Numerical Analysis of Switching and Current-Voltage Characteristics of Graphene Nano-Ribbon Field Effect Transistors

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ABSTRACT: Graphene is one of the recently discovered materials from which graphene nano-ribbon (acronym, GNR) is derived. GNR has a great impact on the nano scale field effect transistors (FET) which has been tried to demonstrate in this study. Here, we have illustrated an analytical model of GNRFET for spatial distributions of the electric potential along the channel. The analysis is performed to find out the dependences of the source-drain current on the drain voltages as well as on the back gate and top gate voltages for different geometric parameters of the device. The switching on and switching off characteristics have been demonstrated in terms of current density, channel length and gate-voltage.

Keywords: GNRFET, Gate Voltage, Energy Bandgap, Channel Length, Current Density, Potential Distribution

INTRODUCTION

It has not been many a year since graphene was discovered. Graphene is, basically, a carbon-based purely two dimensional material with large amount of captivating properties, especially, the prospect of ultrahigh carrier mobilities beyond those of the conventional semiconductors [1], [2], [3]. The fascinating prospects of graphene have aggravated rigorous work focused on the intensification of graphene metal-oxide-semiconductor field-effect transistors (MOSFETs) [4], [5].

Nano-ribbons (also called nano-graphene ribbons or nano-graphite ribbons), often abbreviated GNRs, are strips of graphene with ultra-thin width (<50 nm). Graphene ribbons were introduced as a theoretical model by Mitsutaka Fujita and co-authors to examine the edge and nano-scale size effect in graphene [6], [7], [8]. If graphene is narrowed into slender graphene nano-ribbon (GNRs), a sizeable band gap opens [9], [10].

A large number of applications of GNRFET have maneuvered the advantages of GNR such as nano switch [11], tunneling transistor [12] etc in the recent times. GNRFET has powerful ability to control the electrostatics and hence is expected to reduce the short channel effect [13], which depends on the device electrostatics. Pei et al. [14] exploited an analytical modeling of the current at ballistic limit to evaluate GNRFET. Thiele et al. [15] applied a quasi-analytical modeling approach to analyze the current-voltage characteristics.

In this paper, we have tried to study the current density of GNRFET varying the carrier density in the device. An analytical GNRFET model [16] has been used for numerical simulation. The equations of the model include Poisson’s equation in the weak non-locality approximation. Room temperature (300K) has been considered for electrical measurement. The source-drain current density versus drain voltage dependencies have been exhibited for different gate-voltages. The condition for ON-OFF with the current-voltage characteristics have been demonstrated.

THEORETICAL ANALYSIS

2.1 Device Structure

As the name suggests, GNRFET is field effect transistor (FET) with graphene nano-ribbon (GNR) as the channel material. In this experiment, we have used GNRFET common source circuit for simulation and hence examined the DC characteristics. Different gate geometries have been applied to investigate the effect of gate geometry on GNRFET performance. The analytical model has been used to calculate the potential distributions in the GNRFET as a function of the back gate, top gate, and drain voltages, $V_b$, $V_g$, and $V_d$ respectively. We consider GNRFET with n-type channel. Therefore, positive back gate voltage, i.e., $V_b>0$ and...
negative top gate voltage, i.e., $V_g < 0$ have been assumed. Schematic of the device structure is shown in Fig. 1 and Fig. 2 (a), (b) show the side view and top view of the device respectively.

2.2 The Analytical Model of GNRFET

The energy spectrum of Graphene strips (nano-ribbons) with a gap between the valence and conduction bands mainly depends on the nano-ribbon width $d$:

$$\varepsilon_{p,n} = \pm v \sqrt{\left(p^2 + \left(\frac{m}{\hbar^2}\right)^2 n^2\right.}$$  \hspace{1cm} (1)

Here, $v = 108 \text{cm}^{-1}$ is the characteristic velocity of the electron and hole spectra, $P$ is the momentum in along the nano-ribbon, $\hbar$ is the reduced Planck constant, and $n = 1, 2, 3, \ldots$ is the sub and index.

The appearance of the band gap between the valence and conduction bands and specifically the density of states (DOS) as a function of the energy is the issue of the quantization corresponding to eqn. (1) of the electron and hole energy spectra in nano-ribbons since the electron and hole are confined in one of the lateral directions. The electron (hole) gas become degenerated at fairly high back gate voltages in relatively narrow nano-ribbons with quantized energy spectrum and the pertinent energy gap between the sub bands in the valence and conduction band which is exactly opposite to graphene with zero energy gap. In this paper we have regulated the model such that the GNR-FET operates under the condition that the electron gas in the channel is non-degenerate. Hence, the back gate voltage is considered to be reasonably high in order to keep the electron density moderated, the electron gas in the channel non-degenerated and also for the electrons to occupy only the lowest ($n = 1$) sub and in the conduction band nano-ribbons. The highly conducting channel sections adjacent to the source and drain are equipotential. The potentials are equal to the potentials of the source and drain contacts, i.e., equal to $V = 0$ and $V = V_d$, respectively. The GNR-FET region under the top gate (the device active region) defined as follows: $L_g \leq x \leq L_g/2, W_b \leq z \leq W_g$, where $L_g$ is the length of the top gate, and $W_b$ and $W_g$ are the thicknesses of the layers between graphene and the back and top gates, respectively. The following equation is used to find the potential distribution along the channel.
with the boundary conditions,
\[ \psi|_{x = -L_g/2} = 0, \quad \psi|_{x = L_g/2} = V_d. \]  
(3)

Here, \( \Sigma_- \) and \( \Sigma_+ \) are the electron and hole sheet densities in the channel, \( e \) is the electron charge, and is the dielectric constant of the material separating the channel from the gates. Eqn. (2), which governs the electric potential, \( \psi(x) = \psi(x, 0) \), is a consequence of the two-dimensional Poisson equation for the electric potential \( \psi = \psi(x, z) \) for the active region under consideration in the weak non-locality approximation \[16, 18\]. This equation provides solutions, which can be obtained from the exact solution of the two-dimensional Poisson by the expansion in powers of the parameter \[ W_b^2 + W_g^2 / 45(W_b + W_g) \delta^2 \], which is much smaller than unity in the situation under consideration. The relationship between the potential in the channel and the electron and hole charge becomes limited by neglecting the first term in the left-hand side of eqn. (2) where the lowest approximation in such an expansion corresponding to the so-called gradual channel approximation proposed by W. Shockley. Thus, eqn. (2) can be used when \( \delta < 1 \) and, hence, when \( L_g \geq \max(W_b, W_g) \). In particular, it is possible to study essentially non-uniform potential distributions in the GNR-FET channel and the short-gate effects analytically because of the harnessing of the approximation under consideration. The spacing, \( d_s \) between the nano-ribbons being small, a small scale non-uniformity of the electric potential distribution in the in-plane direction \( y \) can be neglected.

In the right-hand side of eqn. (2), \( \Sigma_+ \) is considered to be neglected since the electron density markedly exceeds the hole density at \( V_b > 0 \). Considering the relationships between the electron density and the electric potential, from Eq.(2) one can arrive at the following equations governing the potential distribution in the active region:

\[ \frac{\partial^2 \psi}{\partial x^2} = -\frac{3}{W_b W_g} \phi = -\frac{3}{W_b W_g} \left( W_b + W_g \right) \exp \left( \frac{\phi}{K_BT} \right) \]  
(4)

\[ \frac{\partial^2 \psi}{\partial x^2} = -\frac{3}{W_b W_g} \left( 0 - V_d \right) = -\frac{3}{W_b W_g} \left( W_b + W_g \right) \exp \left( \frac{\phi}{K_BT} \right) \]  
(5)

If the electron gas is non-degenerate, the dependences of the exponential term in the right-hand sides of (4) and (5) are valid. The threshold value of the back gate voltage, at which the degeneration of the electron gas occurs, can be estimated as \( V_b V_F \).

### 2.3 Potential Distributions at Low Top-Gate Voltages

Since the modulus of the potential \( |\phi| \) can be not that large when the top gate is negative \((V_f < 0)\) and its absolute value \( |V_d| \) is sufficiently small, (4) and (5) can be linearized and presented in the following form:

\[ \frac{\partial^2 \psi}{\partial x^2} = \frac{1}{W_b W_g} \left( 1 + \frac{W_b}{W_b + W_g} \right) \phi = -\frac{1}{W_b W_g} \frac{\phi}{K_BT} \]  
(6)

\[ \frac{\partial^2 \psi}{\partial x^2} = \frac{1}{W_b W_g} \left( 1 + \frac{W_b}{W_b + W_g} \right) \left( V_d + V_b \right) = -\frac{1}{W_b W_g} \frac{\phi}{K_BT} \]  
(7)

\( V_f < 0 \). At \( V_d = 0 \), (6) and (7) yield

\[ \phi = \frac{V_b W_g}{W_b + W_g} \left( \frac{\phi}{K_BT} - 1 - \cos \left( \frac{\phi}{K_BT} \right) \right) \]  
(8)

where

\[ \lambda = \sqrt{1 - \frac{W_g}{W_b + W_g} \phi} \]  
(9)

Is the effective screening length and \( \Lambda = \sqrt{\frac{W_b W_g}{3}} \). At \( \phi = 0 \) as follows from eqn. (6), the function exhibits a minimum \( \phi = \phi_{\text{min}} \) at \( x = 0 \) with

\[ \phi_{\text{min}} \approx \frac{V_b W_g}{W_b + W_g} \left( \frac{1}{\sqrt{1 - \frac{W_g}{W_b + W_g} \phi}} \right) \approx \frac{V_b W_g}{W_b + W_g} \]  
(10)

Here, we have taken into account that normally \( V_b \gg K_BT/e \) and \( L_g \gg (\text{with} <) \).

\[ \phi_m \approx \phi_{\text{min}} \approx \frac{V_b W_g}{W_b + W_g} \]  
(11)
Comparing $|\phi_m|$ given by eqn. (9) with $K_B T$, we find that (4), (5), (6), (7) and (8) are valid when $|V_d| \lesssim V_b W_d/W_b$.

2.4 Potential Distributions at High Top-Gate Voltages

At high back gate voltages, the quantity playing the role of the screening length is rather small. In this case, the length of the regions near the points $x = L_g/2$, in which the potential changes from $\phi = 0$ to $|\phi| > K_B T$ and from $\phi = V_d$ to $|0\phi V_d| > K_B T$, is small in comparison with the top gate length $L_g$. In such short regions near $x = \pm L_g/2$, the potential distribution can still be described by eqn. (7). However, in a significant part of the active region the electron charge, which is in such a situation exponentially small, can be disregarded and the last (exponential) terms in (4) and (5) can be omitted.

Thus at high top gate voltages, (4) and (5) in the central region can be presented in the following form:

$$\left(\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\nu_b \nu_g} \phi = - \left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \left(1 - \frac{\nu_g}{\nu_{gb} + \nu_{gg}}\right) \frac{V_d}{2}\phi\left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \frac{\sinh(\nu_{gb}/2)}{\sinh(\nu_{gb}/L_g)}\right) \frac{\nu_{gb} + \nu_{gg}}{V_d} \phi(\nu_{gb} + \nu_{gg})$$

Solving (11) using boundary conditions given by eqn. (3), we arrive at

$$\phi = \left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \left(1 - \frac{\nu_g}{\nu_{gb} + \nu_{gg}}\right) \frac{V_d}{2}\phi\left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \frac{\sinh(\nu_{gb}/2)}{\sinh(\nu_{gb}/L_g)}$$

At $V_d = 0$, $\phi$ exhibits a minimum at $x = 0$ and

$$\phi_{m0} \approx \left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \left(1 - \frac{\nu_g}{\nu_{gb} + \nu_{gg}}\right) \frac{V_d}{2}\phi\left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \frac{\sinh(\nu_{gb}/2)}{\sinh(\nu_{gb}/L_g)}$$

At reasonable values of the drain voltage $V_d$ (sufficiently small compared to $V_b$), eqn. (11) yields

$$\phi_m \approx \left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \left(1 - \frac{\nu_g}{\nu_{gb} + \nu_{gg}}\right) \frac{V_d}{2}\phi\left(\frac{\nu_d}{\nu_{gb} + \nu_{gg}}\right) \frac{\sinh(\nu_{gb}/2)}{\sinh(\nu_{gb}/L_g)}$$

Fig. 3 and Fig. 4 show examples of the spatial distributions (along the channel, i.e., in the $x$ direction) of the electric potential in the active region (under the top gate) calculated for a GNR FET with $W_b = 100\text{nm}$, $W_g = 30\text{nm}$, and $L_g = 300\text{nm}$ at the back gate voltage $V_b = 2.0\text{V}$, assuming different values of the top gate voltage $V_g$ and the drain voltage $V_d$.

As seen from Fig. 3, the potential distribution markedly sags and the height of the barrier for electrons near the center increases with increasing absolute value of the top gate potential $|V_g|$.

![Fig.3: Spatial distributions of the potential at different top gate voltages [16]](image-url)
Fig. 4: Spatial distributions of the potential at different drain voltages [16]

Fig. 4 reveals that in the GNR-FET with chosen parameters the minimum value of the potential $\phi_m$ and, hence, the height of the barrier $-\phi_m$ for the electrons propagating from the source are virtually insensitive to the drain voltage. The approach used in this subsection is valid when $e|\phi_m| > k_B T$, i.e., when $|V_g| < V_b W_g/W_b \approx k_B T = e$. In the limit $L_g > \Lambda > \lambda$ the equations for $\phi_m$ obtained above [18].

III. CALCULATION AND ANALYSIS OF CURRENT-VOLTAGE CHARACTERISTICS

The source-drain current can be determined by the electrons overcoming the potential barrier under the top gate. The density of this current (per unit length) can be derived from the following formula:

$$J = \frac{2e}{\pi h^2} \left( \int_{P_m^d} \frac{dp}{dE} \right)$$

(16)

where

$$v_p = \frac{e}{\hbar} \frac{p}{\sqrt{(p^2 + \omega_n^2)}}$$

(17)

is the velocity of the electron with momentum $p$ in the lowest subband of the nanoribbon conduction band and $P_m^d$ and $P_m^s$ are the momenta of the electrons with the energies $e|\phi_m|$ and $e(\phi_m + V_d)$, respectively. We have taken into account that the nanoribbon array is dense: $d \ll d$. Integrating in $e^{-q n}$ (16), we arrive at:

$$J = n \left( \frac{e}{\pi h^2} \right) \sqrt{\frac{2\pi m^*}{\hbar^2}} \exp \left( \frac{\phi_m}{k_B T} \right) \times \left[ V_b - (V_b - V_d) \exp \left( - \frac{eV_d}{k_B T} \right) \right]$$

(18)

3.1 Current versus Drain Voltage

The dependence of the source-drain current on the drain voltage is associated with the dependence of $\phi_m$ on this voltage given by (21) and (25) and the voltage dependence of the last factor in the right-hand side of eqn. (29). At low top gate voltages ($|V_g| \ll V_b W_g/W_b$), using (21) and (29), we get:

$$J \propto [V_b - (V_b - V_d) \exp \left( - \frac{eV_d}{k_B T} \right)]$$

(19)

i.e., $J \propto V_d$ if $V_d \ll k_B T/e$, and $J = \text{const.}$ If $(k_B T/e \ll V_d \ll V_b W_g/W_b)$. At moderate and high top gate voltages when $\phi_m$ is given by eqn. (16), we arrive at the following dependence

$$J \propto \exp \left[ \frac{-eV_d}{2k_B T \cosh \left( \frac{V_b}{k_B T} \right)} \right] \times \left[ V_b - (V_b - V_d) \exp \left( - \frac{eV_d}{k_B T} \right) \right]$$

(20)

At $V_d \ll k_B T/e$, eqn. (21) yields the same linear dependence on the drain voltage as that described by (19). When $V_d \gg k_B T/e$, we obtain:

$$J \propto \exp \left[ \frac{-eV_d}{2k_B T \cosh \left( \frac{V_b}{k_B T} \right)} \right]$$

(21)

3.2 Current versus Gate Voltages

According to (10), (14), and (17), the source-drain current as a function of the top gate voltage is described by the following relations:

$$J \propto \exp \left[ \frac{V_d}{V_g} \frac{V_g}{V_b} \right]$$

(22)

At $|V_g| \ll V_b W_g/W_b$. 

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According to (22) and (23), the dependence of the source-drain current on the top gate voltage is much steeper in the range high top gate voltages \(|V_g| > W_g/W_b\), than at \(|V_g| \approx V_b W_g/W_b\), i.e., when the central region of the channel becomes essentially depleted. The source-drain current versus the back gate dependence at high top gate and drain voltages is given by

\[
J \propto V_b \exp \left[ \frac{-eV_b}{k_BT} \left( 1 - \frac{1}{\cosh \left( \frac{V_b}{2k_BT} \right)} \right) \right]
\]

(24)

The source-drain current increases with increasing backgate voltage due to the pertinent increase in the electron density in all regions of the channel.

3.3 Description of Numerical Simulation

The description of all the inputs and outputs used for numerical simulation has been presented in this section. The input symbols used in the theoretical part have been described in Table 1. In Table 2, the output symbols have been listed.

### Table 1: Description of the inputs of all the symbols used in the theoretical part [19]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit (S.I.)</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_g)</td>
<td>Length of the top gate</td>
<td>nm</td>
<td>(L_g)</td>
</tr>
<tr>
<td>(W_b)</td>
<td>Thicknesses of layers between graphene and the back gate</td>
<td>nm</td>
<td>(W_b)</td>
</tr>
<tr>
<td>(W_g)</td>
<td>Thicknesses of layers between graphene and the top gate</td>
<td>nm</td>
<td>(W_g)</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Dielectric constant of materials between the channel and gates</td>
<td>F/m</td>
<td>(\varepsilon)</td>
</tr>
<tr>
<td>(V_b)</td>
<td>The back gate voltage</td>
<td>Volt</td>
<td>(V_b)</td>
</tr>
<tr>
<td>(V_g)</td>
<td>The top gate voltage</td>
<td>Volt</td>
<td>(V_g)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>The electrostatic potential</td>
<td>V</td>
<td>(\phi)</td>
</tr>
<tr>
<td>(e)</td>
<td>elementary charge</td>
<td>Coulombs</td>
<td>(e)</td>
</tr>
<tr>
<td>(\psi)</td>
<td>The electric potential</td>
<td>V</td>
<td>(\psi)</td>
</tr>
<tr>
<td>(\upsilon)</td>
<td>the characteristic velocity of the electron</td>
<td>cm/s</td>
<td>(\upsilon)</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>Energy Bandgap</td>
<td>eV</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>(k_B)</td>
<td>Boltzmann Constant</td>
<td>m kg/s K(^{-1})</td>
<td>(k_B)</td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature</td>
<td>Kelvin</td>
<td>(T)</td>
</tr>
</tbody>
</table>

### Table 2: Description of the outputs of all the symbols used in the theoretical part [19]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit (S.I.)</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J)</td>
<td>Current density</td>
<td>(mA/cm)</td>
<td>(J)</td>
</tr>
<tr>
<td>(I_{on})</td>
<td>Current in On condition</td>
<td>Amber</td>
<td>(I_{on})</td>
</tr>
<tr>
<td>(I_{off})</td>
<td>Current in Off condition</td>
<td>Amber</td>
<td>(I_{off})</td>
</tr>
</tbody>
</table>

IV. RESULTS AND ANALYSIS

The current density versus drain voltage dependences for GNRFET calculated for different top gate voltages \(V_g\) are shown in Fig. 5. The calculation is done for \(L_g = 300\) nm at the back gate voltage \(V_g = 2\) V and \(\Delta = 0.4\) eV, \(\varepsilon = 4\), \(W_b = 100\) nm, \(W_g = 30\) nm, and \(T = 300\) K. From Fig. 5, it is seen that the source-drain current as a function of the drain voltage in a GNRFET with relatively long gate (\(L_g = 300\) nm) exhibits saturation starting rather low drain voltages. This is a consequence of very weak sensitivity of the potential barrier for the electrons propagating from the source to the drain voltage [13]. The transformation of the \(J\) vs \(V_d\) dependences with varying top gate voltage in the range of the latter \([V_g] > V_b W_g/W_b\). These indicate strong sensitivity of the source-drain current to the gate voltages.

Now considering (20) and (23), it is seen that the source-drain current saturates when \(V_d\) becomes larger that \(k_B T e\) in a GNRFET with a long top gate, whereas in a GNRFET with \(L_g\) comparable with \(\Delta\), the source- drain current markedly increases with increasing \(V_d\) even at rather large values of the latter. This phenomenon is shown in Fig.6 (a), (b) by the calculation of current-voltage characteristics for GNRFETs with long (\(L_g = 300\) nm) and short (\(L_g = 100\) nm) top gates.
Fig. 5: The source-drain current density versus drain voltage dependencies at fixed back gate voltage ($V_b=2.0V$) and different top gate voltages $V_g$.

Fig. 6: (a), (b) The source-drain current density as a function of the top gate voltage at different back gate voltages. [19]

Now, we examine the switching behavior of GNRFET. Similar to the previous section, we assume $\Delta = 0.4$ eV, $\varepsilon = 4$, $W_b = 100$ nm, $W_g = 30$ nm, and $T = 300$ K. For switching on and switching off condition, we have considered drain voltage $V_d= 0.5V$ and back-gate voltage $V_b= 2V$. The gate voltage is varied within -0.7V to -0.8V and the switch on and switch off curve is shown in Fig. 7 (a) and (b) respectively. It is seen that current density varies reciprocally under particular length for different top gate voltages for On condition. From another point of view, the current density decreases as the channel length increases for fixed value of gate-voltage. On the other hand, the switching off condition remains same for all gate voltages applied. Therefore, reducing the channel thickness can result in better performance of the device. The ratio of current in On condition and current in Off condition for different gate voltages $V_g$ are shown in Fig. 8.
Fig. 7: (a) Current in on condition, $I_{\text{ON}}$; (b) Current in off condition, $I_{\text{OFF}}$

Fig. 8: Ratio of $I_{\text{ON}}$ and $I_{\text{OFF}}$ for different gate voltages, $V_g$

V. CONCLUSION

In this study, we have examined the current-voltage relationship of GNRFET and the switching on and switching off properties in terms of current density, channel length and gate-voltage. Our experiment includes the dependencies of the source-drain current versus the drain voltage as well as the back gate and top gate voltages. The shortening of the top gate can result in a substantial modification of the GNRFET current-voltage characteristics. The band gap of the GNR channel strongly depends on its width, which significantly affects the on current and off current. Although our analysis has not been performed under excessively high back gate voltages due an assumption that the electron gas in the channel is non-degenerate, the results show that the analysis can be utilized for GNRFET optimization.
REFERENCES


