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$\left\lfloor \frac{p}{2} \right\rfloor$- Cordial labeling of Some Cycle Related Graphs

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ABSTRACT: *A* $\left\lfloor \frac{p}{2} \right\rfloor$*- **cordial labeling** of a graph G with p vertices is a bijection f: V(G) →{1, 2, 3,..., p} defined by* $f(e = uv) = \begin{cases} 1 & \text{if } |f(u) - f(v)| \le \left\lfloor \frac{p}{2} \right\rfloor \\ 0 & \text{otherwise} \end{cases}$ *and* $|e_f(0) - e_f(1)| \le 1$*. If a graph has* $\left\lfloor \frac{p}{2} \right\rfloor$*- cordial labeling then it is called* $\left\lfloor \frac{p}{2} \right\rfloor$*- cordial where* $\left\lfloor \frac{p}{2} \right\rfloor$ *represents the nearest integer less than or equal to* $\frac{p}{2}$

In this paper we prove C_n *is* $\left\lfloor \frac{p}{2} \right\rfloor$*- cordial graph for n ≥ 3, except for n = 4 and the graph obtained by duplication of an arbitrary vertex by a new edge in cycle* C_n *is* $\left\lfloor \frac{p}{2} \right\rfloor$*- cordial and the graph obtained by duplication of an arbitrary edge by a new vertex in cycle* C_n *is* $\left\lfloor \frac{p}{2} \right\rfloor$*-cordial. By a graph we mean a finite, undirected graph without multiple edges or loops. For graph theoretic terminology, we refer to Harary [2] and Bondy [1].*

Keywords: $\left\lfloor \frac{p}{2} \right\rfloor$ *- cordial, cycle.*

Definition1.1: Duplication of a vertex v_k by a new edge e = v'v'' in a graph G produces a new graph G' such that N(v') = {v_k, v''} and N(v'') = {v_k,v'}.
Definition1.2: Duplication of an edge e = uv by a new vertex w in a graph G produces a new graph G' such that N(w) = {u, v}.

Theorem1.1: C_n is $\left\lfloor \frac{p}{2} \right\rfloor$ - cordial graph for n ≥ 3, except for n = 4.

Proof:
Case(i): n = 4m, m = 2, 3, 4, …
Let the vertices be $u_1, u_2, \ldots, u_n$.

Define f(u_i) = $\begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le \frac{n}{2} \\ n-i \quad if \quad i \quad is \quad odd \quad and \quad i \le \frac{n}{2} \\ n+1-i \quad if \quad i \quad is \quad even \quad and \quad \frac{n}{2} < i < n \\ i+1 \quad if \quad i \quad is \quad odd \quad \frac{n}{2} < i < n \\ 1 \quad if \quad i = n \end{cases}$

Then the labels of the vertices u_1, u_2, u_3, …, $u_{n/2}$ are respectively

n-1, 2, n-3, 4,…n-$\frac{n}{2}$+1, $\frac{n}{2}$,

Hence $\left|f(u_i) - f(u_{i+1})\right|$, i = 1, 2, 3, …, $\frac{n}{2}$-1, are respectively

n-3, n-5, n-7, …, 7, 5, 3, 1.

Thus the corresponding edge labels are 0, 0, 0, … $\frac{n}{4}$-1 times, 1, 1, 1, … $\frac{n}{4}$ times.

And the labels of vertices $u_{\frac{n}{2}+1}$, $u_{\frac{n}{2}+2}$, …, u_{n-1}, u_n are respectively

$\frac{n}{2}$+2, $\frac{n}{2}$-1, $\frac{n}{2}$+4, $\frac{n}{2}$-3, …, n, 1

And $\left|f(u_i) - f(u_{i+1})\right|$, i = $\frac{n}{2}$, $\frac{n}{2}$+1, $\frac{n}{2}$+2, $\frac{n}{2}$+3, …, n-3, n-2, n-1, are respectively 2, 3, 5, 7, …, n-5, n-3, n-1.

Thus the corresponding edge labels are 1,1,1, … $\frac{n}{4}$ times, 0, 0, 0, … $\frac{n}{4}$ times.

And $\left|f(u_n) - f(u_1)\right| = n-2$

The label of the corresponding edge is 0.

Hence $e_f(0) = e_f(1) = \frac{n}{2}$

Case(ii): n = 4m+1, m = 1, 2, 3, …

Let the vertices be u_1, u_2, …, u_n.

Define f(u_i) = $\begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le \left[\frac{n}{2}\right] \\ n-i \quad if \quad i \quad is \quad odd \quad and \quad i \le \left[\frac{n}{2}\right] \\ n+1-i \quad if \quad i \quad is \quad odd \quad and \quad \left[\frac{n}{2}\right] < i < n \\ i+1 \quad if \quad i \quad is \quad even \quad \left[\frac{n}{2}\right] < i < n \\ 1 \quad if \quad i = n \end{cases}$

Here $e_f(1) = \frac{n+1}{2}$, $e_f(0) = \frac{n-1}{2}$

Case (iii): n = 4m + 2, m = 1, 2, 3, …,

Let the vertices be $u_1, u_2, \ldots, u_n$.

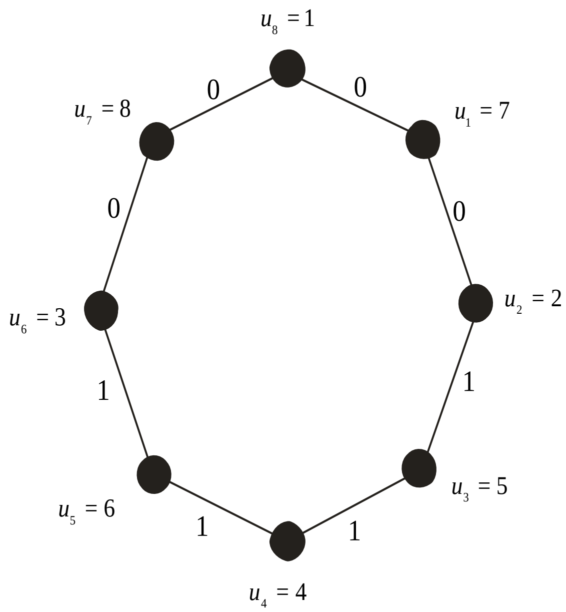
$$\text{Define } f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le \frac{n}{2}+1 \\ n+1-i \quad if \quad i \quad is \quad odd \quad and \quad i \le \frac{n}{2} \\ n+1-i \quad if \quad i \quad is \quad even \quad and \quad \frac{n}{2}+1 < i < n \\ i \quad if \quad i \quad is \quad odd \quad \frac{n}{2} < i < n \\ 1 \quad if \; i = n \end{cases}$$

Hence $e_f(0) = e_f(1) = \frac{n}{2}$

Case (iv): n = 4m+3, m = 1, 2, 3, …,
Let the vertices be $u_1, u_2, \ldots, u_n$.

$$\text{Define } f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ n-i \quad if \quad i \quad is \quad odd \quad and \quad i \le \left\lceil \frac{n}{2} \right\rceil \\ n+1-i \quad if \quad i \quad is \quad odd \quad and \quad \left\lceil \frac{n}{2} \right\rceil < i < n \\ i+1 \quad if \quad i \quad is \quad even \quad \left\lceil \frac{n}{2} \right\rceil \le i < n \\ 1 \quad if \quad i = n \end{cases}$$

$$e_f(0) = \frac{n+1}{2}, e_f(1) = \frac{n-1}{2}$$



The $\left[\frac{p}{2}\right]$ - cordial labeling of c_8

Theorem1.2: The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is $\left[\frac{p}{2}\right]$– cordial.

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, e_3, \ldots, e_n$ be edges of cycle C_n

Without loss of generality we duplicate v_n by an edge e_{n+1} with end vertices v' and v''.

Let the graph so obtained be G.

Then |V(G)| = n+2 and |E(G)| = n+3.

To define f: V(G) $\rightarrow$ {1, 2, 3, …, n+2} we consider the following cases.

Case(i): n = 4m, m = 1, 2, 3, …,

Let f(v') = 2m+1, f(v'') = 2m+2

$$\text{Define } f(u_i) = \begin{cases} i & if \quad i \quad is \quad even \quad and \quad i \leq 2m \\ 4m+3-i & if \quad i \quad is \quad odd \quad and \quad i < 2m \\ 4m+1-i & if \quad i \quad is \quad even \quad and \quad 2m < i < n \\ i+2 & if \quad i \quad is \quad odd \quad 2m < i < n \\ 1 & if \quad i = n \end{cases}$$

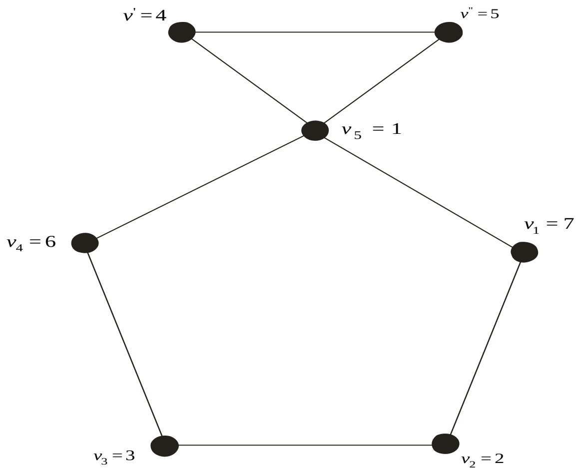
Here $e_f(0)$ = 2m+1, $e_f(1)$ = 2m+2

Case(ii): n = 4m+1, m = 1, 2, 3, …,

Let f(v') = 2m+2, f(v'') = 2m+3

$$\text{Define } f(u_i) = \begin{cases} i & if \quad i \quad is \quad even \quad and \quad i \leq 2m \\ 4m+4-i & if \quad i \quad is \quad odd \quad and \quad i < 2m \\ i+2 & if \quad i \quad is \quad even \quad and \quad 2m < i < n \\ 4m+2-i & if \quad i \quad is \quad odd \quad 2m < i < n \\ 1 & if \quad i = n \end{cases}$$

$e_f(0)$ = 2m+2, $e_f(1)$ = 2m+2



Case(iii): n = 4m+2, m = 1, 2, 3, …,

Let f(v') = 2m+1, f(v'') = 2m+4

$$\text{Define } f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m+2 \\ 4m+5-i \quad if \quad i \quad is \quad odd \quad and \quad i < 2m \\ 4m+3-i \quad if \quad i \quad is \quad even \quad and \quad 2m+2 < i < n \\ i+2 \quad if \quad i \quad is \quad odd \quad 2m+1 \le i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+2$, $e_f(1) = 2m+3$

Case(iv): n = 4m+3, m = 1, 2, 3, …,

Let f(v') = 2m+2, f(v") = 2m+3

$$\text{Define } f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m \\ 4m+6-i \quad if \quad i \quad is \quad odd \quad and \quad i \le 2m+1 \\ 4m+4-i \quad if \quad i \quad is \quad odd \quad and \quad 2m+1 < i < n \\ i+2 \quad if \quad i \quad is \quad even \quad 2m < i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+3$, $e_f(1) = 2m+3$

Theorem1.3: The graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is $\left[\frac{p}{2}\right]$-cordial expect for n = 3.

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, e_3, \ldots, e_n$ be edges of cycle C_n

Without loss of generality we duplicate the edge u_nu_1 by a vertex v'.

Then $|V(G)| = n+1$ and $|E(G)| = n+2$.

To define f: $V(G) \rightarrow \{1, 2, 3, \ldots, n+1\}$ we consider the following cases.

Case(i): n = 4m, m = 1, 2, 3, …,

p = 4m+1, q = 4m+2

Let f(v') = 2m+1

$$Define \quad f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m \\ 4m+2-i \quad if \quad i \quad is \quad odd \quad and \quad i < 2m \\ 4m+1-i \quad if \quad i \quad is \quad even \quad and \quad 2m < i < n \\ i+1 \quad if \quad i \quad is \quad odd \quad 2m < i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+1$, $e_f(1) = 2m+1$

Case(ii): n = 4m+1, m = 1, 2, 3, …,

p = 4m+2; q = 4m+3

$$Define \quad f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m \\ 4m+3-i \quad if \quad i \quad is \quad odd \quad and \quad i < 2m \\ 4m+2-i \quad if \quad i \quad is \quad odd \quad and \quad 2m < i < n \\ i+1 \quad if \quad i \quad is \quad even \quad 2m < i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+1$, $e_f(1) = 2m+2$

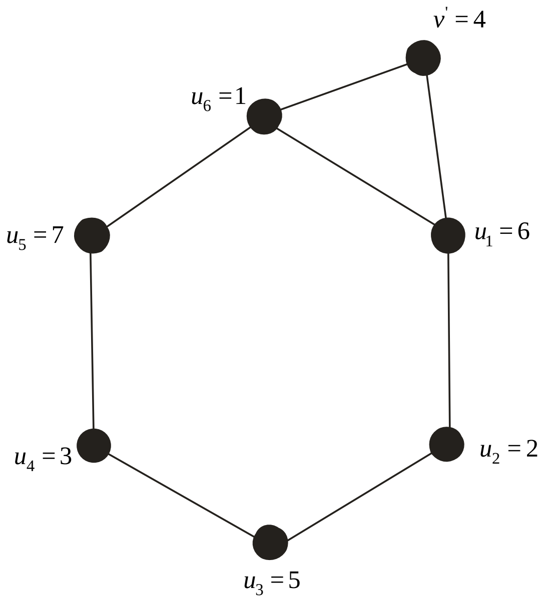
Case(iii): n = 4m+2, m = 1, 2, 3, …,

p = 4m+3; q = 4m+4.

Let f(v') = 2m+2

$$Define \quad f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m \\ 4m+3-i \quad if \quad i \quad is \quad odd \quad and \quad i < 2m \\ 4m+3-i \quad if \quad i \quad is \quad even \quad and \quad 2m < i < n \\ i+2 \quad if \quad i \quad is \quad odd \quad 2m < i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+2$, $e_f(1) = 2m+2$



Case(iv): n = 4m+3, m = 1, 2, 3, …,
p = 4m+4; q = 4m+5
Let f(v') = 2m+2

$$Define \quad f(u_i) = \begin{cases} i \quad if \quad i \quad is \quad even \quad and \quad i \le 2m \\ 4m+4-i \quad if \quad i \quad is \quad odd \\ i+2 \quad if \quad i \quad is \quad even \quad and \quad 2m < i < n \\ 1 \quad if \quad i = n \end{cases}$$

$e_f(0) = 2m+2$, $e_f(1) = 2m+3$.

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