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Optimal Control of Dc Motor Using Performance Index of Energy

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ABSTRACT: *DC motors are widely used in the mechanisms that require control of speed. PID controller are widely used for speed control. But they aren't suitable for high performance cases, because of the low robustness of PID controller. Many researchers have been studying various new control techniques in order to improve the system performance. In dynamic regimes, the conversion efficiency is diminished to about half while during the stationary regime the efficiency is increased. The main drawback of these drives is the obtained diminished electrical energy during the transient regimes, i.e. starting, braking or reversing. The paper objective is to validate a new control, optimal type, which minimizes the consumption of the necessary energy to perform a given state trajectory. Two problems are presented in this paper: the problem formulation and the energy saving control solution, the numerical simulation. The numerical results confirm the performances of the proposed control method.*
Keywords: optimal control, energy, DC Motor

I.   INTRODUCTION

The DC motors are comprehensively used in various industrial applications such as electrical equipment, computer peripherals, robotic manipulators, actuators, steel rolling mills, electrical vehicles, and home appliances. Its applications spread from low horse power to the multi-mega watt due to its wide power, torque, speed ranges, high efficiency, fast response, and simple and continuous control characteristics. The speed of DC motor can be changed by controlling the armature and field voltages. The dynamic regimes of DC motor, such as starting and stopping, occur very often during normal work. These frequent dynamic regimes diminish considerably the conversion's efficiency, which is higher in the stationary state. The conventional control using P and/or PI controllers or other advanced techniques has as objectives a good dynamic behaviour and realization of the given trajectory with a minimum error. There are theoretical results validated by numerical simulation concerning the energetic optimal control of DC and AC electric drive systems[7], which take into consideration the minimization of the consumption energy during the transient periods, *e.g.* starting, stopping and reversing. The numerical results indicate the certain possibilities to decrease the consumption energy [3,5]. There are many approaches regarding the conversion's efficiency. Thus multi-objective optimization methods can be applied for the slow dynamic system or for stationary regimes. Also the differential evolution method could be applied for the case of steady-state conditions. Loss minimization techniques and other methods are developed for AC. motors [7,8] or for real application. Taking into consideration these requirements and achivements, an optimal control approach oriented to minimize the expenditure energy for a DC motor is developed in this paper. Using a model of a DC motor, an adequate performance functional criteria, and optimal control theory [3], the non-recursive solution of matrix differential equation, Riccati type, MRDE, is developed as optimal control law.These approaches are based on very strong mathematical instrument, optimal control theory, being formulated various problems as minimum time or energy.Obviously, there are constraints generated by magnitude limits for the electric and mechanic parameters. The introduction of the constraints transforms the problem into a nonlinear one. The solution, assessed using numerical procedures, is applied to a DC motor. The numerical results confirm the utility as well as the properties of the solution, high dynamic performances, without overshoots and good robustness. A schemeis proposed based on calculus of the optimal control law.

II.   PROBLEM FORMULATION

DC motor is a power actuator which transforms electrical energy in to mechanical energy. DC motor is widely used in many industrial applications where wide range of speed –torque characteristics required. In this paper, the separated excited DC motor model is chosen according to his good electrical and mechanical performances more than the other DC motor models. A DC motor, controlled by armature voltage, constant field is an invariantcontrollable dynamic system described by the differential equations.

$$\frac{d\omega(t)}{dt} = -\frac{b}{J}\omega(t) + \frac{k}{J}i(t) - \frac{1}{J}m_s(t)$$

$$\frac{di(t)}{dt} = -\frac{k}{L}\omega(t) - \frac{R}{J}i(t) + \frac{1}{L}u(t)$$

(1)

In the space state the differential equations (1) get the form

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \tag{2}$$

where the state is given by

$$x(t) = [\omega(t) \quad i(t)]^T \tag{3}$$

The control problem consists in finding of an admissible control, armature voltage$u(t)$, which transfers the system (2) from the initial state

$$x(0) = [0 \quad 0]^T \tag{4}$$

to a desired state

$$x_1(t) = [\omega_1 \quad i_1]^T \tag{5}$$

in the fixed time t_1, ω_1 being the desired final speed, with minimization of the consumption energy. In order to minimize the consumption energy, the index performance criteria of energy in the form

$$J = \frac{1}{2}[x(t_1) - x_1]^T S[x(t_1) - x_1] + \frac{1}{2}\int_0^{t_f}[x^T(t)Qx(t) + u^T(t)Ru(t)]dt \tag{6}$$

Is associated to system (2), where weighting matrices S and Q are 2×2 positive semidefined matrices and R is a 1×1 positive defined matrix. The optimal control problem is: with free end point, fixed time and unconstrained. The restrictions of the magnitude for the control and state could be solved by the adequate choice of the weighting matrices. The direct minimization of drawn energy can be achieved by introducing input power

$$P_1 = u(t)i(t) = u(t)x_2(t) \tag{7}$$

in index performance criteria (6). There are more power losses which are determinated by the angular speed: eddy current losses

$$p_F = k_F \omega^2(t) \tag{8}$$

hysteresis power losses

$$p_H = k_H \omega(t) \tag{9}$$

viscous power losses

$$p_V = b\omega^2(t) \tag{10}$$

also the output mechanic power has the form

$$P_O = k_H \omega(t) \tag{11}$$

wherek_F , k_H , b are constant coefficients.

III.   METHODOLOGY

The solution of the problem exists and is unique if the system (1) is controllable and completely observable and the weighting matrices carry out the above conditions [2,5].By using the variational method, the Hamiltonian ofthe optimal control problem is

$$H(x,u,\lambda,t) = g(x(t),u(t),t) + \lambda^T(t)f(x(t),u(t),t) = \frac{1}{2}[x^T(t)Qx(t) + u^T(t)Ru(t) + \lambda^T(t)x(t)] \tag{12}$$

λ(t) being costate vector.
Therefore the following canonic system is obtained

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T} \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} w(t) \tag{13}$$

The well known boundary conditions result from initial state x(0)=x$_0$ transversalitycondition

$$\lambda(t_1) = S[x(t_1) - x_1] \tag{14}$$

The solution of the problem is given by

$$u^{*}(t) = -R^{-1}B^{T}y(t) \tag{15}$$

where x(t) and y(t), the cost vector being the solution of the canonical Euler-Lagrange system.
The optimal control law at any moment 't' is

$$u^{*}(t) = -R^{-1}B^{T}P(t_1-t)x(t) + R^{-1}B^{T}K_1(t_1-t)x_1 + R^{-1}B^{T}K_2(t_1-t)w(t) \tag{16}$$

in which P(t$_1$-t) is the solution of differential matrix equation, Riccati type (RDME).
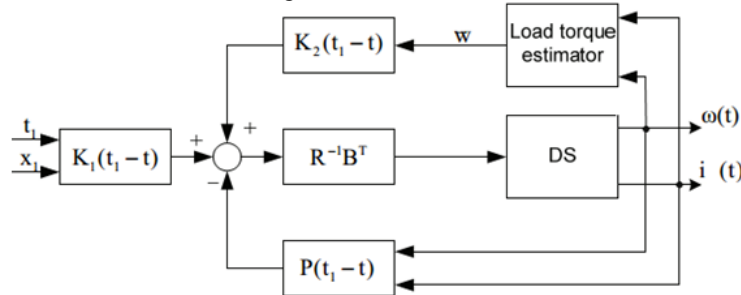The structure of optimal control is as follow in Fig.1

Fig.1 Structure of optimal control law

The optimal control solution (16) has three components: the state feedback, the reference to achieve the desiredfinal state **x$_1$**, the compensating feedforward of the perturbation **w**(*t*) effect, Fig.1. The solution is a non-recursive one and can be computed for any value of *t*, from initial time *t=0* to the final time *t1*.

IV.   NUMERICAL RESULTS

The system (1) was numerically simulated using a discretized model and Matlab/Simulink software. The system parameters are:1.5 kW, 1200rpm, 15A.The simulation results are presented in Fig.2, Fig.3 and Fig.4 and concretely we have optimal control for speed, current and voltage of DC motorby accepting that:
x$_1$(t)=ω(t) and x$_2$(t)=i(t).
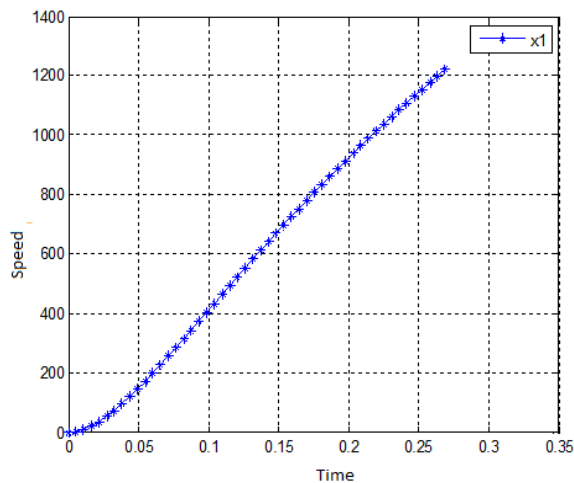
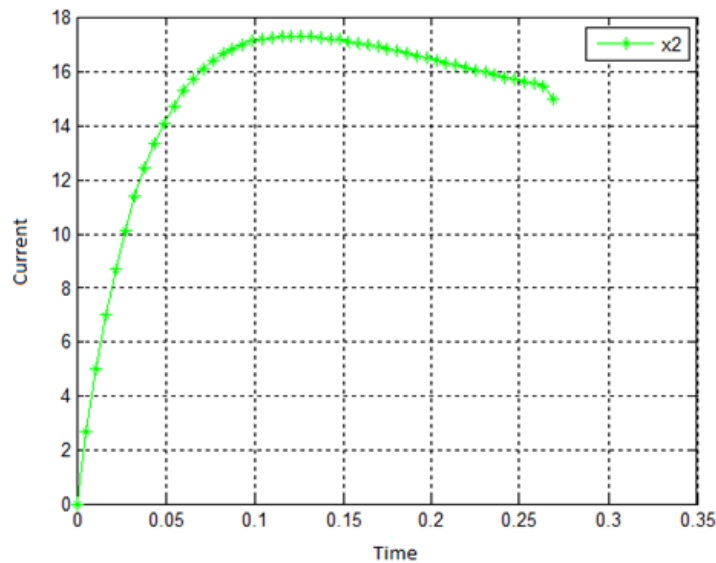**Fig.2**Optimal control law for speed
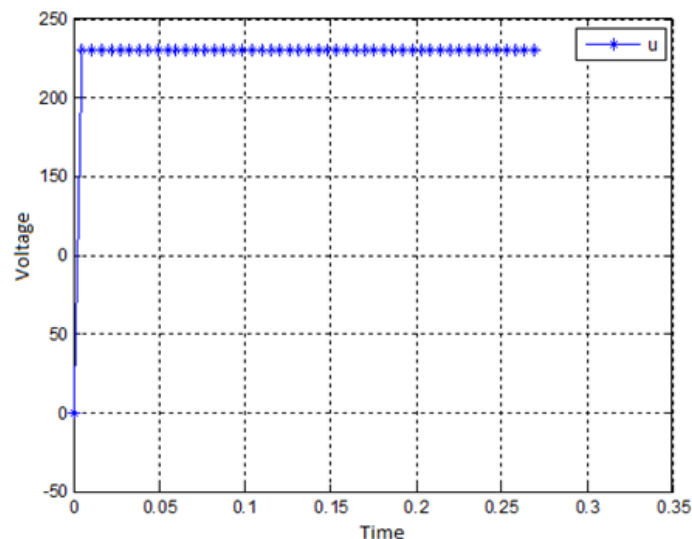
Fig .3 Optimal control law for current

Fig .4 Optimal control law for voltage

V.　CONCLUSION

In order to enhance the conversion efficiency of the DC drive systems a new optimal control law, which minimizes the drawn energy, was developed. The solution properties have been tested via simulation, using MATLAB-SIMULINK. With respect to conventional control, the optimal proposed control for a DC drive system minimizes the drawn energy to perform a given trajectory. The reduction in the drawn energy leads to the decreasing of the energy expenditure.The main features of the proposed optimal control law are: the solution of control is an analytical one, high dynamic performances, without overshoots and good robustness to the load variation decreasing the drawn energy with about 6% to 10% for a dynamic regime as starting, stopping and reversing; the optimal control is linear and unconstrained being applicable only for linear drive systems; voltage, current and speed limits can be maintained within strict limits by adequate choice of the weighting matrices; the optimal control can be mixed with conventional control in the aim to assure the works, dynamic and steady state regimes.

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