

## Minimizing Utility Time in Mixed-Model Assembly Lines

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**ABSTRACT:** A mixed model assembly line is a production line where a variety of product models are produced. Simulated Annealing has attracted considerable attention due to its potential in dealing with difficult optimization problems. The present paper reports on a new approach to applying this method to minimize the total cost of the utility time and idle time incurred due to different line parameters (launch interval, station length, starting point of work, upstream walk) in closed station. The performance of the method is numerically tested on standard problems. Experimentation demonstrates the relative desirable performance of the presented methodology.

**Keywords:** Mixed-model assembly line, Mixed-integer linear programming, open station

### I. INTRODUCTION

A mixed-model assembly line is a manually operated production line capable of producing a variety of different product models simultaneously and continuously. Each workstation specializes in a certain set of assembly work elements, but the stations are sufficiently flexible to perform their respective elements on different models. While one model is being assembled at one station, a different model is being assembled at the next station. Mass produced consumer products commonly assembled on mixed-model lines include automobile [4] and large and small home appliances. Variations in model styles, options, and sometimes brand names characterize these products.

Assembly lines have traditionally been used to assemble products that have the same physical design. Workers execute a predetermined set of tasks at each workstation as products move through an assembly line. A product is gradually completed, when it moved by conveyor passing through each station. In a workstation, one or more workers work on the same product. In general, there are many issues that must be resolved to set-up and use an assembly line. These issues are:

(1) determination of the number of stations on the line; (2) determination of line cycle time; (3) model sequencing, that is, determine the order of models in which they will flow maximize utilization of workers in the assembly line, And (4) line balance, that is, balancing the work assignments among the workers and minimizing the number of workers required.

In a mixed-model assembly line, various models of a product are intermixed. Since models may require different assembly times at each station, and they are fed into the station at a certain rate, the amount of work required of a worker is unequal in a station. When the conveyor moves the work-piece through the station, workers sometimes can complete their work within the station and wait to work on the next work-piece. Sometimes, they may not finish their work within the station and need the help of an additional worker. In such a condition, the assembly line becomes unbalanced and labor utilization will be low. In order to solve this problem, the sequence of work-pieces, the rate that work-piece move into a station and the length of station are all important factors which need to be chosen judiciously. Objective of this work is to finding the best possible line parameters such as launch interval, sequence of work-pieces, station length to minimize utility time cost in a mixed-model, multi-station assembly line.

### II. GENERAL CONFIGURATION OF THE LINE

The mixed-model assembly line system considered is shown in figure 1. It consists of N workstations linked by a conveyor moving at a constant speed,  $V_c$ . There is only one worker at each station. After passing through each station, the work-pieces are gradually completed. There is only one work-piece in a station at a time.

A work-piece is loaded on the conveyor belt every  $\lambda$  minute. The worker begins work on a work-piece within his allowable work area and moves with the work-piece in the downstream direction while performing his assembly work on it. The time that required for completing assembly of the model (M) at station (I) is  $t_{mi}$ .

When worker reaches his downstream boundary while assembly the work-piece (either completing the assembly or leaving it incomplete), he returns upstream at a constant speed  $V_o$ , to begin work on the next work-piece. If the next work-piece has not entered his allowable limit area when he returns upstream, the worker waits for the work-piece. If the work-piece has not been completed yet when he reaches the downstream boundary, the additional worker will help to finish the work-piece. The utility work is typically handled the use of utility workers who assist the regular workers during work overload. Minimizing the utility work contributes to reducing not only labor cost, but also the risk of stopping the conveyor as well.

An assembly line may consist of closed stations or open stations. The choice of using closed station or open station in an assembly line depends on the type of product, the environment of production, and the production and safety regulations. For example, an assembly line tends to be closed station type in a paint shop, because different paint color operations are not allowed to mix together. However, a mechanical assembly line may be allowed to be open station type as long as the worker who works in one station is not interfered by another worker from another station Fig. 1.

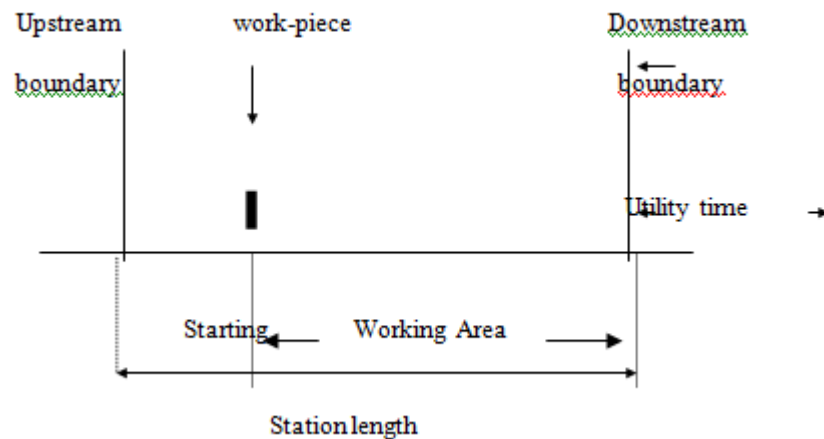


Figure1. Diagram of assembly line in close station

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#### Descriptions Of Some Terminology And Assumptions For The Lines Are As Follow:

(1) Launch interval: is defined as a fixed-time at interval which successive work-pieces are feed into a station. There are two types of launching interval: (a) fixed rate launching in which the launching period is a weighted average of the total assembly time for all products to be assembled over all stations, in this case

production cycle time must be less than or equal model cycle time, and (b) Variable rate launching in which the launching period is the task time of the last product launched at the first station so the worker at the first station can start working on the next product immediately after completing work on the current product. When units are launch at variable rate and in arbitrary order it is necessary for optimum utilization of the line, that worker cycle time must be equal to maximum model cycle time of unit [2, 4].

(2) Starting point of work: the starting point of workplace in a station is upstream position where a worker starts working on it. This point may be either on the zero reference point or within the station in a closed-station system, and may be within or before the zero reference point in an open-station system.

(3) Minimal part set: in this research, a minimal part set will be define as the smallest possible set as parts in the same proportion as the demands mix during the whole working period. Suppose if the model A, B, and C have the demand of 700, 200, and 100 units, respectively, it is difficult to sequence a total of 1000 or more work-piece at one time. This demand set {700,200,100} is divide by its largest common divisor (which is 100 in this case) to obtain the minimal part set a {7, 2, 1}. The problem of scheduling all products during the working period is then reduce to finding the assigned order of models to stations in minimum part set {7, 2, 1} order. The number of times that a minimal part set repeat in order to complete the demand during the entire working period is the largest divisor, called frequency, F. In this case is  $F = 100$ . Motivations for working with the minimum part set are as follows: First, it is becoming common practice in industry to plan for production in terms of the minimum part set, especially in flexible manufacturing. Second, the approach greatly simplifies the computations, thereby permitting the derivation of optimal solution for problems of realistic size. Third, the results obtain from working with the minimum part set MPS rather than the full part are surprisingly better [1, 3]. (4) Utility work: Utility work results when products move to the downstream limit of a worker's working area faster than he can finish work. The amount of work that remains to be completed is called utility work. In this situation, an additional worker may be assigned to assist the worker to complete the unfinished work-piece either within the station or outside the station.

(5) Closed station: In closed station, the worker must work within the limits of the station; worker is not allowed to move out of his work limit area when he assembles the product. This happens, for instance, in pits or in paint booths where the assigned work cannot be accomplished outside.

(6) Open station: In open station, the worker is permitted to move outside his station up to specific limits. The limits are necessary to prevent him from either moving an undesirable distance away from his station or from entering a closed station.

The following assumptions are considered to model the assembly line:

- 1) The assembly line is an open station;
- 2) No buffer exists between any two adjacent stations;
- 3) The layout of the mixed-model line assembly system is one dimensional;
- 4) The worker must work within the work zone assign to him;
- 5) A work-piece is load on the conveyor belt every  $\lambda$  minute (launching interval time).
- 6) Work-pieces are fixed on the moving belt;
- 7) There is no more than one work-piece at a station at a time;
- 8) Each station has only one worker;
- 9) Stations in the system are connects by a moving conveyor.
- 10) Workers perform at a constant rate in either direction of movement.
- 11) No buffer will exist between any two adjacent stations.
- 12) Maximum length of assembly line is given

#### In the development of model, we used the following notation:

A. Input parameters

$i$  = Station,  $i = 1, \dots, N$ ;

$N$  = Number of station;

$J$  = Position of a work-piece in the MPS,

$j = 1, \dots, K$ ;

$K$  = Total number of work-pieces to be sequence

$$K = \sum_{m=1}^M D_m ;$$

$m$  = Model index ( $m = 1, \dots, M$ );

$M$  = Number of models in minimal part set;

$V_c$  = Downstream speed of the conveyor;

$V_o$  = Upstream speed of worker;

- $D_m$  = Minimum part set Demand for model m (m= 1,..., M);
- F = Number of times that minimal part set is repeat;
- $L_{max}$  = Maximum available line length, feet;
- $t_{mi}$  = Assembly time for model m at ith station, (i= 1,...,N);
- $CO_i^{UT}$  = Cost of utility work time at the ith station;
- TCM = Time conveyor move;
- $\Delta$  = Distance that worker can move along the line.

**Decision Variables**

- $X_{mj}$  = Binary Decision variable equal to 1 if model m is the jth sequence work-piece in MPS; 0
- $\lambda$  = Launch interval between two successive work-pieces, minutes;
- $L_i$  = Station length;  $UT_{ij}$  = Utility worker time on the jth work-piece at the ith station minutes;
- $ST_{ij}$  = Starting point of assembly of work-piece in the jth sequence at the ith station on the line;
- $UD_{ij}$  = Upstream distance by a worker at the ith station for the jth work-piece, feet.

**The Open Station System**

In the open station, the worker begins to work on work-piece within the stations or at upstream stations. If a worker starts to works at upstream station, he is not allowed to interfere with another worker who is working at the upstream station. Following are the formulations of the utility time, upstream distance and the constraints for starting points of open station type system.

**The utility time**

In the open station, worker rides downstream and moves at a speed of  $V_c$  from his starting point  $ST_{ij}$  while performing the jth sequenced work-piece. The distance that worker works on the work-piece is (figure 1):

The time that conveyor move through this distance is:

$$TCM = [(L_1 + L_2 + \dots + L_i) - ST_{ij}] / V_c$$

$$\Delta_1 = (L_1 + L_2 + \dots + L_i) - (ST_{ij})$$

When a worker is not able to complete the work-piece by the time he reaches the downstream position, he returns upstream. A utility worker is assigned to work on the incomplete work-piece for a time, which is:

$$UT = [t_{mi} - [(L_1 + L_2 + \dots + L_i) - ST_{ij}] / V_c]$$

minutes. Because each sequenced position can only have one model, utility work at station i for the jth sequence work-piece is:

$$UT_{ij} = Max \{0, \sum_{m=1}^M X_{mj} t_{mi} - [(L_1 + L_2 + \dots + L_i) - ST_{ij}] / V_c \}$$

**STARTING POINT**

When the worker starts assembling the work-piece in upstream position in an open station, he is not allowed to interfere with the worker who works in the upstream station i. If  $ST_{i+1,j}$  is the starting point at (i+1) station for the jth assembled work-piece, then  $ST_{i+1,j}$  must be beyond the stop assembly point at the upstream ith station for the same work-piece on the coordinate. So the constraint of the starting point is:

$$ST_{i+1,j} \geq ST_{ij} + V_c \sum_{m=1}^M X_{mj} (t_{mi} - UT_{ij}) = ST_{ij} + V_c \sum_{m=1}^M X_{mj} t_{mi} - V_c UT_{ij}$$

**THE UPSTREAM DISTANCE**

Upstream distance for a worker walk in station no waiting time, then his new starting point will be within the boundary. Under assumed launching interval  $\lambda$  and the conveyor downstream moving speed  $V_c$ , we can find the distance between any two adjacent work-piece as  $\lambda V_c$ . If worker's upstream moving speed is

$V_o$  The time required to reach the work-piece upstream is  $\frac{UD_{ij}}{V_o}$ . Similarly, the time for the work-piece moving

downstream to meet the worker is  $(\lambda V_c - UD_{ij}) / V_c$  so, we have formulation as follows:

$$\frac{UD_{ij}}{V_o} = \frac{\lambda V_c - UD_{ij}}{V_c} \quad UD_{ij} = \lambda V_o \left( \frac{V_c}{V_o + V_c} \right) \quad \text{When } V_o \gg V_c \text{ we have } UD_{ij} = V_c \lambda$$

**The Open Station Mixed-Integer Linear Programming Model**

The mathematical model with objective of minimizing the total cost of the utility time for the open station assembly line can be written as:

Minimize  $Z = F \sum_{i=1}^N (CO_i^{UT} \sum_{j=1}^K UT_{ij})$

Subject to:

$$\sum_{m=1}^M X_{mj} = 1 \quad \text{For } j=1, \dots, K \tag{8}$$

$$\sum_{j=1}^K X_{mj} = D_m \quad \text{For } m=1, \dots, M \tag{9}$$

$$ST_{i,j+1} = ST_{ij} + V_c \sum_{m=1}^M X_{mj} t_{mi} - V_c UT_{ij} - UD_{ij} \quad \text{For } i=1, \dots, N \text{ and } j=1, \dots, K \tag{10}$$

$$\sum_{i=1}^N L_i = L_{max} \tag{11}$$

$$ST_{i+1,j} \geq ST_{ij} + V_c \sum_{m=1}^M X_{mj} t_{mi} - V_c UT_{ij} \quad \text{For } i=1, \dots, N-1 \text{ and } j=1, \dots, K \tag{12}$$

(12) Ensures that the worker at (i+1) th station will not interfere with the worker at ith station.

$$UT_{ij} \geq \sum_{m=1}^M X_{mj} t_{mi} - [(L_1 + L_2 + \dots + L_i) - ST_{ij}] / V_c \quad \text{For } i=1, \dots, N \text{ and } j=1, \dots, K \tag{13}$$

(13) Indicates the positive utility time.

$$UD_{ij} = \lambda V_o \left( \frac{V_c}{V_o + V_c} \right) \tag{14}$$

$$UD_{ij} \leq ST_{i,j+1} - ST_{ij} + V_c \sum_{m=1}^M X_{mj} t_{mi} - V_c UT_{ij} \quad \text{For } i=2, \dots, N \text{ and } j=1, 2, \dots, K \tag{15}$$

$$UD_{ij} \leq \lambda V_c \tag{15}$$

$$X_{ij} \in \{1,0\}, L_i \geq 0, UT_{ij} \geq 0, \lambda \geq 0, ST_{ij} \geq 0, UD_{ij} \geq 0 \tag{16}$$

**III. MODEL FORMULATION**

$$\text{Minimize } Z = F \sum_{i=1}^N (CO_i^{UT} \sum_{j=1}^k UT_{ij})$$

Subject to

$$\sum_{m=1}^M x_{mj} = 1 \quad \text{For } J= 1, \dots, k. \tag{1}$$

Constraint one assures that each scheduled order is occupied by exactly one model.

$$\sum_{j=1}^K x_{mj} = D_m \quad \text{For } m= 1, \dots, M \tag{2}$$

(2) All minimum part set demands are satisfied.

$$\sum_{i=1}^N L_i = L_{\max} \tag{3}$$

(3) Indicates the available station length will be used completely.

$$ST_{i,j+1} = ST_{ij} + V_c \sum_{m=1}^M X_{mj} (t_{mj} - UT_{ij}) - UD_{ij} \text{ For } i=1,\dots,N \text{ and } j=1,\dots,K \tag{4}$$

(4) Keep track of the locus of starting points at station i.

$$UT_{ij} \geq \sum_{m=1}^M X_{mj} t_{mj} - \frac{Li - ST_{ij}}{V_c} \text{ For } i=1,\dots,N \text{ and } j=1,\dots,K \tag{5}$$

1) Equation (5) shows that utility time depends on the sequenced work-piece and the available working distance.

$$UD_{ij} = \lambda V_o \left( \frac{V_c}{V_o + V_c} \right) \tag{6}$$

$$UD_{ij} \leq \lambda V_c \text{ For } i=1,\dots,N \text{ and } j=1,\dots,K \tag{7}$$

(6) And (7) ensures the operator's minimum upstream distance.

$$X_{ij} \in \{1,0\}, \lambda, L_i, ST_{ij}, UT_{ij} \geq 0, UD_{ij} \geq 0 \text{ For } i=1,\dots,N \text{ and } j=1,\dots,K$$

#### IV. PROPOSED ALGORITHM

The complete proposed algorithm for the annealing process is described here:

**STEP 0:** Determine an initial solution, which is selected from a population of 200000 randomly generated solutions, and calculate  $T_0$  and  $T_f$  by equations  $T_0 = \Delta f_{\min} + \frac{1}{10}(\Delta f_{\max} - \Delta f_{\min})$ ,  $T_f = \Delta f_{\min}$ , respectively. The temperature  $T$  is initialized to  $T_0$  and the iteration counter  $m$ , and equilibrium counter ( $r$ ), are set to 0.

**STEP 1:** Compute the objective function, select efficiency frontier and randomly select starting point, set the temporary solution  $a^* = a^0$ , and the temporary function  $E = f(a^0)$ . In executing SA, the temperature tuning

$$\beta \text{ and } M \text{ are calculated by } T_{i+1} = \frac{T_i}{1 + \beta T_i}, \beta = \frac{T_0 - T_f}{MT_0 T_f}, M = \frac{N(N-1)}{2}, \text{ respectively.}$$

**STEP 2:** Generate a feasible neighborhood search by "pairwise exchange". For pairwise exchanging, two unique products are randomly selected and exchanged. This new sequence, which obtained after exchange is referred to as the present solution and its objective value, is determined ( $f_p$ ).

**STEP 3:** Evaluate the value of the objective function after pairwise exchange:  $\Delta f = f_p - f_c$ . If  $\Delta f$  is less than or equal zero go to step 5; otherwise go to Step 4.

**STEP 4:** Exchange acceptance process: (a) If  $\Delta f \leq 0$ , then go to STEP 4b; otherwise go to STEP 4d. (b) Accept the pairwise exchange and increment the iteration counter  $m = m + 1$ , and go to step 5. (c) If  $\Delta f \geq 0$ ,

then go to STEP 4d, otherwise go to step 5. (d) Compute (Metropolis)  $P(\Delta f) = e^{\frac{\Delta f}{k_b T}}$  and select a uniform distribution with the range [0, 1]. If the random number less than  $P(\Delta f)$ , then go to STEP 4b; otherwise return to STEP 2.

**STEP 5:** Equilibrium test process

(a) If the value of objective function after exchange ( $E$ ) is less than the best value found so far ( $f_p$ ) go to step 5b; otherwise go to step 5c. (b) Change the temporary solution, and if  $m < e$  go to step 5c; otherwise go to Step

$$2. \text{ (c) If } \frac{|f_e - f_g|}{f_g} \leq \varepsilon, \text{ go to step 6; otherwise go to step 2.}$$

**STEP 6:** If  $m$ , the number of pairwise exchange examined, is greater than or equal  $M$ , then Go to step 7; otherwise change the temperature according to equation (9), increment the equilibrium counter  $r = r + 1$ , and go to step 2

**STEP 7:** Stop.

## V. COMPUTATIONAL RESULTS

To evaluate the problem in order to observe the effect of different launch intervals on the utility time and idle time, different launch intervals are selected and tested. We obtain a set of optimal line parameters. The results of total cost, total utility time and total idle time are also optima.

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