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Three-Dimensional Hydromagnetic Flow Arising In a Porous Flat Slider

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ABSTRACT : *The problem considered here is the injection of a viscous fluid through a moving flat plate in the presence of a transverse uniform magnetic field. The solution of such a flow model has applications in fluid-cushioned porous sliders, which are useful in reducing the frictional resistance of moving objects. The governing equations are reduced to a system of nonlinear ordinary differential equations by means of appropriate transformations for the velocity components. The resulting boundary value problem is solved numerically using the Matlab routine bvp4c. The influence of the magnetic field on the velocity components, load-carrying capacity and friction force are discussed in detail with the aid of graphs and tables.*

Keywords: *Hydromagnetic flow, porous slider, load-carrying capacity, friction force*

I. INTRODUCTION

Within the past sixty years, there has been remarkable interest in the flow thought channels with porous walls owing to their applications in various branches of engineering and technology. Familiar examples are boundary layer control, transpiration cooling and gaseous diffusion. In addition, blowing is used to add reactans, prevent corrosion and reduce drag. Suction is applied to chemical process to remove reactans. Much work has been done in order to understand the effect of fluid removal or injection through channel walls on the flow of various fluids. Berman [1] made an initial effort in this direction. Further contributions have been made since then by many researchers. We refer the reader to the studies by Cox [2] and Choi et al. [3], and references cited there in regarding detailed analysis of various results on this subject. In the above mentioned case the flow is two dimensional. Skalak and Wang [4] were the first study a three dimensional flow arising between moving porous flat plate and the ground. The calculations of such flows are interesting in the mechanical engineering research. Practical examples of flowsof this type include hydrostatic thrust bearings, air-cushioned vehicles and porous sliders. It is well known fact that fluid-cushioned porous sliders are useful in reducing the frictional resistance between two solid surfaces moving relative to each other. Previous studies include the porous circular slider [5] and porous elliptic slider [6, 7]. Later, for a second-order viscoelastic fluid, fluid dynamics analysis of lift and drag of a porous elliptic slider was done by Bhatt [8] obtaining the first-order perturbation solution for the case of a very low cross-flow Reynolds number. Ariel [9] extended Skalak and Wang's analysis to a Walter's B viscoelastic fluid. In this study, the perturbation and exact numerical solution were obtained. Recently, Khan et al. [10] obtained a series solution of the long porous slider problem using the homotopy perturbation method. In their subsequent research [11], they solved the same problem using Adomian decomposition method. Faraz [12] studied the circular porous slider problem using variational iteration algorithm. More recently, Wang [13] has investigated the effect of slip on the performance of the porous slider. Baris and Dokuz[14] have presented a theoretical study for the elliptic porous slider using an elastic-viscous fluid. A literature survey clearly indicates that no solutions have been given for the three dimensional flows of this type in the presence of a uniform magnetic field. Therefore, the present study aims to solve such a problem involving the porous flat slider by introducing a constant magnetic field, and to assess qualitatively the effect of the magnetic field on the components of velocity, lift and drag.

II. PROBLEM FORMULATION

We consider the steady, incompressible laminar flow of a Newtonian viscous fluid between a porous flat slider and the ground in the presence of a uniform magnetic field. Figure 1 shows the physical model and the coordinate system for the problem under discussion.

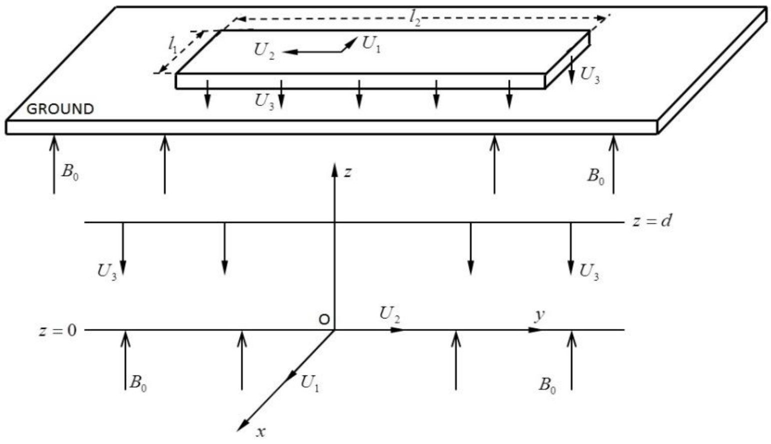

Figure 1 Sketch of flow geometry and coordinate system

A fluid is forced through the porous bottom of the slider and thus separates the slider from the ground. The supply pressure is assumed to be large enough to cause a nearly constant injection velocity U_3 through the slider. The slider is moving laterally with constant velocities U_1 and U_2 along the negative x- and y-directions, respectively. We fix Cartesian coordinates x, y, z on the slider such that the slider is motionless and the ground moves laterally with constant velocities U_1 and U_2 along the positive x- and y-directions, respectively. We have further assumed $l_2 \gg l_1 \gg d$ such that end effects can be neglected.

In a reference frame translating with slider, let u, v, w be the velocity components of the fluid in the directions x, y, z, respectively. Following Skalak and Wang [4], we look for a solution, compatible with continuity equations, of the form

$$u = U_1 g(\eta) + \frac{U_3 x}{d} f'(\eta)$$
$$v = U_2 h(\eta) \qquad (1)$$
$$w = -U_3 f(\eta)$$

Where $\eta = z/d$ is the similarity variable. The prime above denotes the differentiation with respect to η .

An external uniform magnetic field B_0 is applied in the z-direction. The magnetic Reynolds number is assumed to be very small. In this case, the induced magnetic field produced by motion of fluid can be ignored in comparison to the applied one. In addition, the imposed and induced electric fields are assumed to be negligible, thus the electromagnetic body force per unit volume simplifies $\mathbf{F}_{em} = \sigma_0 (\mathbf{v} \times \mathbf{B}) \times \mathbf{B}$, where $\mathbf{B} = (0, 0, B_0)$ is the magnetic field vector and σ_0 is the electrical conductivity. Due to the assumption stated above, Maxwell's equations become redundant.

The equations expressing conservation of momentum are as follows:

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] - \sigma_0 B_0^2 u \qquad (2)$$

$$\rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] - \sigma_0 B_0^2 v \qquad (3)$$

$$\rho \left[u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] \qquad (4)$$

Where ρ is the density, p the pressure, μ the dynamic viscosity. The last terms on the right hand sides of Eqs.(2) And (3) result from electromagnetic body forces. Note that we neglect non-magnetic body forces.

The boundary conditions of the problem are

$$\begin{aligned} &u(0)=U_1 \;,\; v(0)=U_2 \;,\; w(0)=0 \\ &u(1)=0 \;,\; v(1)=0 \;,\; w(0)=-U_3 \end{aligned} \qquad (5)$$

Under the above assumptions, substituting Eq.(1) into Eqs.(2)-(4)and eliminating the pressure term from these equations we obtain

$$f'''+R\left(f f''-f'^2\right)-M^2 f'=C \qquad (6)$$

$$g''+R\left(f g'-g f'\right)-M^2 g=0 \qquad (7)$$

$$h''+R f h'-M^2 h=0 \qquad (8)$$

Where $R=\dfrac{U_3 \rho d}{\mu}$ is the cross flow Reynolds number, $M=B_0 d\sqrt{\sigma_0/\mu}$ is the magnetic parameter and C is an unknown constant.

The boundary conditions transform to

$$\begin{aligned} &\eta=0 : f(0)=0 \;,\; f'(0)=0 \;,\; g(0)=1 \;,\; h(0)=1 \\ &\eta=1 : f(1)=1 \;,\; f'(1)=0 \;,\; g(1)=0 \;,\; h(1)=0 \end{aligned} \qquad (9)$$

It is recorded that in the absence of magnetic parameter M Eqs. (6)-(8)together with the associated boundary conditions (9) are the same as those obtained by Skalak and Wang [4].

For the problem under consideration, it is important to find the load-carrying capacity L and friction force components D_x and D_y. These physical quantities can be calculated by integrating pressure and shear stress components on the slider. The dimensionless expressions for the load-carrying capacity and friction force components are given through the following equations:

$$L^*=\frac{12\nu^2}{\rho U_3^4 l_2 l_1^3}\iint_S (p-p_A)\,dS=-\frac{1}{R^3}f'''(0) \qquad (10)$$

$$D_x^*=\frac{1}{\rho U_1 U_3 l_1 l_2}\iint_S -\tau_{zx}\,dS=-\frac{1}{R}g'(1) \qquad (11)$$

$$D_y^*=\frac{1}{\rho U_1 U_2 l_1 l_2}\iint_S -\tau_{zy}\,dS=-\frac{1}{R}h'(1) \qquad (12)$$

Where p_A is the pressure at the edge of the slider.

III. NUMERICAL RESULTS AND DISCUSSION

The system of nonlinear ordinary differential equations (6)-(8) under the relevant conditions given in Eqs. (9) Constitute a two-point boundary value problem with no analytical solution. This is why we have decided to obtain numerical solution for the problem under discussion. To numerically solve the above boundary value problem we have used to Matlab solver boundary value problem (bvp4c). This solver employs a collocation method which produces continuous solution on an appropriate mesh. Mesh selection and error control are based on the residual of the continuous solution. The approximate solution satisfies the set of ODEs at both ends also at the midpoint of each interval $[\eta_i,\eta_{i+1}]$ using a fourth-order accurate Lobatto IIIA formula. We set the relative and absolute tolerance equal to 10^{-6} .For more information about bvp4c solver and its performance in solving boundary value problems, the reader is referred to Ref.[15]. To validate the numerical method used in the present work, we compared our results for the values of $f''(0), g'(0), h'(0)$ and C with those of Skalak and Wang [4]. We saw excellent agreement with existing results in [4] for the case of $M=0$.

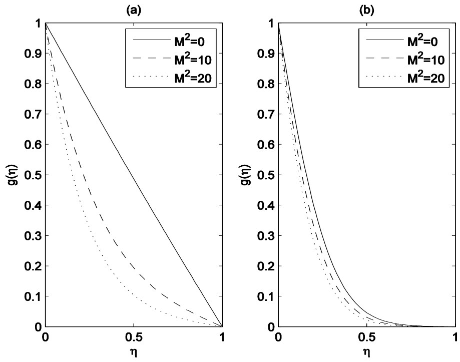

Figure 2 (a) Lateral velocity profile in the x-direction for $R = 0.1$ **(b)** Lateral velocity profile in the x-direction for $R = 10$

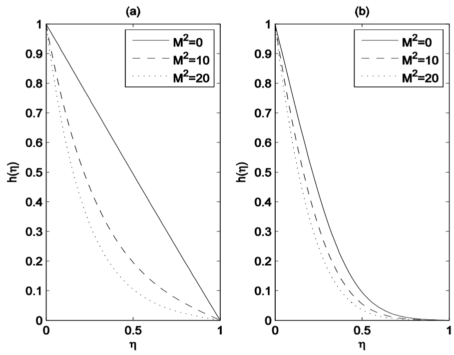

Figure 3 (a) Lateral velocity profile in the y-direction for $R = 0.1$ **(b)** Lateral velocity profile in the y-direction for $R = 10$

The predictions based on the foregoing analysis are displayed graphically in Figures 2 and 3. In these figures the functions which correspond to the lateral velocity components are plotted versus η for two different values of the cross-flow Reynolds number R, with M^2 as a parameter. It is clear from these figures that the lateral velocity profiles is linear for $R = 0.1$ in the non-magnetic case while these profiles become highly nonlinear for increasing values of R and M.

Again from these figures we observe that with an increase in the value of the cross-flow Reynolds number, the lateral velocity components decrease. Moreover, increasing the magnetic parameter decreases the lateral velocity components further. This result qualitatively agrees with expectation since the application of a transverse magnetic field normal to the lateral flow directions has a tendency to create a drag-like Lorentz force. This force decreases the lateral velocity components.

For a porous slider, the important physical quantities lift L^* and drag components D_x^* and D_y^*. It is interesting to note that the lift L^* is independent of translation, but the drag components D_x^* and D_y^* depend on the cross flow. Table 1 provides the lift L^*, the drag components D_x^* and D_y^*, and the ratios of friction forces to lift (D_x^*/L^*, D_y^*/L^*). It can be easily seen from these tables that both lift and drag components increase rapidly, although at different rates, when the strength of cross flow decreases. Physically this can be explained as follows: if everything else is held fixed, the decrease in the value of cross flow Reynolds number results only from the decrease in the gap width. In this case, since the changes in the values of the velocity components occur in the smaller distance, velocity gradients become larger. It is for this reason that both stress components in the fluid layer lift and drag components on the slider increase considerably as the cross-flow Reynolds number decreases.

The efficiency of a porous slider can be increased by making the ratio of fiction force to lift smaller. As pointed out by Wang [6], porous slider should be operated at cross-flow Reynolds number R less than unity for optimum efficiency. Table 1 shows that the fact that porous sliders should be operated at small values of R still remains valid even when an external uniform magnetic field is applied. Moreover, from the optimum efficiency point of view, it is more efficient to move a flat slider on a fluid subject to a magnetic field with high intensity.

Finally, we observe from Table 1 that the ratio of friction forces to lift increases with an increase in R , up to a critical value of R (say, R_c) in the internal $3 < R_c < 5$ and thereafter decrease with increasing R . Therefore, it is desirable to operate the slider beyond the critical cross-flow Reynolds number R .Note that for a porous flat slider R is approximately 4 in the absence of the magnetic field [4].

Table 1 Lift and drag components for some values of R and M^2

R	M^2	L^*	D_x^*	D_y^*	D_x^*/L^*	D_y^*/L^*
0.2	0	1558,13	4,47950	4,65900	0,002875	0,002990
	10	3039,23	1,22850	1,25900	0,000404	0,000414
	20	4494,66	0,47200	0,48100	0,000105	0,000107
0.6	0	62,0718	1,19917	1,34333	0,019319	0,021642
	10	116,708	0,34300	0,36900	0,002939	0,003162
	20	170,454	0,13383	0,14183	0,000785	0,000832
1	0	14,3658	0,57790	0,69380	0,040227	0,048295
	10	26,1166	0,17200	0,19390	0,006586	0,007424
	20	37,6892	0,06820	0,07510	0,001810	0,001993
3	0	0,71689	0,06427	0,10213	0,089647	0,142468
	10	1,14149	0,02263	0,03157	0,019828	0,027654
	20	1,56253	0,00973	0,01277	0,006229	0,008171
4	0	0,34293	0,02770	0,04905	0,080776	0,143034
	10	0,51967	0,01048	0,01603	0,020157	0,030837
	20	0,69557	0,00468	0,00665	0,006721	0,009561
5	0	0,19667	0,01268	0,02462	0,064473	0,125184
	10	0,28594	0,00512	0,00850	0,017906	0,029727
	20	0,37511	0,00236	0,00362	0,006291	0,009651
6	0	0,12617	0,00602	0,01263	0,047687	0,100129
	10	0,17715	0,00257	0,00462	0,014489	0,026061
	20	0,22824	0,00123	0,00203	0,005404	0,008909
10	0	0,03810	0,00037	0,00096	0,009711	0,025197
	10	0,04860	0,00019	0,00043	0,003909	0,008848
	20	0,05923	0,00010	0,00021	0,001688	0,003546

IV. CONCLUSIONS

In this paper, we are concerned with a theoretical investigation of steady three-dimensional flow of a viscous fluid between a porous flat slider and ground in the presence of a transverse uniform magnetic field. By means of appropriate similarity transformations, the governing equations are reduced to a set of ordinary differential equations. The transformed nonlinear ordinary differential equations were solved numerically using the Matlab routine bvp4c. The effects of values physical parameters like the cross-flow Reynolds Number R and magnetic parameter M on the lateral velocity profiles, lift and drag components were presented in graphical and tabular forms. It was found that the relevant parameters have a strong influence on the results. It is hoped that the results of present study may be useful for understanding of various technological problems related to porous sliders.

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