American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-6, pp-30-37
www.ajer.org
Research Paper Open Access

Experimental Modal Analysis of a Flat Plate Subjected To Vibration

Owunna Ikechukwu[1], *Ikpe Aniekan[1], Satope Pau[1] & Ememobong Ikpe[2]
[1]Department of Mechanical Engineering, Faculty of Engineering and Computing, Coventry University, West Midlands, UK.
[2]Department of Instrumentation and Control, Exxon Mobil Producing Nigeria, Ibeno, Nigeria

ABSTRACT: *Modal analysis is significant in evaluating the mode shapes generated by a component under vibrational excitation, as the mode shapes can be used to determine the displacement or response of the component under the influence vibration in real life application. Result obtained from the modal analysis will generate a number of resonances which the frequency and damping effect can be determined by measurement. However, determining the accuracy of modal analysis result is somewhat difficult as the experimental results and the results generated by Finite Element Analysis (FEA) solvers can be affected by a number of factors pointed out in this paper. In this study, a flat plate was mounted on an electromagnetic shaker which enabled the excitation of the plate, while results of the response were measured using a transducer attached to the plate. The plate was also modelled using CATIA software and the files transferred to the different FEA solvers such as HYPERMESH, ANSYS 6 Degree of Freedom (DOF) as well as ANSYS 5 degree of freedom, in which the same analysis was carried out to obtain a set of results other than the experimental results. Each FEA solver generated results that were in close proximity with the experimental results, particularly the results generated by ANSYS 5 Degree of freedom. Hence, to ascertain the accuracy of the results obtained from modal analysis experimental procedure, it is important to match up the results generated from different FEA solvers with the experimental results.*

Keywords: *Experiment, Modal analysis, Vibration, Flat plate, Modal shapes*

I. INTRODUCTION

Vibrations ensue when a body is subject to any arrangement of forces. In other words, high intensities of strain, stress and noise are set up in the body as a result of vibrations. Unlike static deformations, the degree of vibration in a body under a system of force is reliant on both the magnitude and the period of the exciting force. Vibrations in different structures occur at different frequencies. An inherent property of any structure as related to vibration is the natural frequency of the structure. If a structure vibrates at frequencies higher or closer to the natural frequency of the body, the vibration may be exceptionally high (Ivan et al., 2014). It is therefore necessary to evaluate the natural frequencies of structures under loading. Modal analysis is a process to describe various natural features of a geometry (structure) which may include damping, mass, modes shapes (dynamic properties) and frequency (Polytech, 2001). Generally resonant vibrations can be characterised by simple and effective use of Modes. Modes are known to be inherent properties of a structure. This within a structure is caused by interaction between the elastic properties and inertial of the materials. In analysis of structure, Modal analysis helps to identify areas of weakness in design that needs improvements.

In many engineering applications, the natural frequencies at which a body vibrates is of utmost importance. Estimating the natural frequencies of a vibrating body is a common aspect of dynamic analysis and can be referred to as an 'eigenvalue analyses. The un-damped free vibration response of a structure called mode shapes is also an important inherent property of a structure. The free vibration response is caused by initial disturbance from the static equilibrium position.

1.1 Mode Shapes and Natural Frequency

In order to understand vibration problems, identification and quantification of the resonance of a structure needs to be understood. There are two types of vibration namely: Forced vibration and resonant vibration. Forced vibration can be caused by external loads, internally generated forces/loads, unbalances and ambient excitation while resonant vibration is majorly caused by natural modes of vibration of the body (Brain, 1999). The mode shape of the resonance dominates the total vibration shape (operating deflection shape) of structures at or near the natural frequency (Brain, 1999). The Operating deflection shape is defined as any forced motion of two or more points on structure. The shape is defined by specifying the motion of two or more points noting that motion is a vector quantity which indicates that it possesses magnitude and direction. Degree of freedom is better expressed as a motion at a point in a direction. It is worth noting that the measured deflected shapes determine all experimental modal parameters. There are two kinds of Modes which are rigid and flexible body modes (Brain, 1999). Furthermore, all structures possess up to six rigid body modes which are three rotational modes and three translational modes. The fundamental properties of a freely vibrating, un-damped system are mode shapes and natural frequencies. For a multi-degree-of-freedom system at the choice of co-ordinates do not affect the values of the natural frequencies of the system. It is however not the case for mode shapes as mode shapes is dependent on co-ordinate. However, the motions of the physical system defined by the mode shapes will be identical. The natural frequencies are thus said to be fundamental properties that are not dependent on the choice of coordinates. The mode shapes are often determined by finding the Eigen values and the Eigen vectors of the system.

1.2 Frequency, Eigen Values and Eigen vectors

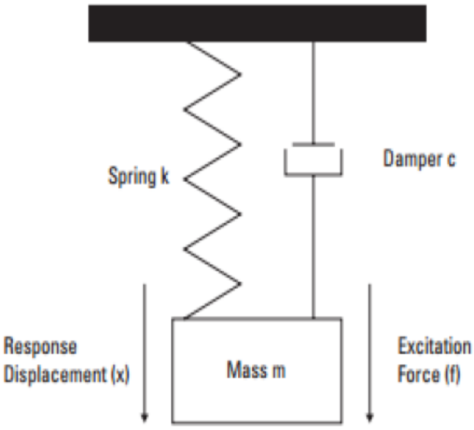

Figure 1: Single degree of freedom systems

The natural frequency, ω, is in units of radians per second (rad/s). The typical units displayed on a digital signal analyzer, however, are in Hertz (Hz). To analyze equation the system shown in the Figure 1, the force acting on the system is summed up.
This becomes;

$$m\ddot{x} + c\dot{x} + Kx = F \qquad (1)$$

Where
$\ddot{x}$ is acceleration
$\dot{x}$ is velocity
x is displacement
c is damping constant
F is excitation force
k is spring
If the displacement and the Force are is written as a cosine function. Such that:

$$x = Xcos\omega_n t \qquad (2)$$

And

$$F = F_o cos\omega_n t \qquad (3)$$

The equation becomes:

$$m\omega^2 * cos\omega_n t + c\omega Xsin\omega_n t + KXcos\omega_n t = F_o cos\omega_n t \qquad (4)$$

For un-damped vibrations, c=0
The equation becomes;

$$-m\omega^2 Xcos\omega_n t + KXcos\omega_n t = F_o cos\omega_n t \qquad (5)$$

$$m\omega^2 X + KX = F_o \qquad (6)$$

For free vibrations; $F_o = 0$,

The equation becomes:

$$-m\omega^2 X + KX = 0 \qquad (7)$$

$$m\omega^2 - K = 0$$

$$m\omega^2 = K \quad (8)$$

Thus,

$$\omega^2 = \frac{K}{m} \quad (9)$$

$$\omega = \sqrt{\frac{K}{m}} \quad (10)$$

ω_n Is the natural frequency of the body.

1.3 Objective:

i. To learn how to excite a system at different frequencies
ii. To understand the response of a plate subjected to a periodic load
iii. To compare the measured natural frequencies with those calculated
iv. To study the influence of different boundary conditions on the plate's response
v. To present the findings as a formal laboratory report

1.4 Apparatus Supplied:

i. Shaker
ii. Square plate
iii. Transducer
iv. Impact hammer
v. LDS Vibration Test Equipment
vi. Data acquisition system
vii. Vernier caliper
viii. Steel rule
ix. Flat Metal plate

The data acquisition system employed in this experiment is shown in Figures 2 and 3

Figure 2: Final Setup of the Experiment

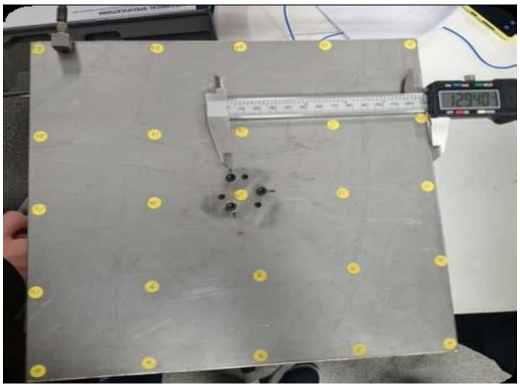

Figure 3: Square Plate for analysis

1.5 Experimental Procedures

With the aid of Vernier calipers, the different dimensions of the metal plate to be excited are taken. These measurements are needed for the modelling of the plate in any FEA analyzer for comparism. The plate is mounted on the data acquisition system. This would be able to process the vibrations in the plate and process them into frequencies. The shakers are then mounted on the plate. This would help to notice the mode shapes at the different vibrating frequencies of the plate. All equipment's are then observed to ensure they are in good condition and zero error on measuring equipment's is removed. The plate is divided into sixteen (16) different nodes. Each of which will be excited to completely understand the natural modes of vibration of the plate. The impact hammer is used to excite the plate at the first node. The impact is a very minor one as it is intended to create a response that is small and traceable. The data acquisition system uses the results of the vibration motion to plot a curve of frequency. The process is repeated for the other 15 nodes of the plate. The total results are then processed to obtain the natural frequencies and mode shape. Record the frequencies obtained and the Eigen mode shapes of the plate for different frequencies.

1.6 Precautions

There are a few set of precautions necessary in this experiment. To avoid errors in the whole experimental results at large, the precautions to take include:

i. Ensure at every time that there are enough grains on the metal to have a good view of the mode shapes.
ii. Ensure that the impact hammer is not metallic to ensure that the exciter does not deform the flat plate under study.
iii. Ensure that correct safety wears are in place and used appropriately.
iv. Avoid applying to much force to help ensure stability of the whole instrument being engaged.

II. CALCULATIONS

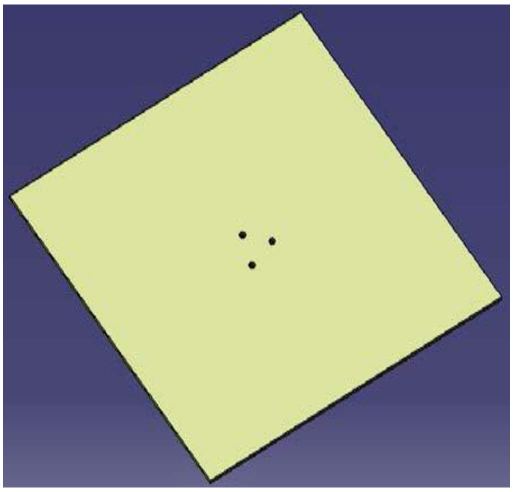

Figure 4: Model of Flat plate

For plates under free vibration, the natural frequency is written in term density, young's modulus, second moment of area and the plate dimensions.

Recall,

$$m = \rho V \tag{11}$$

Thus,

$$m = \rho l^3 \tag{12}$$

Also the stiffness (k) is written as;

$$k = \frac{EI\varepsilon}{l(1-\nu^2)} \tag{13}$$

The natural frequency can then be re-written as;

$$\omega = \sqrt{\frac{EI\varepsilon}{\rho l^3 * l * (1-v^2)}} \quad (14)$$

But $I = \frac{lh^3}{12}$ (15)

Therefore, ω becomes,

$$\omega = \sqrt{\frac{Elh^3\varepsilon}{\rho l^4 * 12 * (1-v^2)}} \quad (16)$$

$$= \sqrt{\frac{l\varepsilon}{12}}\sqrt{\frac{Eh^3}{\rho l^4\,(1-v^2)}} \quad (17)$$

$$\omega = B\sqrt{\frac{Eh^3}{\rho l^4\,(1-v^2)}} \quad (18)$$

B is a factor that is dependent on the constraints on the body

2.1 Design Parameters

The plate shown in Figure 4 is modelled to be a cantilever beam with loads at its free end. The various properties of the material are presented below;

L = 250mm = 0.25m, H = 1.5mm = 0.0015m, v=0.3, E = 210GPa, ρ = 7850kgm^{-3}

Substituting the design parameters into equation 18 we have

$$\omega = 4.07\sqrt{\frac{210 * 10^9 * 0.015^3}{7850 * 0.25^4\,(1-0.3^2)}}$$

ω = =20.51Hz

The other results are produced in Table 1;

Table 1: Calculated results of the natural frequencies of the flat plate

Mode	**ωHz**
1	20.51188
2	29.93625
3	34.82483

The experimental results obtained are shown in Table 2

Table 2: Experimental results obtained for Natural Frequencies

Mode	Experimental Results
1	20.1
2	28.3
3	97.9
4	
5	145.6
6	226.3
7	273
8	346

III. RESULTS AND ANALYSIS

The two methods for obtaining the natural frequencies and the mode shapes of the body gave the results shown in Table 1 and 2. Comparatively, this showed that the results in Table 1 and 2 were very close. However, the results obtained from the experiment had a wider difference compared to the calculated values of the frequencies. It can be assumed that the shaker on which the plate was mounted for the experimental results caused the vibration. Based on the calculation, it was considered that the stiffness of the plate is negligible while in the experiment, the shaker was considered to have some level of stiffness which may have interfered with the experimental results. To further demonstrate the process of determining the natural frequencies and mode shape, Finite Element Approach (FEA) was used. Three different FEA solvers were employed. The plate was modelled using CATIA software and the files transferred to the different solvers. The first model of the design was constrained at the holes in all degrees of freedom. A pictorial view of the different constrains is shown in Figure 5.

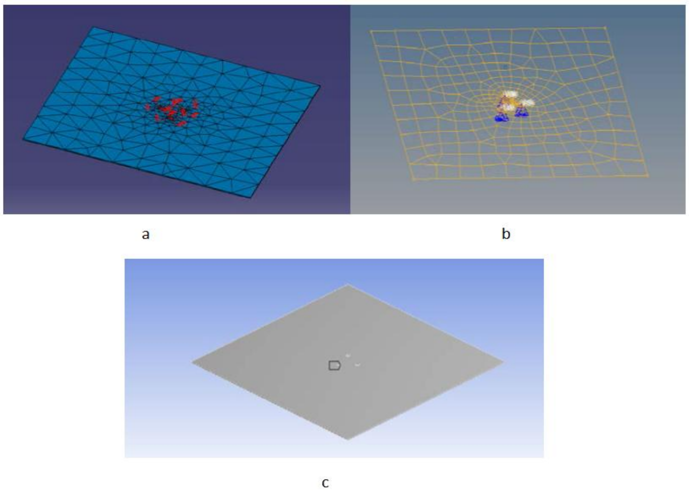

Figure 5: Constrained Model of the flat Plate: **(a)** CATIA Model, **(b)** HYPERMESH Model, **(c)** ANSYS Model

With this constrains, the results obtained is shown in Table 3

Table 3: Tabulated Results of Natural Frequencies

Mode	ω (Hz)				
	Experimental Results	HYPERMESH 6DOF	ANSYS	CATIA	Calculations
1	20.1	55.88	57.16	69.54	20.51
2	28.3	56.91	58.15	70.18	29.94
3	97.9	71.87	71.979	77.123	34.82
4		80.64	80.821	84.934	
5	145.6	117.72	118.83	126.83	
6	226.3	210.13	214.17	249.92	
7	273	211.10	214.32	266.57	
8	346	281.10	288.66	401.07	
9		387.17	403.66	508.46	
10		395.42	411.87	525.79	

The result shown in Table 3 is graphically presented in Figure 6. The graph shows that the HYPERMESH and ANSYS results are similar but relatively lower than the values obtained in the experimental results. In all cases, the curves look alike.

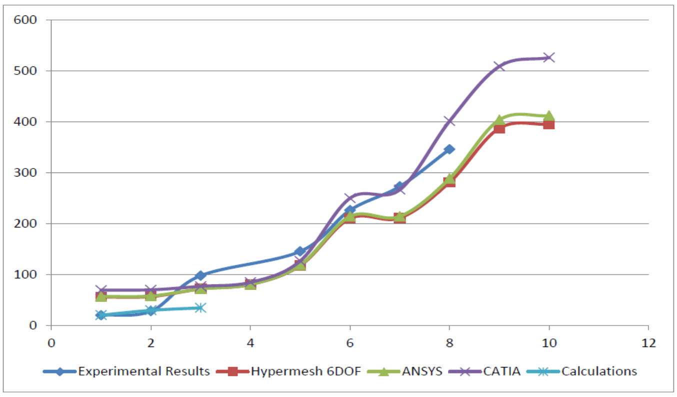

Figure 6: Plot of the Natural Frequencies of the flat Plate

The variation in result may be said to have result from the fact that the experiment was done such that the body is not constrained in the z-direction. It is possible to simulate this in HYPERMESH. The first analysis in HYPERMESH was done such that the plate was constrained in all the 6-degrees of freedom. In the second design, only 5 degrees of freedom (DOF) were constrained. The plate was allowed to move freely in the z-directions. This puts the model closer to that obtainable in the real life experiment mounted on a shaker. The results from the second analysis showed a considerable difference from that of the first analysis. The results are tabulated as shown in Table 4.

Table 4: Natural Frequency with 5 DOF Included

Mode	*ω* (Hz)			
	Experimental Result	HYPERMESH 6DOF	HYPERMESH 6DOF	Calculations
1	20.1	55.88	48.4	20.51
2	28.3	56.91	48.86	29.94
3	97.9	71.87	79.89	34.82
4		80.64	116.41	
5	145.6	117.72	142.45	
6	226.3	210.13	207.43	
7	273	211.10	208.16	
8	346	281.10	364.79	
9		387.17	381.08	
10		395.42	384.21	

The result in Table 4 showed that the movement in the z axis is considerably important and that it has a significant effect on the natural frequency of the plate. To further aid this investigation. A plot of the values from both simulations is shown in Figure 7. The plot showed that the result from the 5 DOF were more acceptable and resembles that from the experiment.

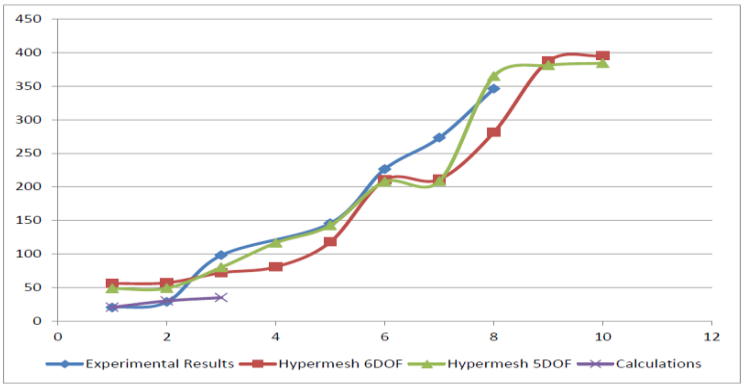

Figure 7: Plot of the natural frequencies of the flat plate with 5 DOF included

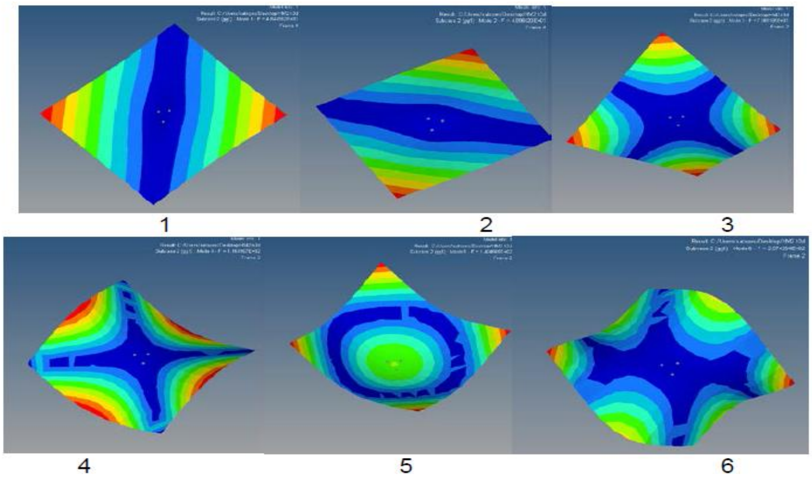

Figure 8: H3D Plots of Mode Shapes of a Flat Plate

The mode shapes of the flat plate are plotted in H3D as shown in Figure 8. All the mode shapes for the different finite element analysis Solvers are shown in the appendix. A critical look at the mode shapes from different solvers show that the results are very close to each other. There is a very great resemblance between the plots from the experiment and those obtained in the finite element analysis.

IV. FACTORS THAT AFFECTED THE NATURAL FREQUENCY OF THE PLATE

From the experiment and the simulations carried out in this work, it was observed that natural frequency of the plate was affected by two major factors which included the shape or rather said dimensions of the plate and the loads exerted on it.

4.1 Sources of Errors

Sources of error in experimental modal analysis can be categorized in three groups which are the experimental date acquisition errors, signal processing errors and modal analysis errors. The quality of the frequency response functions measured is adversely affected by the various factors especially noise and systematic errors which determines the accuracy and reliability of the experimental modal analysis (Karakan, 2008). The factors affecting the response functions therefore include the following;

I. Noise measurement

i. Noise from power supply
ii. Cable motion, rattles, cabling problem

II. Structure boundary condition

i. Fixed
ii. Free

III. Calibration (Error from the operator)

i. Transducer calibration
ii. Complete system calibration

IV. Hammer tip

i. Soft tip, super soft tip, hard tip, soft tip.

V. Amplifiers and transducers

i. Sensitivity and accelerometer
ii. Location and attachment of transducers

VI. Errors and solutions from digital signal processing

i. Leakage
ii. Filtering
iii. Averaging
iv. Aliasing
v. Zooming

VII. Non-linearity

The FEA software(s) used does introduce some errors when modelling which might be as a result of the following;

i. Computational error while rounding off
ii. Boundary conditions poorly approximated
iii. Individual shape functions selection
iv. Mesh generation quality
v. System damping properties poorly modelled

V. CONCLUSION

The results of the various analysis showed that any of the methods of analysing modal frequencies of a body can be used. However, the limitations of each method in describing the overall real life situation must be taken into consideration. In the instance of this experiment, the shaker introduced stiffness in the z-direction which was well modelled in HYPERMESH. The results from HYPERMESH gave the closest results to the real life situation in the experiment. The shape of the body and the load on the system as well as the degree of freedom of the body determine majorly the natural frequency of any system whose modes is being analysed. In all, The FEA analysis, the hand calculations and experimental result were close enough to conclude that minimum error was introduced during the conduct of the experiment. In all, the FEA analysis, the hand calculation and the experimental results were close enough to conclude that minimum errors were introduced during the experimental exercise.

REFERENCES

[1]. Brain, J. S. (1999) Experimental Modal Analysis. [online] available from <http://systemplus.co.jp/support/data/techpaper/mescope/tech/28.pdf> [01 January 2015]
[2]. Ivan B., Marco, A. and Mathias, L. (2014) Physically and Geometrically Non-linear Vibrations of Thin Rectangular Plates. International Journal of Non-Linear Mechanics 58: 30-40
[3]. Polytech, G. (2001) Basic of Experimental Modal Analysis. Polytec GmbH, Polytec-Platz 1-776337 Waldbronn Germany
[4]. Karakan, E. (2008) Estimation of Frequency Response Function for Experimental Modal Analysis. [online] available from <http://openaccess.iyte.edu.tr:8080/xmlui/bitstream/handle/11147/3697/T000699.pdf?sequencee=1> [13 January 2015].