American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-3, pp-12-21
www.ajer.org
Research Paper Open Access

A Novel Convergence Approach for an Adaptive Equalizers

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ABSTRACT : *Linear equalizers were derived either on the deterministic Zero Forcing (ZF) approach for equalizers of ZF type or on the stochastic Minimum Mean Square Error (MMSE) approach for equalizers of the MMSE type. We present a new formulation of the equalizer problem based on a Weighted Least Squares (WLS) approach. Here, we show that it is possible and in our opinion even simpler to derive the classical results in a purely deterministic setup, interpreting both equalizer types as Least Squares solutions. This, in turn, allows the introduction of a simple linear reference model for equalizers, which supports the exact derivation of a family of iterative and recursive algorithms with optimize behavior. Due to this reference approach the adaptive equalizer problem can equivalently be treated as an adaptive system identification problem for which very precise Statements are possible with respect to convergence, optimization and l_2-stability.*

Keywords: *Zero Forcing (ZF), Minimum Mean Square Error (MMSE), Least Squares (LS), WeightedLeastSquare (WLS) and SingleInput SingleOutput (SISO).*

I. INTRODUCTION

Linear equalizers were designed either on the deterministic ZF approach for equalizers of ZF type or on the stochastic MMSE approach for equalizers of the MMSE type. We proposed a new formulation of the equalizer problem based on a weighted least squares (LS) approach. This deterministic concept is very much in line with Lucky so original formulation [11], leaving out all signal and noise properties (up to the noisevariance) but at the same time offers new insight into the equalizer solutions, as they share common LS or thogonality properties. This novel LS approach allows very general formulation to cover a multitude of equalizer problems, including different channel models, multiple antennas as well as multiple users [1].

In practice, the equalizer problem is not yet solved once the solution is known, as it typically involves a matrix inversion, a mathematical operation that is highly complexandchallenginginlow-costfixed-pointdevices.Adaptivealgorithmsarethus commonly used to approximate the results. Suchadaptive algorithmsforequalization purposescomeintwoforms, aniterative (alsooff-lineorbatch process) approach as well asarecursiveapproach (alsoon-lineordata-drivenprocess) that readjusts its estimates oneachnewdata elementthat isbeingobserved. Both approaches have their benefits and drawbacks. Ifchannelestimation isperformedinapreviousstep (forvarious reasons), then the iterative algorithm based onthe channelimpulseresponsemaybemost effective.Onthe other hand, itisnot required tocompute firstthe channelimpulseresponseifonlythe equalizer solution isofinterest. In particularintime-variantscenarios, onemay not have the chance to continuously estimate thechannelandthen compute equalizersolutionsiteratively andtherefore, arecursive solution that isableto track changes, may betheonlyhopeforgood results[2],[3].

However, such adaptive algorithms require a deep understanding of their properties as selecting their free parameter, the step-size, turns out to be crucial. While forward cascades adaptive filter designers were highly satisfied when they able to prove convergence in the mean-square sense, more and more situations now become known, in which this approach has proved to be insufficient, since, despite the convergence in them enquire sense, the worst case sequences exist that cause the algorithm to diverge. This observation has started with Feintuchs adaptive IIR algorithm and the class of adaptive filters with a line are filter in the error path [4],[5]but has recently found in other adaptive filters[6],[7],as well as in adaptive equalizers[8]. A robust algorithm design, on the other hand, is much more suited to solving the equalization problem as it can guarantee the adaptive algorithm will not diverge in any case. In this contribution we show how to design robust, adaptive filters for linear equalizers [9],[10].

II. Formulation For Transmission Model

Throughout this paper, we adopt that the separable transmit signal elements s_k have unit energy $E[|s_k|^2]=1$,and the noise variance at the receiver is given by $E[|v_k|^2]=N_0$ .We are considering several similar but distinct transmission schemes:

2.1 Single User (SU) Transmission for Frequency Selective SISO Channels

The following SU transmission defines frequency selective (also called time dispersive)single-input single-output (SISO)scenarios:

$$\boldsymbol{r}_k = \boldsymbol{H}\boldsymbol{s}_k + \boldsymbol{v}_k \tag{1}$$

Here, the vector $s_k = [s_k, s_{k-1}, \ldots, s_{k-S+1}]^T$ consists of the current and $S-1$ past symbols according to the span $L < S$ of the channel H , which is typically the Toeplitz form as describe in (2). The received vector is defined as $r_k = [r_k, r_{k-1}, \ldots, r_{k-R+1}]^T$. Let the transmission be disturbed by additive noise v_k being of the same dimension as r_k .

$$\begin{bmatrix} \boldsymbol{r}_k \\ \boldsymbol{r}_{k-1} \\ \vdots \\ \boldsymbol{r}_{k-R+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_0 & \boldsymbol{h}_1 & \cdots & \boldsymbol{h}_{L-1} & & & \\ & \ddots & \ddots & & & \ddots & \\ & & \boldsymbol{h}_0 & \boldsymbol{h}_1 & & \cdots & \boldsymbol{h}_{L-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{s}_k \\ \boldsymbol{s}_{k-1} \\ \vdots \\ \boldsymbol{s}_{k-S+1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_k \\ \boldsymbol{v}_{k-1} \\ \vdots \\ \boldsymbol{v}_{k-R+1} \end{bmatrix} \tag{2}$$

Note that for a toeplitz form channel H we have $R < S$. A linear equalizer applies an FIR filter f on the received signal r_k so that $f^H r_k$ is an estimate of $s_{k-\tau} = e_\tau^T s_k$ for the delayed version of s_k . A unit vector $\mathbf{e}_\tau = [0,\ldots,0,1,0,\ldots,0]^T$ with a single one at position τ facilitates the description.

2.2 Single User (SU) Transmission for Frequency Selective MIMO Channels

The transmissions follow the same form as described in equation (5), although with a different meaning as we transmit over N_T antennas and receive by N_R . Suchmultiple input multiple- outputsystems are generally referred toasMIMO systems. The transmit vector $s_k = \left[s_{1,k}, s_{2,k}, \ldots, s_{N_T,k}\right]^T$ is of dimension $1\times N_T$, the channel matrix H ,and thus the receive vector and the additive noise vector are of dimension $1\times N_R$. Here, N_T is the number of transmit antennas. As in the previous case, we assume each entry of the transmit vector to have unit power. Unlike the earlier situation, however, we have to distinguish $N_R > N_T$ (under determined LS solution) and $N_R < N_T$ (over determined LS solution). For $N_R = N_T$ both solutions coincide. In order to detect the various entries of the transmit vector s_k , we again employ a unit vector $e_t : e_t^T s_k = s_{t,k}$. Note however that in contrast to the previous channel model, a set of N_T different vectors $e_t ; t = 1,2,\ldots,N_T$ will be employed in order to select all N_T transmitted symbols while in the frequency selective SISO case a single vector e_τ is sufficient. Early works on linear MIMO equalization can befound in [15] and [16]. Note that precoding matrices are often applied in particular in modern cellular systems such as HSDPA and LTE. In this case the concatenation of the precoding matrix and the wireless channel has to be considered as a new compound channel. Such precoding matrices can also have an impact on the dimension of the transmit vector s_k as in many cases fewer symbols than rank are transmitted at each time instant k . A particular form of this is given when the precoding matrix shrinks to a vector, in which case we talk about beamforming where only one symbol stream is transmitted.

2.3 Maximizing SIR and SINR

To understand the vast amount of research and information available on this subject, one has to ask the question “What is the purpose of an equalizer?” While Lucky’s original work focused on the SU scenario, attempting a minimax approach to combat intersymbolinterference (ISI), today we typically view the equalizer in terms of signal-to-interference ratio (SIR) or signal-to-interference and noise ratio (SINR). If a signal, say s_k , is transmitted through a frequency selective channel, a mixture of ISI, additive noise and signals from other users multiuser interference (MUI) is received. If signals are transmitted by multiple antennas, then additional so-called spatial ISI (SP-ISI) is introduced. The ratio of the received signal power P_s and all disturbance terms before an equalizer indicated by the index 'be') is easily described as

$$SINR_{be} = \frac{P_s}{P_{ISI} + P_{SP-ISI} + P_{MUI} + N_0} \tag{3}$$

The task of the equalizer is to improve the situation, i.e., to increasethis ratio. A linear filter applied to the observed signal can for example result in an increased $P_S^{'} > P_s$,utilizing useful parts of P_{ISI} and P_{SP-ISI} , while the remaining and/or $P_{SP-ISI}^{'} < P_{SP-ISI}$ and/or $P_{MUI}^{'} < P_{MUI}$ is decreased. Unfortunately, the noise power N_0 as well as its power spectraldensity is in general also changed when an equalizer filter is applied.At best it can be considered possible to achieve the post equalizationSINR (the index 'ae' denotes after equalization)

$$SINR_{ae} \leq \frac{P_s + P_{ISI} + P_{SP-ISI}}{N_0} \tag{4}$$

Where the equalizer manages to treat the ISI and SP-ISI as useful signal whilst at the same time eliminating the MUI (for example by successive interference cancellation). The ratio of $SINR_{be}$ to the eventually achieved $SINR_{ae}$ is considered as the equalizer gain. The purpose of this paper is to develop a unified view of the SINR and SNR relation to the MMSE and ZF equalizer, which permits the introduction of a simple linear reference model as well as its application in an adaptive system identification framework.

III. A Reference Model for An Adaptive Equalizers

While classical literature views the equalizer problem as minimizing a mean square error, we show in the following section that this is in fact not required and a purely deterministic approach based on a least squares modeling is possible. This approach in turn leads to the novel interpretation of the adaptive equalizer problem in terms of a classic system identification problem. For such problems, however, a much stronger l_2 -stability and robustness has been derived in the past to ensure convergence of the adaptive algorithms under worst case conditions. In order to apply such robust techniques, we first have to show the equivalent system identification approach for equalizers. We start with the ZF equalizer and then continue with its MMSE counterpart [8].

3.1 Zero Forcing (ZF) Equalizer

A solution to the ZF equalizer problem is equivalently given by the following LS formulation:

$$\begin{aligned} f_{\tau,t,m}^{ZF} &= \arg\min \| H^H f - e_{\tau,t,m} \|_2^2 \\ &= \arg\min \| H^H [f - (HH^H)^{-1} H e_{\tau,t,m}] \|_2^2 \end{aligned} \tag{5}$$

With $e_{\tau,t,m}$ indicating a unit vector with a single one entry at Position τ , for transmit antenna t of user m , thus $e_{\tau,t,m}^T s_k = s_{k-\tau,t,m}$, the transmit signal at antenna t of user m that will be decoded at delay lag τ . Note that this form of derivation requiresno signal or noise information, focusing instead only onproperties of linear time-invariant systems of finite length (FIR);it thus entirely ignores the presence of noise. This is identicalto Lucky's original formulations [14], where system propertieswere the focus and the particular case of $N_T = 1, M = 1$ was considered. If $RN_R < SN_T M$ (for example, in Lucky's SISO frequency selective scenario, we have $R < S$) the solution to this problem is obviously given by

$$f_{\tau,t,m}^{ZF,o} = (HH^H)^{-1} H e_{\tau,t,m} \tag{6}$$

Commonly known as the ZF solution. Note that this is a so-called overdetermined LS solution as we have more equations than entries in $f_{\tau,l,m}^{ZF}$. When $RN_R > SN_T M$ an alternative so-called underdetermined LS solution exists, as long as $rank(H) = SN_T M$

$$f_{\tau,t,m}^{ZF,o} = H(HH^H)^{-1} e_{\tau,t,m} \tag{7}$$

And requires independent consideration as will be provided further on in this section.
Let us first consider the overdetermined case of (7). As ISIdoes not vanish for finite length vectors, we propose the followingreference model for ZF equalizers

$$e_{\tau,t,m} = H^H f_{\tau,t,m}^{ZF,0} + v_{\tau,t,m}^{ZF,0} \tag{8}$$

With the modeling error vector

$$v_{\tau,t,m}^{ZF,o} = (I - H^H (HH^H)^{-1} H) e_{\tau,t,m} \tag{9}$$

The term $v_{\tau,t,m}^{ZF,o}$ models ISI, SP-ISI, and MUI. The larger the equalizer length RN_R, the smaller the ISI, e.g., measured in $\| v_{\tau,t,m}^{ZF,o} \|_2^2$. The cursor position τ also influences the result.

3.2 Minimum Mean Square Error (MMSE) Equalizer

MMSE solutions are typically derived on the basis of signal and noise statistics [21], e.g., by

$$f_{\tau,t,m}^{MMSE} = \arg\min E[| f^H r_k - s_{k-\tau,t,m} |^2] \tag{10}$$

However, the linear MMSE solution can alternatively be defined by

$$\begin{aligned} f_{\tau,t,m}^{MMSE} &= \arg\min(\| H^H f - e_{\tau,t,m} \|_2^2 + N_0 \| f \|_2^2) \\ &= \arg\min \| (HH^H + N_0 I)^{\frac{1}{2}} [f - (HH^H + N_0 I)^{-1} H e_{\tau,t,m}] \|_2^2 + MMSE \end{aligned} \tag{11}$$

With an additional term, according to the additive noise variance N_0. We consider here white noise; alternative forms with colored noise, as originating, for example from fractionally spaced equalizers, is straightforward; one only has to replace $N_0 I$ with R_{vv}, the autocorrelation matrix of the noise.

This formulation of the equation (11) has now revealed that the MMSE problem equivalently can be written as a weighted LS problem with the

$$MMSE = e_{\tau,t,m}^T [I - H^H (HH^H + N_0 I)^{-1} H] e_{\tau,t,m} \tag{12}$$

Defines the minimum mean square error. As the term is independent of f, it can thus be dropped when minimizing equation (11). The well-known MMSE solution is now obviously

$$f_{\tau,t,m}^{MMSE} = (HH^H + N_0 I)^{-1} H e_{\tau,t,m} \tag{13}$$

Similarly to the ZF equalizer, an over determined solution for $RN_R < SN_T M$ also exists here.

$$f_{\tau,t,m}^{MMSE,o} = H(HH^H + N_0 I)^{-1} e_{\tau,t,m} \tag{14}$$

Under white noise both solutions are in fact identical $f_{\tau,t,m}^{MMSE,o} = f_{\tau,t,m}^{MMSE}$, which is very different to the ZF equalizer. Correspondingly, to thereference model for ZF equalizers in equation (8), we can now alsodefine a reference model for MMSE equalizers

$$e_{\tau,t,m} = H^H f_{\tau,t,m}^{MMSE} + v_{\tau,t,m}^{MMSE} \tag{15}$$

With the modeling error

$$v_{\tau,t,m}^{MMSE} = (I - H^H (HH^H + N_0 I)^{-1} H) e_{\tau,t,m} \tag{16}$$

Note, however, that unlike in the case of the ZF solution themodeling error is not orthogonal to the MMSE solution, i.e., $v_{\tau,t,m}^{MMSE} H^H f_{\tau,t,m}^{MMSE}$ is not equal to zero. MMSE equalizers are typically designed on the basis of observations rather than system parameters.Multiplying the signal vector with $e_{\tau,t,m}$ we obtain

$$e_{\tau,t,m}^T s_k = s_{k-\tau,t,m} = f_{\tau,t,m}^{MMSEH} H S_k + v_{\tau,t,m}^{MMSEH} S_k \tag{17}$$

How does a received signal look after such MMSE-equalization? We apply on the observation vector and obtain

$$\begin{aligned} f_{\tau,t,m}^{MMSE,H} r_k &= s_{k-\tau} - v_{\tau}^{MMSE,H} s_k + f_{\tau,t,m}^{MMSE,H} v_k \\ &= s_{k-\tau} + \tilde{v}_{k,t,m}^{MMSE} \end{aligned} \tag{18}$$

From classic equalizer theory it is well known that the remainingISI energy of the ZF equalizer is smaller than that ofthe MMSE parts.The weighted LS solution $f_{\tau,t,m}^{MMSE}$ of equation (17),applied to the observation vector r_k, defines a linear referencemodel in which the desired output signal is $s_{k-\tau}$, originating from a transmitted signal over antenna t of user m, corrupted by additive compound noise $\tilde{v}_{k,t,m}^{MMSE}$. The compound noise is defined by $f_{\tau,t,m}^{MMSE,H} v_k$ as well as by the modeling noise $v_{\tau}^{MMSE,H} s_k$, defined by the modeling error vector $v_{\tau}^{MMSE,H}$ in equation (16)

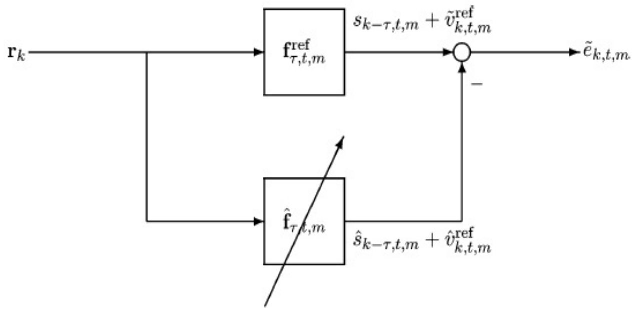


Fig 1: Adaptive Equalization as System Identification problem.

In conclusion, the adaptive equalizer problem has thus taken on the form of an identification problem as depicted in Fig 1. The linear system with impulse $f^{ref}_{\tau,t,m}$ response is estimated as $\hat{f}_{\tau,t,m}$ by an adaptive equalizer algorithm. Here, 'ref' stands for either MMSE or ZF. The outcome of the reference system is disturbed by the compound noise $\tilde{v}^{ref}_{k,t,m}$ (see equation (18)) and constructs a noisy reference symbol $s_{k-\tau,t,m}$. The adaptive filter with its output $\hat{s}_{k-\tau,t,m} + \hat{v}^{ref}_{k,t,m}$ tries to resemble $s_{k-\tau,t,m} + \tilde{v}^{ref}_{k,t,m}$. The distorted error signal $\tilde{e}_{k,t,m}$ is applied to the adaptive filter in order to adjust the equalizer solution.

IV. An Iterative Algorithms for An Adaptive Equalizers

Equalizer solutions requiring matrix inverses are highly complex and numerically challenging, in particular when the matrix size is 50 or over. An iterative algorithm, as referred to here, is one that possesses all data and attempts to achieve an optimal solution. In the literature such algorithms are also referred to at times as off-line or batch algorithms since they require no new data during their operation. In this contribution we show convergence conditions for numerous known and novel algorithms, but do not deal with the question of when to stop the iterations [1].

4.1 An Iterative Zero Forcing Equalizer (IZF)

Let Starting with an initial value f_0 (which can be the zero vector), we arrive at the ZF iterative algorithm for $X = H$

$$\hat{f}_l = \hat{f}_{l-1} + \mu H(e_\tau - H^H f_{l-1}), \quad l = 1,2,3,..... \tag{19}$$

With the reference model in equation (8) we can introduce the parameter error vector $\tilde{f}_l = f^{ZF}_\tau - \hat{f}_l$ and obtain we recognize that the noise condition is satisfied, as property $H v^{ZF}_\tau = 0$ for $\bar{v} = v^{ZF}_\tau$. Convergence conditions for the step-size μ are now also readily derived, being dependent on the largest Eigen value of HH^H.

$$0 < \mu < \frac{2}{max\lambda(HH^H)} \tag{20}$$

As computing the largest eigenvalue may be a computationallyexpensive task, simpler bounds are of interest, even thoughthey may be conservative.

1. A classic conservative bound is given by

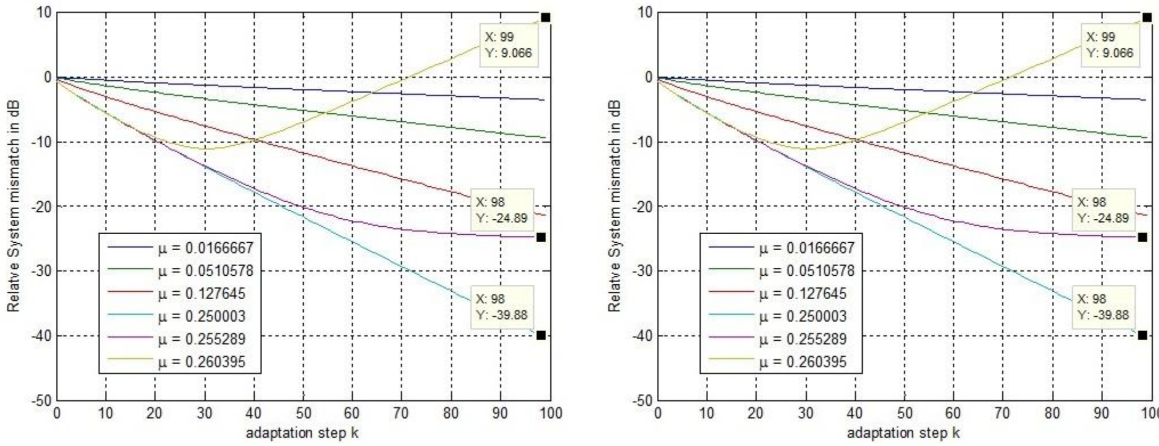
$$0 < \mu < \frac{2}{Trace(HH^H)} \tag{21}$$

And can be computed with low complexity once the matrix H is known.

2. For a Single User in a frequency selective SISO channel, the channel H is defined by a single Toeplitz matrix, the largest eigenvalue of which can also be bounded by $max_\Omega |H(e^{j\Omega})|$, with $H(e^{j\Omega})$ denoting the Fourier transform of the channel impulse response. The corresponding condition for the step-size reads now

$$0 < \mu < \frac{2}{max_\Omega |H(e^{j\Omega})|^2} \tag{22}$$

Such a step-size may be more conservative than the condition in equation (20) but it is also more practical to find.
In the simulation examples presented the bound so obtained is very close to the theoretical value in equation (20).



(i)System Mismatch (ii)Relative Error

Fig.2:I t e r a t i v e Zero Forcing Equalizer

Depending upon different step size conditions we have calculated Relative System mismatch and Error. Here as the number of iterations increases error decreases means we are converging towards desired values of filter weights.

4.2 An Iterative Fast Convergent An Zero Forcing Equalizer (IF-ZFE)

As the convergenceof the previous equalizer algorithm (Iterative ZF Algorithm) is dependent on the channel matrix H , the algorithm exhibits much slower convergence forsome channels than for others, even for optimal step-sizes. Theanalysis of the algorithm shows that the optimal matrix B that ensures fastest convergence is given by $B = [HH^H]^{-1}$, which is exactly the inverse whose computation we are attempting toavoid with the iterative approach. If, however, some a prioriknowledge is present on the channel class (e.g., Pedestrian Bor Vehicular A), then we can precompute the mean value overan ensemble of channels from a specific class, for example

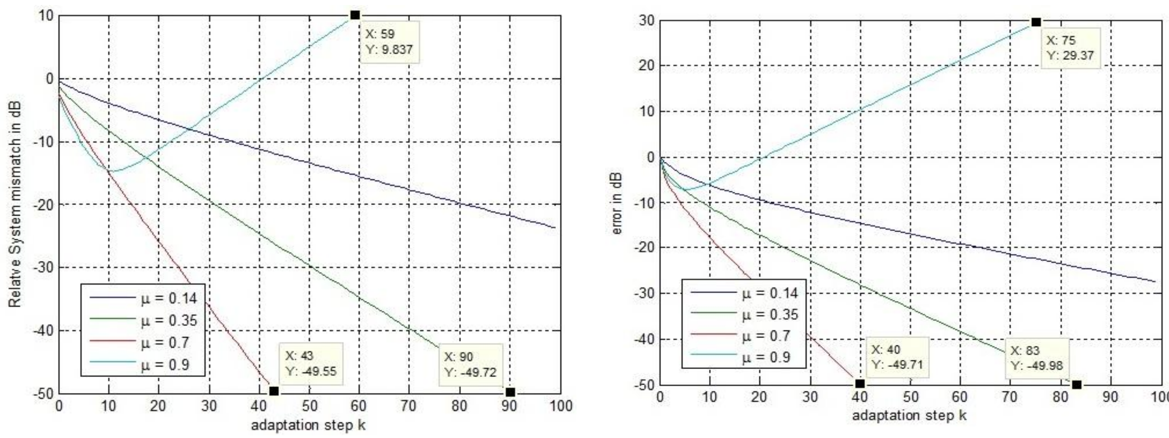
$$E[[HH^H]^{-1}] = \overline{R_{HH}^{-1}} \tag{23}$$

In this case, the algorithm updates read

$$\hat{f}_l = \hat{f}_{l-1} + \mu \overline{R_{HH}^{-1}} H(e_\tau - H^H \hat{f}_{l-1}); \; l = 1,2,..... \tag{24}$$

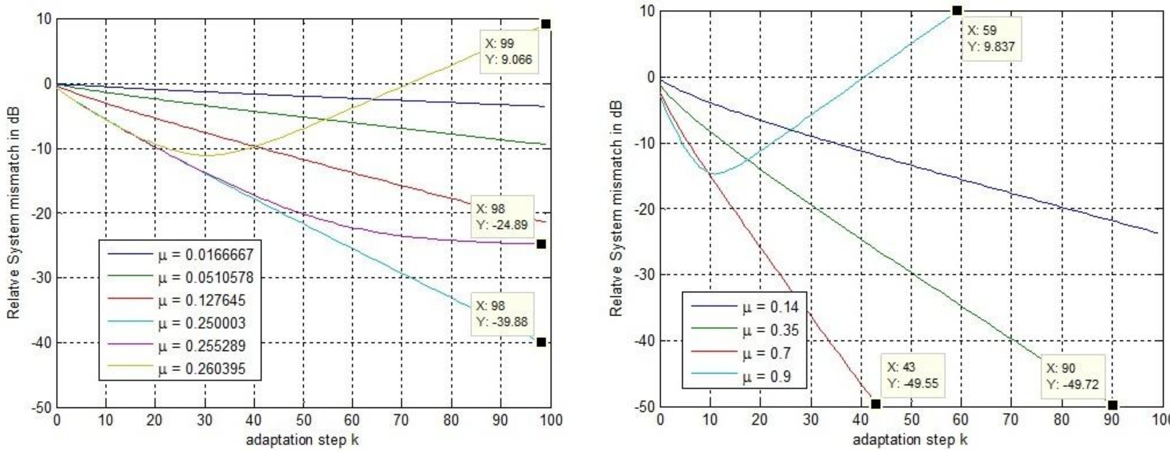
Convergence condition for this algorithm will be

$$0 < \mu < \frac{2}{max\lambda(HH^H)} \tag{25}$$



(i)System Mismatch (ii)Relative Error

Fig.3:Fast Convergent of An Iterative Zero Forcing Equalizer



(i)IZFE (ii)IF-ZFE

Fig.4:SystemMismatch Comparison ofIZFE v/s IF-ZFE

The convergence speed of above Zero Forcing Equalizer depends upon the channel, for some channels it is slowly convergent and for others it is fast convergent. For IF-ZFE, We can observe from results that this algorithm is fast convergent as compared to anIZFEalgorithm as it reaches the desired value in very few iterations.

4.3 An Iterative Minimum Mean Square Error Equalizer (IM^{2}SE^2)

Let's start with our MMSE reference model equation (15), we consider the following update equation

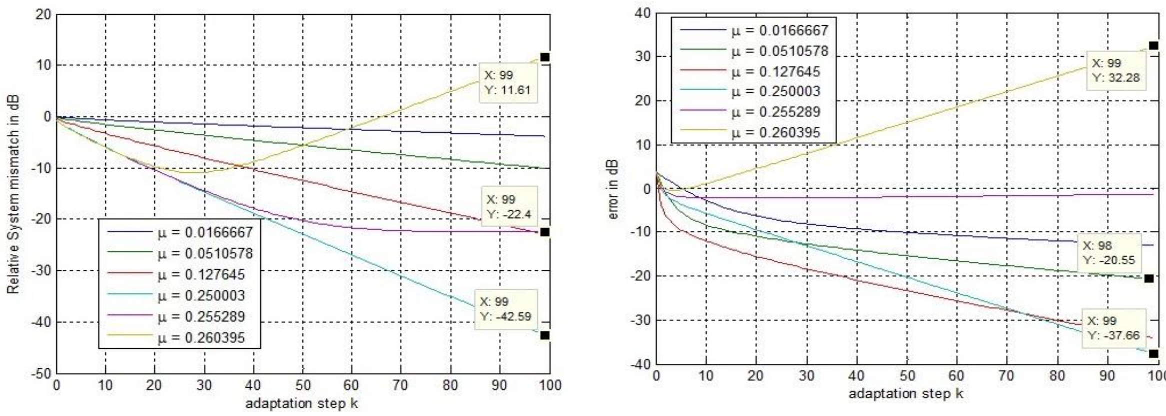
$$\tilde{f}_k = \tilde{f}_{k-1} + \mu X(He_\tau - (HH^H + N_0 I)\tilde{f}_{k-1}) \tag{26}$$

We can thus reformulate equation (26) into

$$\tilde{f}_k = \tilde{f}_{k-1} - \mu X(HH^H + N_0 I)\tilde{f}_{k-1} \tag{27}$$

If we select $X = I$, we can identify $B = HH^H + N_0 I$ and we find as a condition for convergence that

$$0 < \mu < \frac{2}{max\lambda(HH^H + N_0 I)} \tag{28}$$



(i)System Mismatch (ii)Relative Error

Fig.5:IterativeMinimumMeanSquareErrorEqualizer

The an IZF Equalizer does not consider channel noise, it cannot deal with noisy channel, and to deal this problem we designed MMSE Equalizer which considers channel noise in its algorithm for calculating the step size and Equalizer solution. Here also depending upon different step size conditions we have calculated Relative System mismatch and Error. Here as the number of iterations increases error decreases means we are converging towards desired values of filter weights.

4.4 An Iterative Fast Convergent Minimum Mean Square Error Equalizer (IF-M^2SE^2)

As the convergence of the previous equalizer algorithm (Iterative MMSE Algorithm) is dependent on the channel matrix H, the algorithm exhibits much slower convergence for some channels than for others, even for optimal step-sizes. The analysis of the algorithm shows that the optimal matrix B that ensures fastest convergence is given by $B = [HH^H]^{-1}$, which is exactly the inverse whose computation we are attempting to avoid with the iterative approach. If, however, some a priori knowledge is present on the channel class (e.g., Pedestrian B or Vehicular A), then we can precompute the mean value over an ensemble of channels from a specific class, a speedup algorithm is possible with

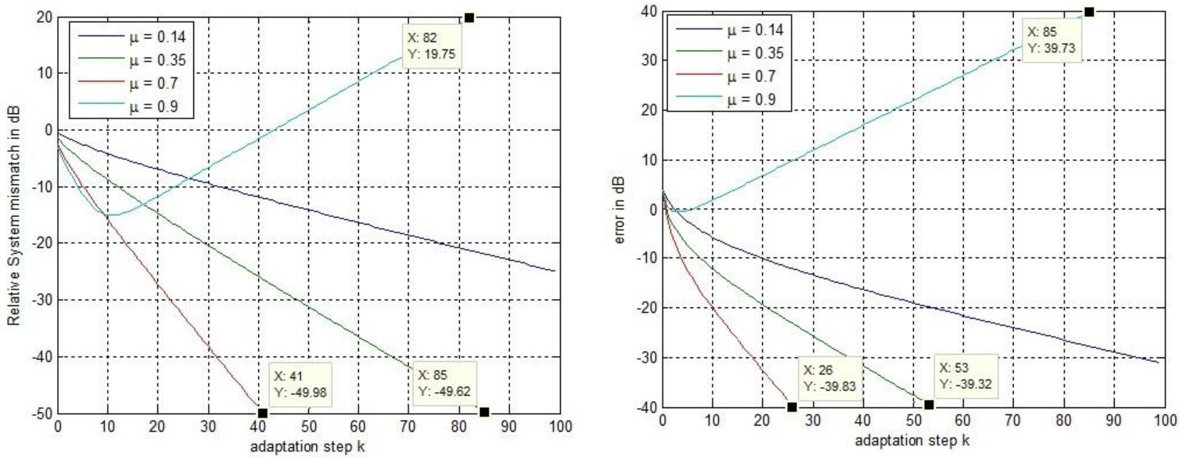
$$X = (R_{HH} + N_0 I)^{-1} \tag{29}$$

In this case our Fast Convergent MMSE Algorithm will become

$$\hat{f}_k = \hat{f}_{k-1} + \mu (R_{HH} + N_0 I)^{-1} (He_\tau - (HH^H + N_0 I)\hat{f}_{k-1}) \tag{30}$$

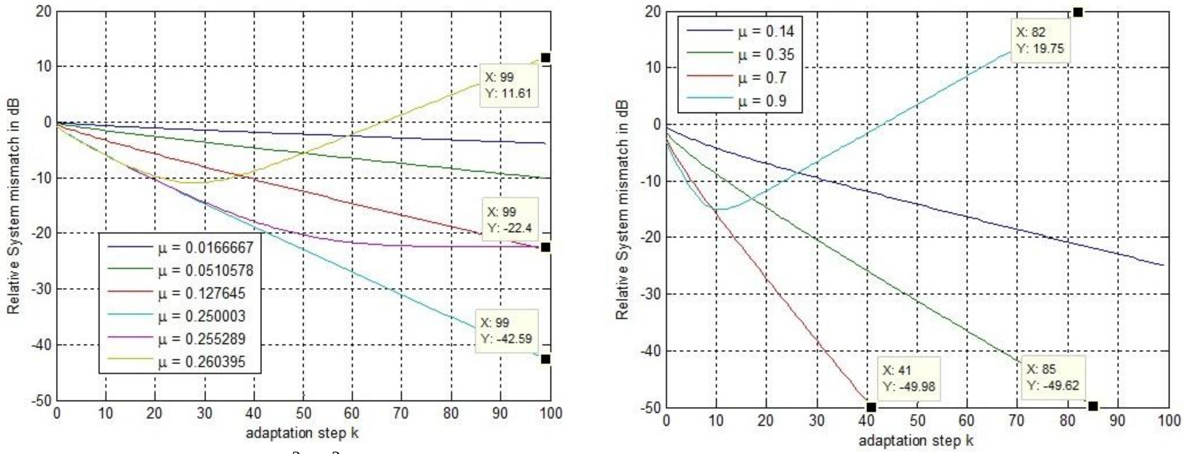
The Convergence condition for this algorithm will be

$$0 < \mu < \frac{2}{max\lambda (HH^H + N_0 I)} \tag{31}$$



(i)System Mismatch (ii)Relative Error

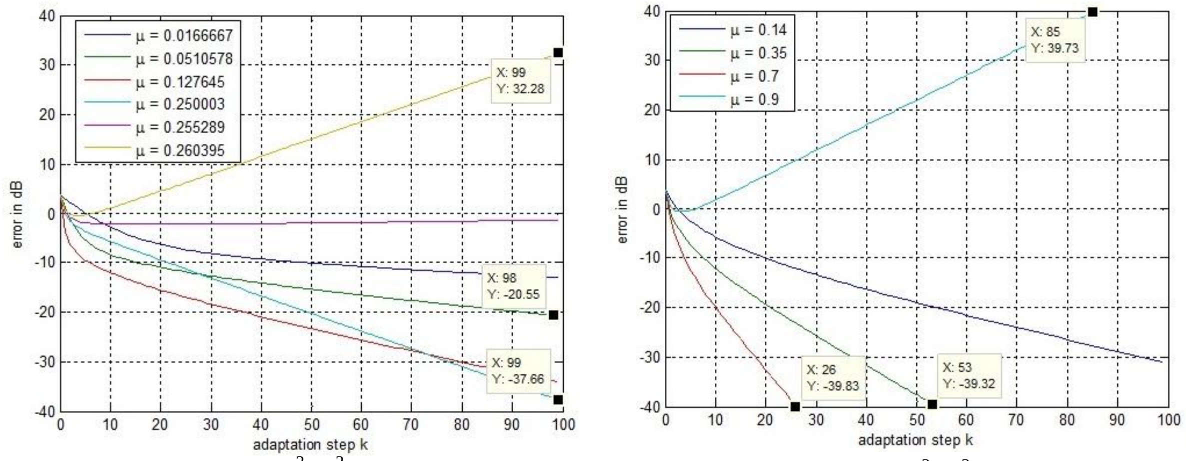
Fig.6:Fast Convergent An Iterative Minimum Mean Square Error Equalizer



(i)IM^2SE^2 (ii)IF- M^2SE^2

Figure7:SystemMismatch Comparison ofIM^2SE^2 v/s IF- M^2SE^2

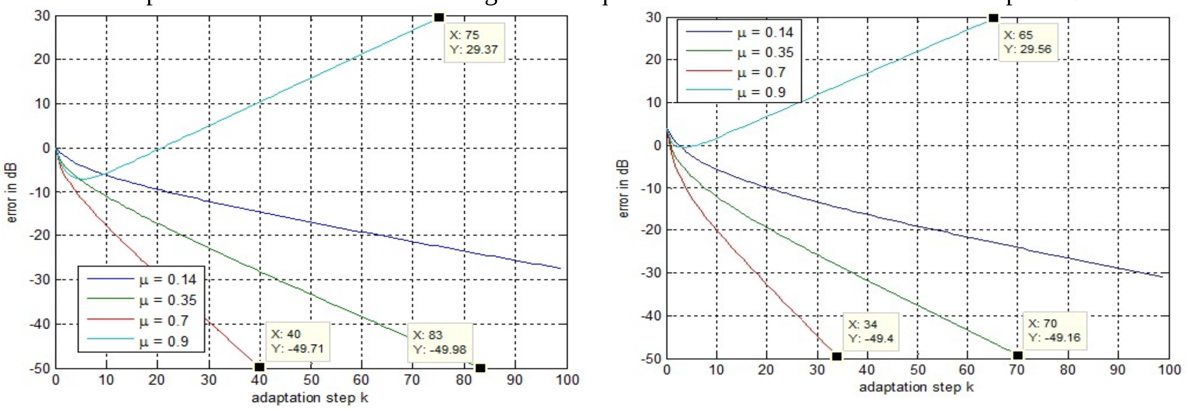
The convergence speed of above IM^2SE^2depends on the channel, for some channels it is slowly convergent and for others it is fast convergent. For IF- M^2SE^2, We can observe from results that this algorithm is fast convergent as compared to aIM^2SE^2algorithm as it reaches the desired value in very few iterations. We have also compared IZF with IM^2SE^2, which is shown in the results. From the results, we can observe that the relative Error of $IM^2SE^2$is less as compared to IZF, because IZF Equalizer can't deal with noisy channels this problem we have overcome using IM^2SE^2 Equalizer which reduces ISI as well as noise power.



(i)IM^{2}SE^2 (ii)IF- M^{2}SE^2

Figure8:Relative Error Comparison ofIM^{2}SE^2 v/s IF- M^{2}SE^2

As compared to an IM^{2}SE^{2}algorithm as it reaches the desired value in very little iteration. We have also compared an IZF with a n IM^{2}SE^2, which is shown in the results. From the results, we can observe that the relative Error of a n IM^{2}SE^{2}isless as compared to an IZF, because an IZF Equalizer can't deal with noisy channels this problem we have overcomeusingIM^{2}SE^{2}EqualizerwhichreducesISIaswellasnoisepower.



(i)Relative Error IZF (ii)RelativeErrorIM^{2}SE^2

Fig.9:RelativeErrorComparison ofIZFandIM^{2}SE^{2}Equalizers

In order to perform out theoretical findings, we present selected Matlab examples, we consider a set of seven channels impulse response of finite length [8] with the length of the channel to be $M = 50$ for which even the first four impulse responses have decayed considerably. If we run an iterative receiver (also of 50 taps), the result for $h_k^{(3)}$ is depicted on the left-hand side (LHS) of Figures, with f_0 denoting the ZF solution and $\hat{f}_i$ denoting its estimate. Based on the convergence condition in equation (20) it is possible to compute the exact step-size bound (0.255), given the channel matrix H . Also shown in the figure are the conservative bound in equation (20), which is the smallest step-size (0.017) in the figure, resulting in the slowest convergence speed and equation (21), which is just a fraction smaller (0.25 vs. 0.255) than the step-size bound. The average inverse autocorrelation R_{HH}^{-1} is computed over all seven channels, and applied in the algorithm's updates. This results in a considerable speed-up in the iterations as proposed and is depicted on the right-hand side (RHS) of Figures.

V. Conclusion

Due to an LS approach it is now possible to derive the classical equalizer types with an alternative formulation, and LS formulation for IZF and a weighted LS formulation for IM^{2}SE^2 equalizers. This in turn resulted in a linear reference model for both. Based on such a linear reference model, it is possible to derive iterative forms of equalizers that are robust. Conditions for their robustness were presented, and in particular ranges for their only free parameter, the step-size, were presented to guarantee robust learning. We have also compared IZF and IM^{2}SE^2 and IF- M^{2}SE^2 Equalizers, it is found that IF- M^{2}SE^2 Equalizer performs better as compared to IZF and IM^{2}SE^{2}Equalizer. Simulation example validates our findings.

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