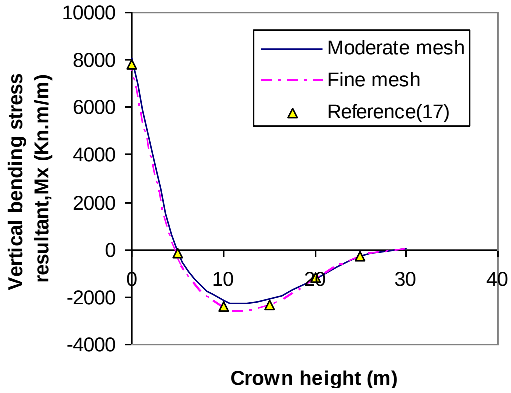


Fig.10. Bending stress resultant (Mx) of the crown line

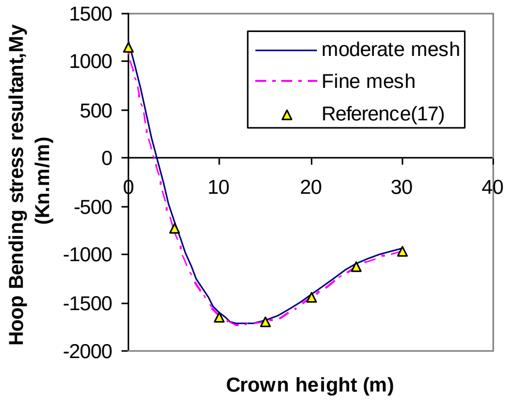


Fig.11. Bending stress resultant (My) at the crown line

2.2. ***The effect of central angle on the stiffness of the cylindrical shell dam.***
In order to investigate the effect of the central angle on the stiffness of cylindrical shell dam several angles from 46 degrees to 180 degrees were considered. Details of these dams are given in Fig.12.

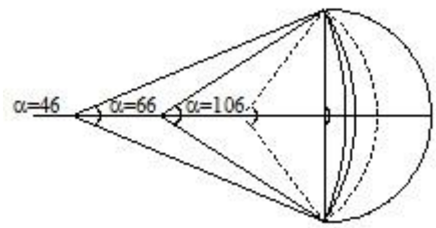


Fig.12. The shape of the central lines of the dam

The variation of radial deflections *w*and stress resultantsNx, Ny,Mx, My with respected to the variation of the central angle of the crownare presented in the Figs.13-17

Fig. 13shows that the increase of the central angle results in a decrease of radial deflection. The maximum variation of radial deflections with respect to the variation of central angles is observed at the crest of the arch dam. Radial deflection decreases quite rapidly for the increase of central angle from 46 degrees to 106 degrees. Further increase of central angle from 106 to 180 degrees, shows deterioration of the rate by which the radial deflection decreases.

The increase of central angle from 106 to 180 degrees only decreases 1.7mm from the radial deflection, but on return it increases the length of the crest of the cylindrical shell dam by 25%, resulting in excessive cost.
The variation of stress resultant Nx, in the *x*-direction, with respect to the variation of central angle, could be studied from Fig. 14, which shows the maximum. Stress at the bottom of the dam and gradually non-linearly decreases toward the crest.

A most interesting point on this figure is the point at about 3 meters from the crest, where all the curves pass through.The variation of stress resultant Ny in the circumferential direction with respect to the variation of central angle could be studied from Fig. 15. From this figure, a tension of about 138 KN per meter at the centre bottom of the dam, and maximum stresses at about middle height of the central cantilever can be observed.Another interesting observation is the point at about 10m from the crest, where all the curves pass through.By increasing the central angle, the circumferential stress does decrease at the crest
The variation of stress resultant Mx in the vertical direction with respect to the variation of central angle could be studied from Fig.16.Variation of central angle has no significant effects on the magnitude of the local max. Stress resultant Mx of the middle height of the crown cantilever. By increasing the central angle, only the location of the pick point drops slightly down. At the centre bottom of the dam, increase of the central angle results in the decrease of stress resultant Mx. Fig. 17 shows the change of central angle has a very significant effect on the stress resultant My. Especially at the crest of the dam, by increasing the central angle, stress resultant My decreases rapidly. Similar effect can be observed at the bottom of the dam on a comparatively very small scale.

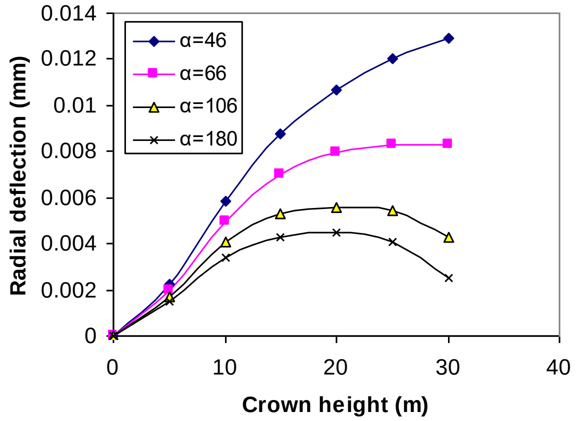


Fig.13. Redial deflection at the crown line

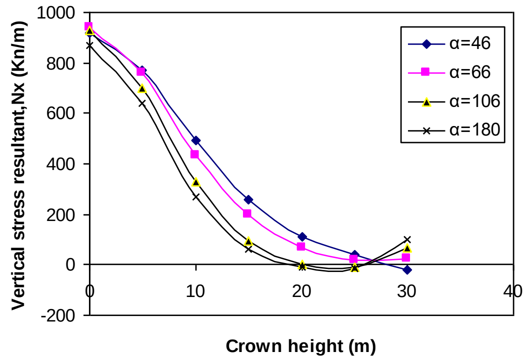


Fig.14. Direct stress resultant (Nx) at the crown line

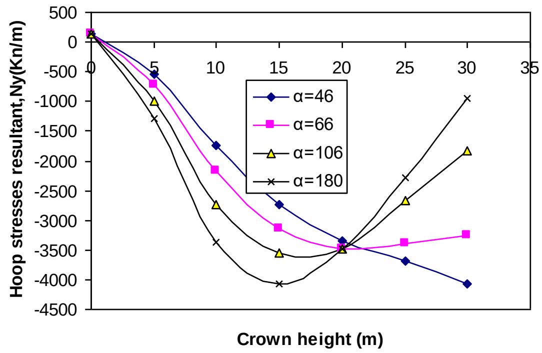


Fig.15. Hoop stress resultant (Ny) of the crown line

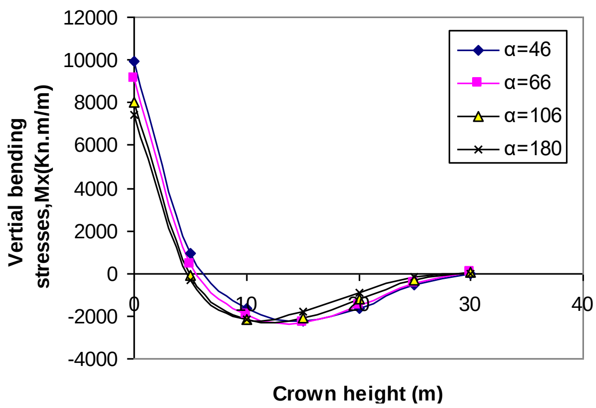


Fig.16. Bending stress resultant (Mx) of the crown line

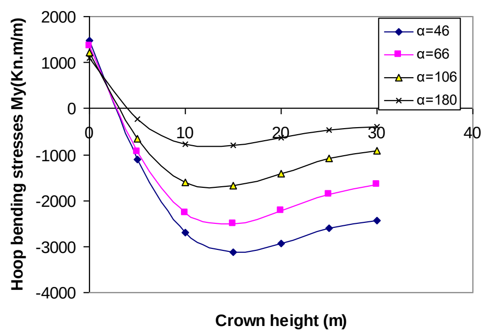


Fig.17. Bending stress resultant (My) at the crown line

V. CONCLUSION

A new rectangular strain-based finite cylindrical shell element has six degrees of freedom at each corner node is developed in the present paper. The element has been applied for linear cylindrical shell problem. The results obtained are compared with other available in the literatures and very good rate of agreements with other solutions were obtained.Small radii and large central angles lead to a strong and stiff cylindrical dam, but there is a limit on the central angle, partly because of the need for the shell tangent to meet the abutments with as large an angle as possible.

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