American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-10, pp-373-380
www.ajer.org
Research Paper Open Access

Free Vibration Analysis Of Circular Cylindrical Shells

Aashish Kumar Jain[1], Aruna Rawat[2], Suresh Singh Kushwah[3]
[1]*M.E. Scholar, Department of Civil Engineering, University Institute of Technology (RGPV), Bhopal, India*
[2]*Asst. Prof., Department of Civil Engineering, University Institute of Technology (RGPV), Bhopal, India*
[3]*Associate Professor & HOD, Department of Civil Engineering, University Institute of Technology (RGPV), Bhopal, India*

ABSTRACT: *The purpose of the present study is to find out the natural frequencies and mode shapes of circular cylindrical shells. The circular cylindrical shell can vibrate in different modes and theoretically infinite modes are possible. The longitudinal mode (m) and circumferential mode (n) in any of their combinations define the modes and the corresponding modal frequencies. A finite element method (FEM) is used for the investigation of vibration behaviour of structural isotropic circular cylindrical shells with variable L/R ratios and for different boundary conditions such as clamped-clamped (C-C), free-free (F-F) and clamped-free (C-F). The mass of the shell is made constant for a particular length to radius (L/R) ratio. The effects of length to radius (L/R) ratio on the modal frequencies of the circular cylindrical shells are studied. The finite element analysis is performed using ANSYS software. The Eigen values of the shells are extracted using block Lanczos iteration method. The natural frequencies of the cylindrical shells increases with decrease in L/R ratios The natural frequencies are also observed to be lowest in the case of the clamped-free (C-F) boundary condition of the cylindrical shell.*

Keywords: *Circular cylindrical shell; Natural Frequency; Modulus of elasticity; Mode shape; Boundary conditions.*

I. INTRODUCTION

Cylindrical shells are widely used in civil structures such as pile foundation, offshore structures, liquid storage tanks, submarine, acoustic of submarines, mechanical and aerospace structures as airplane hulls etc. The vibration analysis of shells has been considering operational scenarios and modal characteristics. Therefore, investigating the natural frequencies and mode shapes characteristics of the cylindrical shell is useful in structural engineering.

The objectives of the present study are:

(a) Calculate the free vibration of the circular cylindrical shells using the FEA approach and compare the results with different shell theories published in the literature
(b) Study the effect of different boundary conditions, such as clamped - clamped (C-C), clamped-free (C-F), and free-free (F-F), on the modal frequencies of the shells
(c) Investigate the effects of different length to radius (*L*/*R*) ratios with different boundary conditions of the circular cylindrical shells.

Mode shapes associated with each natural frequency are combination of flexural (radial), axial (longitudinal) and torsional (circumferential) modes. Natural frequency of cylindrical shells has been extensively studied in many research papers. Many shell theories and methods have been developed for the analyses of the cylindrical shells. The shell theories started from the works done by Love (1888, 1944) and Flügge (1960). Among other shell theories are Timoshenko (1921) theory based on one-dimensional beam theory, and two-dimensional theories include Donnell (1934), Reissner (1940), Sanders (1959) etc. The other theories are based on Rayleigh-Ritz energy methods (Arnold and Warburton, 1949, 1953), closed-form solutions of the governing equations, and iterative solution approaches. Leissa (1973) carried out a review and comparison of various shell theories. The effects of the L/R ratios and different boundary conditions parameter on natural frequencies have been studied.

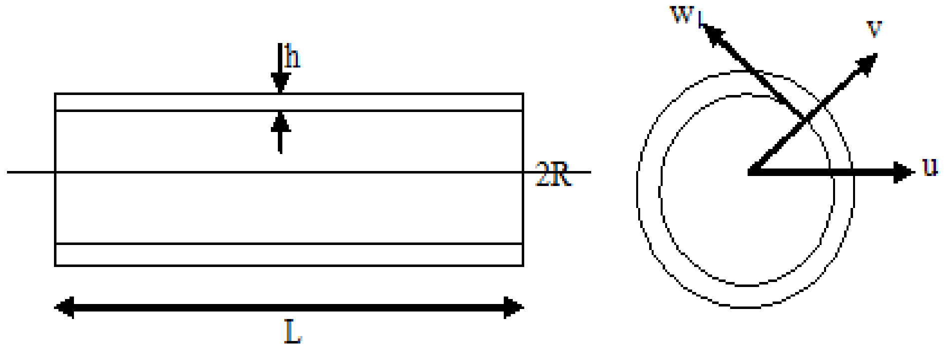


Fig. 1 Co-ordinates system of circular cylindrical shell

II. THEORETICAL FORMULATIONS

Consider an isotropic, circular cylindrical shell with length L and radius R. The shell has mid surface radius r, young's modulus E, poisson's ratio μ and mass density ρ. The displacement fields of the open shell with reference to the coordinate system are denoted by u, v and w in the x, y and radial directions, respectively. For the free vibration of a cylindrical shell, the equations of motion based on Flugge (1960) in matrix form are given as

$$\begin{bmatrix} L11 & L12 & L13 \\ L21 & L22 & L23 \\ L31 & L32 & L33 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = 0 \qquad \text{......................} (1)$$

Where, Lij (i, j = 1,2,3....) are the differential operators with respect to x, θ and t. The first attempt at solving the Eq. (1) involves the assumption of a synchronous motion.

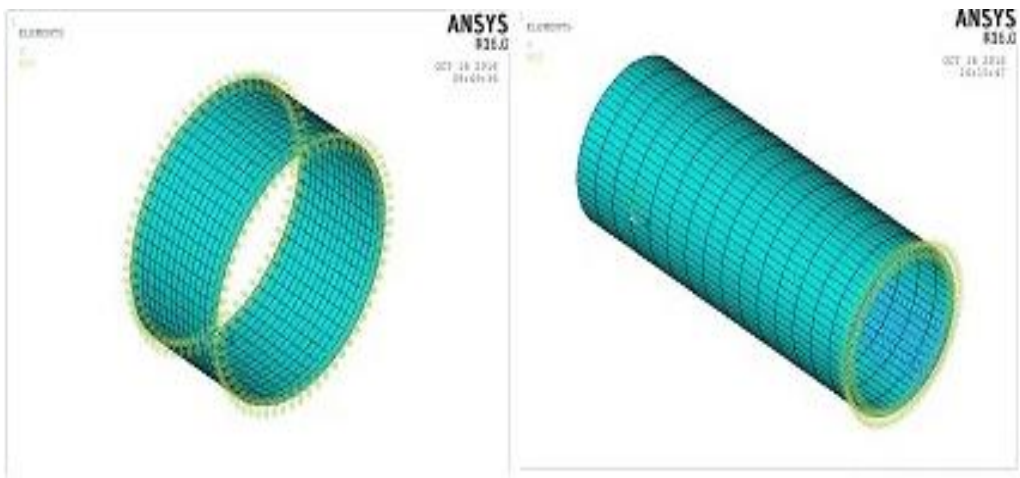
$$\begin{aligned} u(x,\theta,t) &= U(x,\theta)\, f(t) \\ v(x,\theta,t) &= V(x,\theta)\, f(t) \qquad \text{.................} \ \mathbf{(2)} \\ w(x,\theta,t) &= W(x,\theta)\, f(t) \end{aligned}$$

where, f(t) is the scalar model coordinate corresponding to the mode shapes U(x, θ), V(x, θ), and W(x, θ).

III. MESH CONVERGENCE STUDY

Meshing is required to obtain accurate result performed by the cylinder. In the numerical analysis using FE approach, coarse mesh is employed. Therefore, it becomes necessary to determine a good initial mesh density for conversed results to obtain with reduced computational efforts. A set of mesh convergence study is performed for different *L*/*R* ratios.

For this study, *R* = 1 m; *L*/*R* = 1, 5 and 10 are considered for all *L*/*R* ratios and boundary conditions. The corresponding fundamental frequencies are evaluated using Block Lanczos technique. The mesh size along the radius and length of the shell is varied and the converged value of fundamental frequency is considered for all the *L*/*R* ratios.



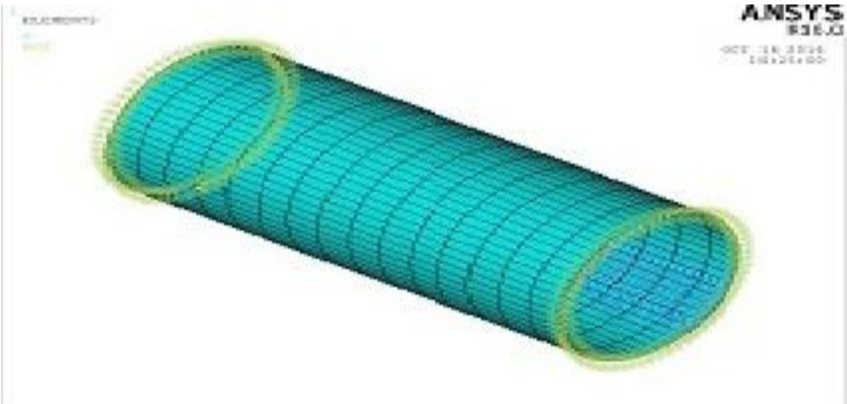


Fig. 2 Meshing in the cylindrical shell element

The effects of different boundary conditions and L/R ratios on the modal frequencies of the shell are investigated. Table 1 shows the comparison of modal frequency (in Hz) in aluminium circular cylindrical shell between ANSYS and result obtained by published paper (ρ = 7,000 kg/m^3, E = 210 GPa, and μ= 0.3, L=600 mm, R=150 mm, h=1 mm), while Table 2 shows the first lowest twenty natural frequencies of aluminium circular cylindrical shell. Table 3 to 5 shows modal frequencies of structural steel circular cylindrical shell for L/R = 1, 5 and 10 for different boundary conditions C-C, C-F and F-F. Fig. 1 shows the co-ordinates system, fig. 2 shows meshing, fig. 3 shows mode shape, fig. 4 shows the graph comparison in mode shape with different boundary conditions in tapered cylindrical shell element.

IV. PROCEDURE MODAL IN ANSYS

To make a modal in ANSYS software in any version and take mechanical APDL or Workbench, following Procedure to be follows in APDL-

1) Preference- Structural
2) Pre Processor
a) Element Type- Add/Edit/Delete- Add- Shell- 3D
b) Material Props- Material Models- Structural- Linear- Elastic- Isotropic (Put Modulus of Elasticity, Poisson’s Ratio and Density of the given material)
c) Sections- Shell-Layup- Add/Edit-Thickness
d) Modeling- Create- Keypoints- In Active CS (Put key points of the given element)
e) Modeling- Create- Line- Straight Line- Draw Lines
f) Meshing- Mesh Tool- Mesh in different conditions
g) Loads- Analysis Type- New Analysis- Modal
h) Loads- Analysis Type- Analysis Option- Block Lanczos
i) Loads- Analysis Type- Analysis Option- No. of modes to extract (put nos. which to be extract)
j) Loads- Analysis Type- Analysis Option- No. of modes to expand (put nos. which to be expand)
k) Loads- Analysis Type- Analysis Option- Start frequency (Take 0) and End frequency (Take 0)
l) Loads- Define Loads- Apply- Structural- Displacement- On line (Choose lines which to be fixed, Simply Supported or free, according to boundary conditions)
3) Solution- Solve- Current CS
4) General PostProc- Result Summary (Check your result as free vibration) and you can take also Read result with Plot results (check deflected shape of the particular vibration and find out the mode shapes also).
5) Take all the results and make and Excel and draw graphs and compare your frequencies with different end conditions.

V. NUMERICAL STUDY AND DISCUSSION

Natural Frequency analysis using FEM, carried out in the present study, aims to compare the performance of the thin circular cylindrical shell. The effects of different length to radius (L/R) ratios and boundary conditions in the FE analysis are investigated. Comparison of the modal frequencies calculated using the present FE approach is made with that published by Lee and Kwak (2015). They presented modal frequencies obtained for the shells by adopting FE approach in ANSYS® and compared with the results obtained from the Flügge (FLH) and the Donnell Mushtari (DM) theories.

The material properties of the cylindrical shell considered are: R = 1 m, ρ = 7,000 kg/m^3, E = 210 GPa, and μ= 0.3. Variations in the modal frequencies of the shell are studied for different height to radius (L/R) ratios varying as 1, 5 and 10.

Table 1: Comparison of modal frequency (in Hz) in aluminium circular cylindrical shell between ANSYS and result obtained by published paper (L=600 mm, R=150 mm, h=1 mm)

Material Properties										
Modulus of Elasticity					71 GPa					
Poisson's Ratio					0.33					
Density					2770 kg/m^3					
Boundary condition	**Mode (m,n)**	**Present ANSYS**	**Aruna Rawat**	**Difference**	**Lee and Kwak method**	**Difference**	**FLH**	**Difference**	**DM**	**Difference**
Clamped-Clamped	(1,5)	376.44	371.00	1.47	371.00	1.47	371.00	1.47	379.00	-0.68
	(1,4)	412.46	411.00	0.36	411.00	0.36	411.00	0.36	415.00	-0.61
	(1,6)	445.04	429.00	3.74	427.00	4.22	429.00	3.74	439.00	1.38
	(1,7)	569.00	546.00	4.21	544.00	4.60	547.00	4.02	558.00	1.97
	(1,3)	587.88	586.00	0.32	586.00	0.32	587.00	0.15	588.00	-0.02
Clamped-Free	(1,3)	145.58	146.00	-0.29	146.00	-0.29	146.00	-0.29	153.00	-4.85
	(1,4)	176.35	175.00	0.77	174.00	1.35	175.00	0.77	185.00	-4.68
	(1,2)	238.76	242.00	-1.34	243.00	-1.74	242.00	-1.34	243.00	-1.74
	(1,5)	267.19	263.00	1.59	261.00	2.37	263.00	1.59	274.00	-2.49
	(1,6)	382.58	382.00	0.15	378.00	1.21	381.00	0.41	391.00	-2.15
Free-Free	(1,1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(1,2)	31.08	30.00	3.60	30.00	3.60	30.05	3.43	31.12	-0.13
	(1,3)	88.91	87.00	2.20	87.00	2.20	87.82	1.24	88.05	0.98
	(1,4)	171.02	171.00	0.01	170.00	0.60	171.65	-0.37	170.85	0.10
	(1,5)	100.91	100.00	0.91	100.00	0.91	100.95	-0.04	100.83	0.08

Table 2: Analysis of modal frequency in Hz obtained by ANSYS in aluminum circular cylindrical shell (L=600 mm, R=150 mm, h=1 mm)

Material Properties			
Modulus of Elasticity		71 GPa	
Poisson's Ratio		0.33	
Density		2770 kg/m^3	
Mode	**Boundary condition**		
	Clamped-Clamped	**Clamped-Free**	**Free-Free**
1	376.74	145.59	0
2	377.16	145.61	0
3	379.80	176.35	0
4	380.05	179.08	0.0008
5	412.47	238.66	0.00161
6	412.58	238.76	0.0029
7	413.92	267.16	31.089
8	413.95	267.19	31.394
9	445.04	349.51	38.101
10	445.06	349.62	38.81
11	450.06	380.8	88.911
12	450.06	382.58	88.911
13	579.39	387.79	100.91
14	579.47	387.91	100.91
15	585.89	389.34	171.02
16	586.74	389.52	172.51
17	586.83	409.62	187.66
18	587.05	409.88	191.9
19	587.33	432.37	279.25
20	587.88	433.43	279.25

Table 3: Study of natural frequency in Hz by ANSYS in structural steel circular cylindrical shell in Cylindrical Shell Element (L=1000 mm, R=1000 mm, h=1 mm, L/R=1)

Mode	**Boundary condition**		
	Clamped-Clamped	**Clamped-Free**	**Free-Free**
1	101.91	40.19	0
2	101.95	40.99	0.000167
3	102.32	41.01	0.000223
4	102.35	42.07	0.000965
5	103.30	44.42	0.00121
6	103.31	44.44	0.06059
7	103.61	44.93	0.64
8	103.62	45.26	0.69
9	104.47	48.62	1.90
10	104.48	52.37	1.90

11	105.01	53.85	2.03
12	105.03	53.87	2.05
13	108.81	58.51	3.55
14	108.84	58.56	3.81
15	109	66.28	4.56
16	109.02	68.01	4.56
17	110.63	68.23	6.01
18	110.68	70.49	6.01
19	111.27	79.60	6.88
20	111.3	79.63	7.19

Table 4: Study of natural frequency in Hz by ANSYS in structural steel circular cylindrical shell in Cylindrical Shell Element (*L*=5000 mm, *R*=1000 mm, *h*=1 mm, *L*/*R*=5)

Mode	**Boundary condition**		
	Clamped-Clamped	**Clamped-Free**	**Free-Free**
1	18.266	7.24	0
2	18.27	7.24	0.0000305
3	18.31	7.60	0.0000616
4	18.31	7.65	0.0001465
5	18.90	8.95	0.0003381
6	18.90	9.21	0.012446
7	18.91	11.72	0.61
8	18.91	11.72	0.62
9	20.63	12.12	0.71
10	20.63	12.13	0.74
11	20.71	15.80	1.75
12	20.71	16.09	1.75
13	21.772	18.10	1.95
14	21.772	18.10	1.95
15	21.789	18.78	3.35
16	21.79	18.78	3.42
17	26.291	18.80	3.61
18	26.291	18.81	3.79
19	26.329	19.52	5.54
20	26.33	20.03	5.54

Table 5: Study of natural frequency in Hz by ANSYS in structural steel circular cylindrical shell in Cylindrical Shell Element (*L*=10000 mm, *R*=1000 mm, *h*=1 mm, *L*/*R*=10)

Mode	**Boundary condition**		
	Clamped-Clamped	**Clamped-Free**	**Free-Free**
1	8.87	3.45	0
2	8.87	3.45	0
3	8.88	3.81	0
4	8.88	3.87	0.0000415
5	9.47	5.73	0.0000564
6	9.47	5.73	0.007868
7	9.47	6.23	0.61
8	9.47	6.23	0.61
9	11.17	8.29	0.65
10	11.17	8.40	0.67
11	11.202	8.85	1.74
12	11.20	8.85	1.74
13	11.874	8.99	1.84
14	11.874	8.99	1.85
15	11.892	9.46	3.34
16	11.892	9.46	3.38
17	15.317	9.69	3.50
18	15.317	9.88	3.61
19	15.319	11.13	5.49
20	15.319	11.13	5.49

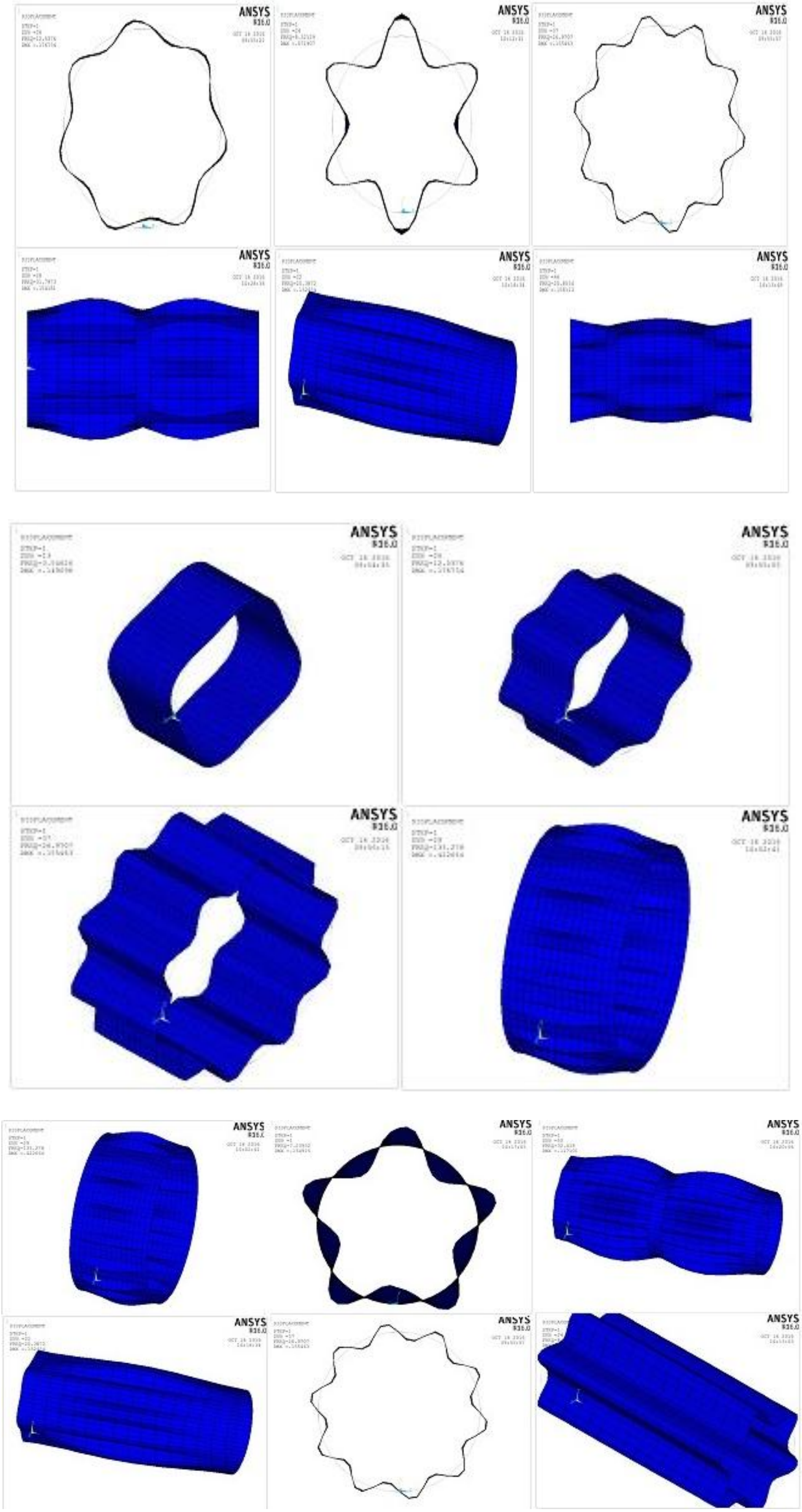


Fig. 3 Mode shape of circular cylindrical shells

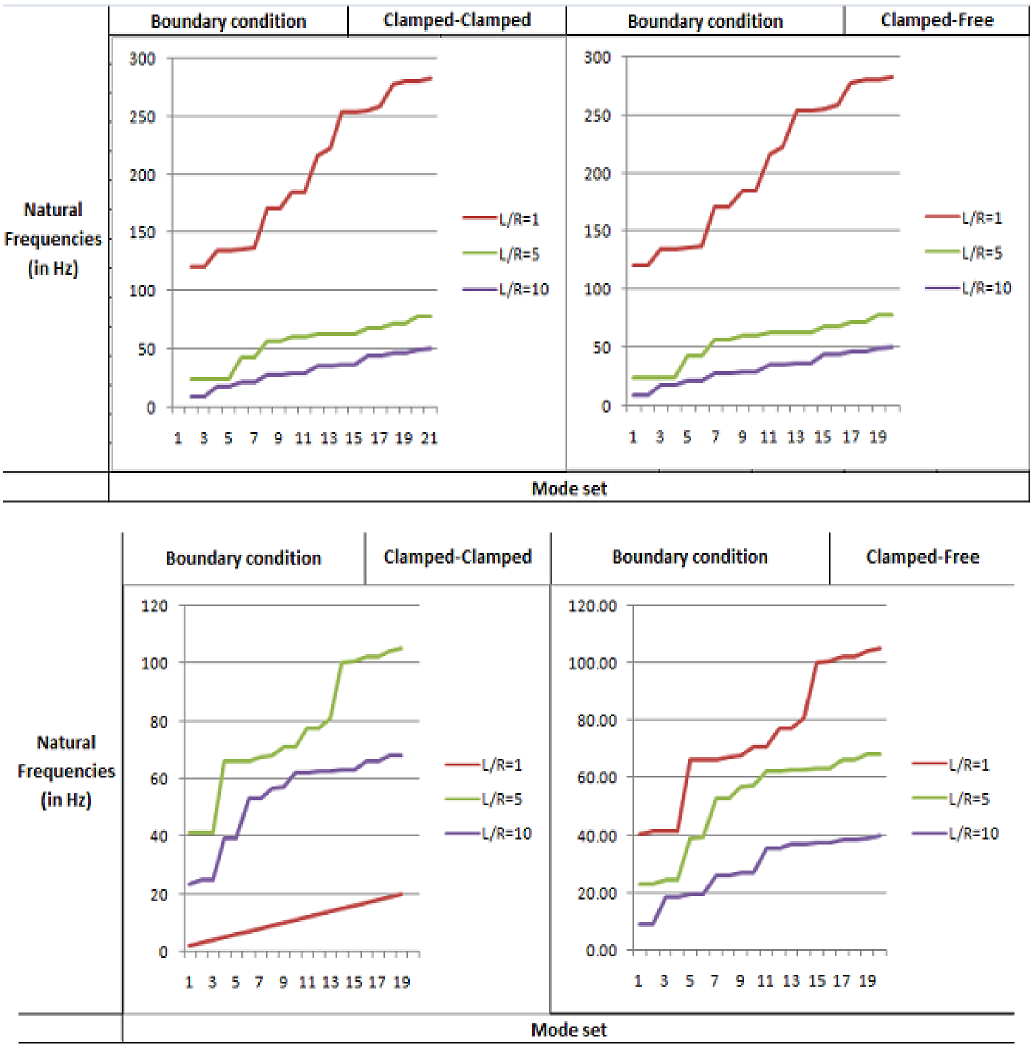


Fig. 4 Comparison in mode shape with different boundary conditions

VI. CONCLUSIONS

A finite element analysis procedure is developed to analyze the natural frequency characteristics of circular cylindrical shells with arbitrary boundary conditions. The natural frequency of circular cylindrical shells is found by using ANSYS software. As the circular cylindrical shell vibrates in different modes, its analysis becomes intricate; especially analytical/closed-form solutions may become intractable. Hence, the FE method may be relied on for conducting natural frequency of the circular cylindrical shells. The numerical results are presented and discussed in above. The broad conclusions that can be made from the present study are summarized as follows

1) The natural frequencies of the cylindrical shells increase with decrease in L/R ratios.
2) The natural frequencies are observed to be lowest in the case of the clamped-free (C-F) boundary condition of the cylindrical shell.
3) The modal frequency of the circular cylindrical shell is influenced mainly by the boundary conditions and L/R ratios.
4) The modal frequency decreases for the C-F boundary condition as compared to the C-C boundary condition.

VII. FUTURE SCOPE OF DISSERTATION

At the conclusion of this thesis, it is appropriate to comment on future research. Shell structures continue to be widely used in pile foundation, aerospace, marine, civil and mechanical engineering. As one of the powerful methods, the state-space method is limited to tapered circular cylindrical shells with variable thickness variations in this thesis. More research seems to be oriented towards developing and using new methods, which can combine the advantage of finite element method and the precision of analytical method for the problems at hand.

REFERENCES

[1]. Ramamurti V and Pattabiraman J (1976) "Free vibrations of circular cylindrical shells *Journal of Sound and Vibration*".
[2]. Sharma C B (1974) "Calculation of natural frequencies of fixed-free circular cylindrical shells *Journal of Sound and Vibration*".
[3]. Forerg K. (1964), "Influence of boundary conditions on the modal characteristics of thin shells". AIAA J. 2(12), 2150-2157.
[4]. Bert C. W., Baker J. L. and Egle D. M. (1969), "Free vibration of multilayer anisotropic cylinder shells".
[5]. Sharma C. B. and D. J. Johna(1972), "Free vibration of cantilever circular cylindrical shell-a comparative study".
[6]. Love A E H (1888) "The small free vibrations and deformation of thin elastic shell"
[7]. Xuebin L (2008) "Study on free vibration analysis of circular cylindrical shells using wave propagation Journal of Sound and Vibration".
[8]. Love A E H (1944) A treatise on the mathematical theory of elasticity Fourth Edition *Dover Publications* New York (NY) USA
[9]. Kurylov Y and Amabili M (2010) Polynomial versus trigonometric expansions for nonlinear vibrations of circular cylindrical shells with different boundary conditions.
[10]. Ross C. T. (1975), "Finite element for the vibration of cones and cylinders".
[11]. Jonin R. F. and Dies P. A. (1979), "Free vibration of circular cylinders of variable thickness".
[12]. Wang J. T. S., Armstrong J. H. and Ho D. V. (1979). "Axisymmetric vibration of prestressed non-uniform cantilever cylindrical shells".
[13]. Takahashi S., Suzuki K., Kosawada T. and Anzai E. (1981), "Vibration of cylindrical shell with varying thickness".
[14]. Chung H. (1981), "Free vibration analysis of circular cylindrical shells".
[15]. Suzuki K., Konno M. and Takahashi S. (1983), "Axisymmetric vibration of cylindrical shell with varying thickness".
[16]. Suzuki K., Takahashi S., Anxai E. and Kosavada T. (1983), "Vibration of a cylindrical shell with variable thickness capped by circular plate".
[17]. Suzuki K. and Leissa A. W. (1986), "Exact solution for the free vibration of open cylindrical shells with circumferentially varying curvature and thickness".
[18]. Leissa A. W. (1973), "Vibration of shells", NASA SP-288.
[19]. Jonin R. F. and Dies P. A. (1979). "Natural Frequency of circular cylinders of variable thickness". Journal of Sound and Vibration, 62:165-180.
[20]. Takahashi, Suzuki K., Kosawada T. and Anzai E (1981). "Vibration of circular cylindrical shells with varying thickness". Bulletin of the Japan Society of Mechanical Engineers, 24:1826-1836.
[21]. Suzuki K., Konno M. and Takahashi S (1981). "Axisymmetric vibration of cylindrical shell with varying thickness". Bulletin of the Japan Society of Mechanical Engineers, 24:2122-2132.
[22]. Suzuki K., Takahashi S., Anzai E. and Kosawada T (1983). "Vibration of a cylindrical shell with variable thickness capped by a circular plate". Bulletin of the Japan Society of Mechanical Engineers, 26:1775-1782.
[23]. Love A. E. H. (1988) "The small Natural Frequency s and deformation of thin elastic shell" Philosophical Transactions of the Royal Society at London 179A 491-546.
[24]. Ganesan N. and Sivadas K. R (1990). "Natural Frequency of cantilever circular cylindrical shells with variable thickness". Computers and Structures, 34:669-671.
[25]. Ganesan N. and Sivadas K.R. (1990). "Natural Frequency of cantilever circular cylindrical shells with variable thickness". Research paper printed in Great Britain, Computers & structures vol. 34, No. 4, pp. 669-677, in press.