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Veselago Chiral Metamaterials

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ABSTRACT: *In this study Veselago chiral metamaterial, VCM, as left-handed chiral metamaterials is examined with a consideration of their optical properties. In particular, it is shown that a negative phase velocity is possible in these new media. In addition, some characteristic features of these media are pointed out and the possibility of negative propagation constants is explained. We generalize the concept of nihility and introduce a more general concept of chiral nihility composite materials where only it appears the chiral parameter in a Veselago chiral metamaterial. The asymmetry in the propagation constants of the two eigenwaves comes to its extreme (the two values differ by sign) in the limiting case of chiral nihility.*

Keywords: *Veselago, left metamaterial, chiral, Born-Fedorov,*

I. INTRODUCTION

In 1968 Veselago [1] analyzed what would happen to electromagnetic waves immersed in a hypothetical medium with negative permittivity and permeability.

The possibility to engineering materials with these exotic properties in some frequency range and practical realizations were proposed and performed, [2-4]. The refractive index in Veselago materials is negative implying that the phase of a wave decreases rather than advances along propagation through these media leading to important implications for nearly all electromagnetic phenomena as pointed out by Veselago himself. Consequences are particularly noticeable in those situations requiring the presence of evanescent waves in the Fourier transform of fields. These waves do not propagate implying a loss of information that could be revive provided to cleverly juxtapose materials with positive and negative indices to annihilate the effects of the evanescent waves [5, 6].

On the other hand, the interest for chiral materials has also been increasing during these last decades and many studies have been devoted to electromagnetic wave propagation in these media [7–12] with a double motivation, theoretical and practical concerning for instance remote sensing. So, a natural question is whether juxtaposing conventional and Veselago chiral materials would also lead to better performances. To discuss this situation, we present in this paper a theoretical analysis of harmonic plane wave propagation in homogeneous, isotropic, unbounded chiral media with negative refractive index endowed with the Drude- Born-Fedorov constitutive relations [13-16] in which permittivity and permeability are negative.

II. CONVENTIONAL CHIRAL MEDIUM

The Maxwell equations for the macroscopic free electromagnetic fields, (without charge and current) are well known. We often write Maxwell's equations in terms of electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$ denoted by bold letters

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \, , \; \nabla \cdot \mathbf{B} = 0 \tag{1a}$$

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{J} \, , \; \nabla \cdot \mathbf{D} = \rho \tag{1b}$$

These equations, however, are not complete. Six more equations, the constitutive relations, have to be added relating the electric field $\mathbf{E}$, the magnetic induction $\mathbf{B}$, the displacement field $\mathbf{D}$ and the magnetic field $\mathbf{H}$ to each other. These constitutive relations are completely independent of the Maxwell equations. The Maxwell equations involve only the fields and their sources. The constitutive relations, however, are concerned with the equations of motion of the constituents of the medium in an electromagnetic field [13-16].

We often write Maxwell's equations in terms of electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, defined by with the non locality definitions of Born-Fedorov, [13]: The Drude-Born-Fedorov constitutive relations of homogeneous, isotropic chiral media are [13]

$$\mathbf{B} = \mu\left[\mathbf{H} + T^c(\nabla\times\mathbf{H})\right] \quad (2)$$

$$\mathbf{D} = \in\left[\mathbf{E} + T^c(\nabla\times\mathbf{E})\right] \quad (3)$$

in which permittivity $\in_c$, permeability μ_c and chirality T^c are function of ω.

With $\exp(i\omega t)$ implicit and the light velocity c = 1, the Maxwell equations become

$$\nabla\times\mathbf{E} + \mathbf{i}\omega\mu\left[\mathbf{H} + T^c(\nabla\times\mathbf{H})\right] = 0 \quad (4)$$

$$\nabla\times\mathbf{H} + \mathbf{i}\omega\varepsilon\left[\mathbf{E} + T^c(\nabla\times\mathbf{E})\right] = 0 \quad (5)$$

It is easy to see that the wave equations are [14]

$$(1-\omega^2\mu\varepsilon T^{c2})\nabla\times(\nabla\times\mathbf{E}) \text{-} 2T^c\mu\varepsilon\omega^2\nabla\times\mathbf{E} \text{-} \omega^2(\mu\varepsilon)\mathbf{E} = \mathbf{0} \quad (5)$$

$$(1-\omega^2\mu\varepsilon T^{c2})\nabla\times(\nabla\times\mathbf{H}) \text{-} 2T^c\mu\varepsilon\omega^2\nabla\times\mathbf{H} \text{-} \omega^2(\mu\varepsilon)\mathbf{H} = \mathbf{0} \quad (6)$$

with $\nabla\cdot\mathbf{E} = 0$, $\nabla\cdot\mathbf{H} = 0$ and $\gamma^2 = k^2(1-\omega^2\mu\varepsilon T^{c2})$ we have

$$\nabla^2\mathbf{E} + 2\gamma^2 T^c\nabla\times\mathbf{E} + \gamma^2\mathbf{E} = \mathbf{0} \quad (7)$$

$$\nabla^2\mathbf{H} + 2\gamma^2 T^c\nabla\times\mathbf{H} + \gamma^2\mathbf{H} = \mathbf{0} \quad (8)$$

We are interested in the plane wave solutions of Eqs.(7, 8) with amplitudes $\mathbf{e}$ and $\mathbf{h}$:

$$\mathbf{E} = \mathbf{e}\exp(i\chi(x\sin\theta + z\cos\theta)) \quad (9)$$

$$\mathbf{H} = \mathbf{h}\exp(i\chi(x\sin\theta + z\cos\theta)) \quad (10)$$

Substituting (9) into (7) for the electric fields the system of equations are

$$e_x - \delta\cos\theta e_y = 0 \quad (11a)$$

$$e_z + \delta\sin\theta e_y = 0 \quad (11b)$$

$$e_y + \delta\cos\theta e_x - \delta\sin\theta e_z = 0 \quad (11c)$$

$$\delta = 2i\chi T^c\gamma^2(\gamma^2 - \chi^2)^{-1} \quad (12)$$

Eliminating e_y from (11c), (9) gives

$$(1 + +\delta^2\cos^2\theta)e_x - \delta^2\sin\theta\cos\theta e_z = 0 \quad (13)$$

$$-\delta^2\sin\theta\cos\theta e_x + (1 + +\delta^2\cos^2\theta)e_z = 0 \quad (14)$$

with a solution if $\delta = \pm i$ and we get from (11) and (12) between the components of the amplitude vector **e** the two relations

$$\sin\theta e_x + \cos\theta e_z = 0 \quad (15)$$

$$\sin\theta e_y \mp ie_z = 0 \quad (16)$$

The same result exists of course for the components of **h**, from here we have

$$\delta^{\pm} = k(1 \pm kT^c)^{-1} \quad (17)$$

If $\delta^{\pm} > 0$ two modes can propagate in this chiral medium. Rename $\delta^{\pm}$ as $\delta^{-} = k(1-kT^c)^{-1} \equiv k_L$, $\delta^{+} = k(1+kT^c)^{-1} \equiv k_R$, and incorporating c the light velocity we have that

$$n_L = \sqrt{\varepsilon\mu}/(1-\omega T^c\sqrt{\varepsilon\mu}/c)^{-1},\ n_R = \sqrt{\varepsilon\mu}/(1+\omega T^c\sqrt{\varepsilon\mu}/c)^{-1} \quad (18)$$

where $k = \omega/c\sqrt{\varepsilon\mu}$, and $k_L(n_L)$ $k_R(n_R)$ correspond to the wave number (refractive index) of LCP and RCP waves in the chiral medium. both n_L and n_R are frequency dependent in Eq. (18), unlike the nondispersive Tellegen constructive relation studied previously [9].

In the past decades, the experimental value of chiral T^c is limited to $T^c = 10^{-12}$ m, and k_L and k_R are always be positive with $\left|\omega T^c\sqrt{\varepsilon\mu}/c\right| < 1$. Recently, the negative refraction with

$\left|\omega T^{c}\sqrt{\varepsilon\mu}/c\right| > 1$ for one branch was suggested in chiral medium near the resonant frequency [16]. By choosing a suitable frequency band in CPPC, we reveal that the above phenomena can occur when the sign of $k_L(n_L)$ changes from positive to negative with an increasing frequency. Same effects would not occur in non dispersive Tellegen constitutive relation [9]. In this case we have that backward wave propagation will occur, but not as metamaterial media. In the next section we analyze this situation.

III. VESELAGO CHIRAL MEDIUM

The characterization of negative refraction by chiral materials is considerably more complicated than of negative refraction by isotropic dielectric-magnetic materials, as more than one refraction wavevector needs to be considered as well as magnetoelectric coupling and possibly directionality [17] . Negative phase velocity (NPV) has been widely adopted as a convenient indicator of negative refraction [18, 19]. While NPV and negative refraction go hand-in-hand for uniform planewave propagation in isotropic dielectric-magnetic materials, this is not generally true for nonuniform plane waves, especially in anisotropic materials. The possibility of NPV propagation in chiral materials was evident some years ago, but only nowadays is it being considered carefully. Here, we analyze the Veselago chiral medium, VCM.

In a isotropic, homogeneous Veselago chiral medium with Drude-Born–Fedorov constitutive relations [13], permittivity ε, permeability μ, refractive index $n = -\sqrt{\mu\varepsilon}$ are negative . For any arbitrary polarized monochromatic plane wave, the solution to Eq. (5) can be expressed as a sum of circularly polarized waves of either left or right handedness. Therefore, left- and right-circularly polarized (LCP and RCP, respectively) waves interact with chiral media.

These solutions yield two propagation constants $k_L = k(1 - kT^{c})^{-1}$ and $k_R = k(1 + kT^{c})^{-1}$.

Note that the subscript *L* and *R* refer to LCP and RCP plane waves with phase velocities $v_L = \omega / k_L$ and $v_R = \omega / k_R$, respectively. Here, the signs of μ, ε and T^{c} are

very important to state positive phase velocity PPV and negative phase velocity NPV.

For example, if μ, ε and T^{c} are all positive then RCP and LCP waves have

PPV when $kT^{c} < 1$. If, however, the condition $kT^{c} > 1$ is satisfied, the LCP wave will have NPV and backward wave propagation will occur. Thus, several conditions can be written to obtain LCP and RCP plane waves with PPV and NPV by arranging the signs and values of μ, ε and T^{c} [28–33]. In the case of left-handed VCM, in which the permittivity and permeability are simultaneously negative, the propagation constants k_L

and k_R have to be modified as

$$k_L = -|k|(1 + |k|T^{c})^{-1} \quad (19)$$

$$k_R = -|k|(1 - |k|T^{c})^{-1} \quad (20)$$

The expressions of $\varepsilon = |\varepsilon|\exp(i\pi)$ and $\mu = |\mu|\exp(i\pi)$ are used in Eqs. (19, 20) to obtain the modified propagation constants given above for left-handed CMTM. Under these permitted expressions of Eqs. (19) and (20), left-handed VCM supports two backward waves and negative refraction will occur if $|k|T^{c} < 1$.

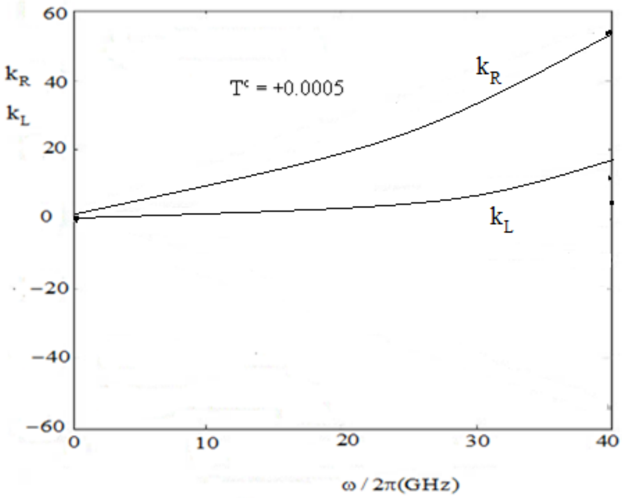
When $T^{c} = 0$, left-handed VCM turns to be left-handed MTM, in which the propagation constants *L* and *R* become identical, as studied in several works [12–13].

Note that Eqs. (19) and (20) state that there are two waves propagating with different negative phase velocity inside the left-handed CMTM. In addition, it is important to mention that the left-handed VCMs now occupy a new place in the family of optically active materials which rotate the plane of the linearly polarized incident wave since they have similar optical properties as in the conventional chiral materials. If

$$|k|T^{c} > 1, k_L = -|k|(1 + |k|T^{c})^{-1}, \quad k_R = -|k|(1 - |k|T^{c})^{-1} = |k|(|k|T^{c} - 1)^{-1} \quad (21)$$

the corresponding phase velocities for LCP and RCP plane waves are given by

$v_L = -\omega / |k|(1 + |k|T^{c})^{-1}$, and $v_R = \omega / |k|(|k|T^{c} - 1)^{-1}$ respectively. In Figure 1, using equations (17), (18), and (19), we illustrate the propagation constants k_L and k_R as a function of the frequency (0-40 GHz) for the conventional chiral and left-handed VCM media having $T^c = +0.0005$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ for the conventional Born-Fedorov chiral medium, and $T^c = -0.0005$, $\varepsilon = -\varepsilon_0$, $\mu = -\mu_0$, for left-handed VCM media. As a comparison between 1a) and 1b) can be seen that in the case of left-handed VCM, we have $k_L < 0$ and $k_R > 0$.



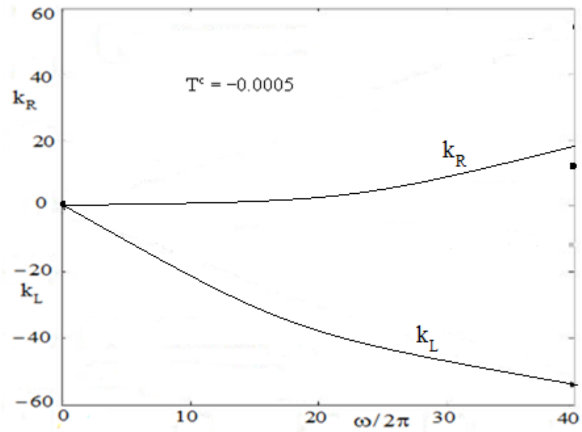


Figure 1. Propagation constants k_L and k_R as a function of the frequency for the conventional Born-Fedorov chiral medium and left-handed VCM media. . a) $T^c = +0.0005$, corresponds to the conventional Born-Fedorov chiral medium., b) $T^c = -0.0005$ corresponds to the VCM medium.

IV. NONRECIPROCAL CHIRAL NIHILITY

Term “nihility” for such medium, whose $\mu = 0$, $\varepsilon = 0$ was introduced by [20] and [21] and reviewed by [21]. Extreme electromagnetic properties of uniaxial non-reciprocal bianisotropic materials in the limiting case of nihility, when both permittivity and permeability of the media tend to zero, so, only the magneto-electric parameters define the material response. Among other interesting effects, we show that the moving nihility materials provide the extreme asymmetry in the phase shift of transmitted waves propagating along the opposite directions.

Furthermore, in this paper, we will generalize the concept of nihility and introduce a more general concept of chiral nihility composite materials where only it appears the chiral parameter in a Veselago chiral metamaterial.

The condition of nonreciprocal nihility is satisfied when the nonreciprocal nihility parameter (NNP) is independent of k which results in the following characteristics of refractive indices $k_L = -1/T^c$, $k_R = +1/T^c$ From equation (18) with $\varepsilon < 0$, $\mu < 0$ we have

$$n_L = -c/\omega T^c,\; n_R = c/\omega T^c \qquad (22)$$

The chirality T^{c} can be dispersive if the Condon model is adopted to consider the quantum transitions in an optically active molecular medium. However, in the limiting case of $\left|kT^{c}\right| \ll 1$, it is found that plane-wave solutions in this medium carry zero power. Wave impedances are purely imaginary. Thus, we can conclude that the nonreciprocal chiral nihility material has properties which are quite different from that of the reciprocal chiral nihility media.

Thus, the concept of chiral nihility lead to understanding of the chiral route to negative refraction and superlensing with the use of chiral structures. We can say that the asymmetry in the propagation constants of the two eigenwaves comes to its extreme (the two values differ by sign) in the limiting case of chiral nihility.

V. CONCLUSION

In this study Veselago chiral metamaterial as left-handed chiral metamaterials was examined with a special consideration of their optical properties. In particular, it is shown that a negative phase velocity is possible in these new media. In addition, some characteristic features of these media are pointed out and the possibility of negative propagation constants is explained. We generalize the concept of nihility and introduce a more general concept of chiral nihility composite materials where only it appears the chiral parameter in a Veselago chiral metamaterial. The asymmetry in the propagation constants of the two eigenwaves comes to its extreme (the two values differ by sign) in the limiting case of chiral nihility.

REFERENCES

[1]. Veselago V.G.The electrodynamics of substances with simultaneous negative values of and μ . Sov.Phys.Usp. 1968, V. 10, P. 509–514.
[2]. Shelbi R.A. Smith D.R. and Schultz S. Experimental verification of a negative index of refraction . Science. 2001, V. 292, P. 77–79.
[3]. Pendry J.B. and Smith D.R. Reversing light with negative refraction. Physics Today. 2004, V. 57, 37–43.
[4]. Mc CallM.W. Lakhtakia A. andWeighlofer,W.S. The negative index of refraction demys tified . Eur.J.Phys. 2002, V. 23, 353–359.
[5]. Pendry J.B. Negative refraction makes a perfect lens. Phys.Rev.Lett. 2006. V. 85 P. 3966–3969.
[6]. PendryJj.B. and Ramakrisna. S.A. Focusing light with negative refractive index . J.Phys. Condens.Matter. 2003. V. 15, 6345-6357.
[7]. Jaggard D.L. Mickelson A.R. and Papas C.H. On electromagnetic waves in chiral media. Appl.Phys. 1979, V. 18, 211–216.
[8]. Bassiri S. Papas C.H.and Engheta N. Electromagnetic wave propagation through a chiral slab. J.Opt.Soc.Am. 1988, V. A5. 1450–1459.
[9]. Q. Cheng and T. J. Cui, Phys. Rev. B 2006, **73**, 113104.
[10]. E. Giorgio, K. Tanaka, L. Verotta, K. Nakanishi, N. Berova, and C. Rosini, Chirality, 2007, 19, 434.
[11]. Mackay T. G., and Lakhtakia A., " Negative reflection in a Faraday chiral medium. ," Microw. Opt. Technol. Lett. 2008, 50, 1368–1371 .
[12]. Mackay T. G., and Lakhtakia A., " Negative refraction, negative phase velocity, and counterposition in bianisotropic materials and metamaterials. ," Phys. Rev. B. 79, 2009 , 235121 .
[13]. H. Torres-Silva**.** Chiral waves in Metamaterial medium, International Journal of Pure and Applied Sciences and Technology, 2011, 2, (2), 54-65.
[14]. G. Oliveri et al. Reconfigurable electromagnetics through metamaterials - A review. Proceedings of the IEEE. 2015, 103 (7): 1034–1056.
[15]. Héctor Torres-Silva Negative Refraction in Chiral Plasma Media, *Journal of Electromagnetic Analysis and Applications*, 2013, 5, 175-181.
[16]. Hector Torres-Silva, Diego Torres Cabezas Chiral Seismic Attenuation with Acoustic Metamaterials, Journal of Electromagnetic Analysis and Applications 2013, Vol.5 No.1, 10-15.
[17]. Qiu C.-W., , Burokur N., , Zouhdi S., , and L.-Li W., " Chiral nihility effects on energy flow in chiral materials. ," J. Opt. Soc. Am. A. 2008, 25, , 55–63 .
[18]. C. Sabah ., " Left-handed chiral metamaterials. ," Central Eur. J. Phys.. 2008, 6, , 872–878 .
[19]. Kwon D.-H., , Werner D. H., , Kildishev A. V., , and Shalaev V. M., " Material parameter retrieval procedure for general bi-isotropic metamaterials and its application to optical chiral negative-index metamaterial design. ," Opt. Exp. 2008 16, 11822–11829 .
[20]. A. Lakhtakia, An electromagnetic trinity from "negative permittivity" and "negative permeability", Int. J. Millimeter and Infrared Waves, 2001, vol. 22, 1731-1734.
[21]. S. Tretyakov, I. Nefedov, A. Sihvola, S. Maslovski, C. Simovski, Waves and energy in chiral nihility, J. Electromagn. Waves Appl. 2003, 17, 695–706.
[22]. Younes Ra'di, Viktar S. Asadchy, and Sergei A. Tretyakov, Nihility in non-reciprocal bianisotropic media, *EPJ Appl. Metamat.* 2015, 2, 6.