Queuing Model for Banking System: A Comparative Study of Selected Banks in Owo Local Government Area of Ondo State, Nigeria

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ABSTRACT: Comparative study of two selected banks (Wema & Skye Bank) in Owo Local Government Area of Ondo state was investigated. The research study employed a queuing model for both banking system to measure the behavioural queuing characteristics of customers in terms of their arrival and service rate respectively. The data for the arrival and service rate of the two banks were collected simultaneously at the space of three days. The data collected was also analysed separately with respect to the queuing theory parameters. Findings showed that Wema bank has a high waiting service time value (0.019%) which may have a negative significant effect on customers on waiting lines. When compared with Skye bank, we realized that Skye bank has an advantageous effect of service time value (0.000022%) which will have a positive significant effect on customers in experiencing little or no queue at all. This clearly indicates that for optimum efficiency in the banks, there is need to increase the number of servers. Also, the result of the respective banks shows that waiting lines reduces when the number of servers increases. The analysis in this study is very effective and practical, and is hereby recommended for the management of the banks to review for better efficiency and performance.

KEYWORDS: Queuing model, Banking system, Waiting lines, Customers, Owo

Notations

- $P_n$: Probability of exactly $n$ customers in the system
- $L_s$: Expected number of customers in the system
- $L_q$: Expected number of customers in the queue
- $W_s$: Average time a customer spends in the system
- $W_q$: Expected waiting time of customers in the queue
- $X$: Number of servers
- $\lambda$: The mean arrival rate (Expected number of arrival per unit time)
- $\mu$: The mean service rate for overall systems

I. INTRODUCTION

Challenges encountered by most commercial or business organisation are the problem of waiting lines. It is so serious that most customers complained of lack of special treatment and care by the management of the commercial organisation. There is need to give customers a special care during service delivery because when a particular customer is satisfied to a certain level, all available customers are attracted to the organisation, this brings about the expected high profit.

Kasum et al., (2006) evaluated queue efficiency in Nigeria banks through comparative analysis of old and new generation banks. Their study employ a primary data collected from selected bank through a well structured questionnaire along the inverted funnel method. The questionnaires were administered by the customers of the banks. Their findings showed that time spent on queue for services in old generation bank is in aggregate longer than in the new generation bank. Also, their findings showed that new generation banks are more efficient in timely service delivery than the old generation banks.
Pei-Chun et al., (2006) examined the service efficiency of 26 banking institutions in Taiwan (including the postal banking services) through the application of queuing theory by evaluating service efficiency of ATMs functions composed of cash withdrawal, fund transferring, password alternations and balance inquiry. They fitted a queuing model for evaluating the service efficiency of ATMs. They collected a field data from July to August 2004. Their findings suggested that some bank should add more ATMs to reduce customer waiting time.

Tian et al., (2011) examined the queuing system of bank based on business process reengineering. They evaluated the bottleneck problems of bank queuing as well as the concept, classification and methodologies of business process reengineering. They used simulation method to analyse the number of open servers of the bank system. Their findings showed that if the bank use simulating method to determine the number of open servers by referring to dynamic statistics; it will improve much in flexibility and make full use of the current resources.

Toshiba et al., (2013) established an optimization model of queuing theory for the improvement of bank service. In their study, they converted the M/M/Z∞/FCFS model into M/M/1/∞/FCFS to know which one is more efficient, a line or more lines. Their findings showed that the efficiency of commercial banks is improved by the queuing number, the service stations number, and the optimal service rate; therefore making the results effective and practical, and increasing customer satisfaction.

II. METHODOLOGY

Optimization Method
The study made use of two selected banks in Owo Local Government Area of Ondo State, Nigeria. The banks are Wema Bank and Skye Bank respectively. The queuing system in the selected banks is the bank service system. The following assumptions were made for the queuing system at the selected banks, in accordance with the queuing theory:

- Poisson arrival rate of λ customers per unit of time.
- Exponential service times of µ customer per unit of time.
- Queue discipline is first come first served basis by any of the server.
- The waiting line has two or more identical servers.
- There is no limit to the number of the queue (infinite).
- The average arrival rate is greater than average service rate.

Model Formulation
All the model adoption in this work is the (M/M/X): (∞/FCFS) Multi Server Queuing Model. This is the extensional form of single server model where customer in a waiting line can be served by more than one server simultaneously. There are n numbers of customers in the queuing system at any point in time. If n<X, (number of customers in the system is less than the number of servers), then there will be no queue. However, X−n number of servers will not be busy. The combined service rate will then be μn = nμ ; n<X. And, if n≥X (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be n−X. The combined service rate will be μn= Xμ ; n ≥X.

Following are the properties of the Multi-Server Queuing Model:
Utilization factors i.e. the fraction of time servers are busy:
\[ \rho = \frac{1}{1 - \rho} \]  
(1)

The probability of having n customers in the system is given by:
\[ P_n = \frac{(\rho^n/n!)}{(1 - \rho)} \]  
(2)

\[ P_n = \begin{cases} \frac{\rho^n/n!}{(1 - \rho)} & n \leq X \\ \frac{\rho^n/n!}{(1 - \rho)} P_0 & n > X \end{cases} \]  
(3)

When n≥X, it is that the number of customers in the system is not smaller than the number of servers, the next customers must wait, that is,
\[ C(X, \rho) = \sum_{n=X}^{\infty} P_n = \frac{\rho^X}{X!(1-\rho)} P_0 \]  
(4)

Expected number of customers waiting on the queue:
\[ L_q = \frac{1}{(X-1)} \left( \frac{\rho}{(1 - \rho)} \right) \frac{X}{\mu} \]  
(5)

Expected number of customers in the system:
\[ L_s = L_q + \frac{1}{\mu} \]  
(6)

Expected waiting time of customers in the queue:
Expecting waiting time of customers in the system:

\[ W_q = \frac{\frac{\mu^2}{\lambda}} \frac{1}{\lambda} \]  \hspace{1cm} (7) 

\[ W_s = \frac{\mu}{1} \]  \hspace{1cm} (8) 

**Data Analysis**

The collected data for the two banks use in this research study were collected simultaneously at the same time range. The data is collected for three days and it comprises the arrival and service rate of customers. The data values for the arrival and service rate for the two banks is shown below:

**Table 1: Wema Bank**

<table>
<thead>
<tr>
<th>Arrival Rate</th>
<th>Service Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>69</td>
</tr>
<tr>
<td>43</td>
<td>36</td>
</tr>
<tr>
<td>49</td>
<td>43</td>
</tr>
</tbody>
</table>

*Source: Authors Computation*

Overall arrival rate of the Wema bank system is:

\[ \lambda_T = \lambda = \lambda_1 + \lambda_2 + \lambda_3 \]
\[ \lambda = 77 + 41 + 49 = 169 \]

Overall service rate of the Wema bank system is:

\[ \mu_T = \mu = \mu_1 + \mu_2 + \mu_3 \]
\[ \mu = 69 + 36 + 43 = 148 \]

**Table 2: Skye Bank**

<table>
<thead>
<tr>
<th>Arrival Rate</th>
<th>Service Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>89</td>
</tr>
<tr>
<td>94</td>
<td>77</td>
</tr>
<tr>
<td>123</td>
<td>85</td>
</tr>
</tbody>
</table>

*Source: Authors Computation*

Overall arrival rate of the Skye bank system is:

\[ \lambda_T = \lambda = \lambda_1 + \lambda_2 + \lambda_3 \]
\[ \lambda = 125 + 94 + 123 = 342 \]

Overall service rate of the Skye bank system is:

\[ \mu_T = \mu = \mu_1 + \mu_2 + \mu_3 \]
\[ \mu = 89 + 77 + 85 = 251 \]

The numbers of server (X) for the two banks are not the same. For Wema bank, the number of servers is 7, while for Skye bank, the number of server is 11.

**Estimating Queuing Parameters for Wema Bank**

Let’s assume 7 waiting lines for the customer in the bank.

When there is a line, \( X = 7, \lambda = 169, \mu = 148, \rho = \frac{169}{148}, n = 0, 1, 2, ..., 6 \).

\[ P_0 = \left[ \sum_{n=0}^{6} \left( \frac{\lambda}{\mu} \right)^n \frac{1}{n!} \right]^{-1} \]

Where:

\[ \sum_{n=1}^{6} \frac{1}{n!} = 3.1322 \]

\[ P_0 = \left[ 3.1322 + \frac{1}{7!} \left( \frac{169}{148} \right)^7 \frac{7 \times 149}{7 \times 148 - 169} \right]^{-1} = 0.3192 \]

\[ L_q = \frac{1}{(7-1)!} \left( \frac{169}{148} \right)^7 \frac{149 \times 149}{7 \times 148 - 169} \times 0.3192 = 0.000038 \]

\[ L_s = L_q + \frac{\lambda}{\mu} = 0.000038 + \frac{169}{148} = 1.1419 \]

\[ W_q = \frac{\mu}{\lambda} = 0.0058 \]

\[ W_s = \frac{\mu}{\lambda} = 0.0000038 = 0.000002 \]

When there are two lines, \( \frac{1}{2} = 84.5, \mu = 148, \rho = 0.5709 \)

\[ L_s = \frac{\rho}{1-\rho} = 1.3305 \]
Similarly, the analysis is same for, when the lines are 3, 4, 5, 6, & 7 respectively.

The Table 3 gives the summary of the queuing parameters of the assumed number of lines to the bank if the number of servers remains unaltered.

Probability that an arriving customer or customers will have to wait for service at the bank is given by the formula:

\[ P_w = \left( \frac{\lambda}{\mu} \right)^X \frac{\lambda}{X!} \frac{R_0}{\mu-\lambda} \]

Where: \( X = 7; \lambda = 169; \mu = 148; P_0 = 0.3192; \) by substituting into the formula.

\[ P_w = 0.00919 or 0.0199\% \]

**Estimating Queuing Parameters for Skye Bank**

Let's assume 11 waiting lines for the customer in the bank.

When there is a line, \( X = 11, \lambda = 342, \mu = 251, \rho = \frac{342}{251}, n = 0, 1, 2, ..., 10 \)

\[ P_0 = \left[ \sum_{n=0}^{11} \left( \frac{\lambda}{\mu} \right)^n \frac{1}{n!} \right]^{-1} = 3.9061 \]

Where:

\[ \sum_{n=0}^{11} \frac{\lambda^n}{\mu^n} = 3.9061 \]

\[ P_0 = \left[ \frac{1}{\left(1+3.9061\right)} \right]^{-1} = 0.2560 \]

\[ L_q = \frac{1}{\left(1+3.9061\right)} \times 0.2560 = 0.00000031 \]

\[ L_s = L_q + \frac{\lambda}{\mu} = 0.00000031 + \frac{342}{251} = 1.3625 \]

\[ W_s = \frac{L_s}{\lambda} = 0.0040 \]

\[ W_q = \frac{L_q}{\lambda} = 0.0000000091 \]

When there are two lines, \( \frac{\lambda}{\mu} = \frac{171}{251}, \rho = 0.6813 \)

\[ L_q = \frac{\rho^2}{1-\rho} = 1.4564 \]

\[ W_s = \frac{L_s}{\lambda} = 0.0125 \]

\[ W_q = \frac{L_q}{\lambda} = 0.0085 \]

Similarly, the analysis is same for, when the lines are 3, 4, 5, 6, 7, 8, 9, 10, & 11 respectively.

The Table 4 gives the summary of the queuing parameters of the assumed number of lines to the bank if the number of servers remains unaltered.

Probability that an arriving customer or customers will have to wait for service at the bank is given by the formula:

\[ P_w = \left( \frac{\lambda}{\mu} \right)^X \frac{\lambda}{X!} \frac{R_0}{\mu-\lambda} \]

Where: \( X = 11; \lambda = 342; \mu = 251; P_0 = 0.2560; \) by substituting into the formula.

\[ P_w = 0.00000022 or 0.000022\% \]

**Table 3: The queuing system characteristics of Wema bank**

<table>
<thead>
<tr>
<th>Number</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( L_s )</th>
<th>( L_q )</th>
<th>( W_s )</th>
<th>( W_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>169</td>
<td>148</td>
<td>1.1419</td>
<td>0.000039</td>
<td>0.0068</td>
<td>0.000002</td>
</tr>
<tr>
<td>2</td>
<td>84</td>
<td>148</td>
<td>1.3305</td>
<td>0.7596</td>
<td>0.0157</td>
<td>0.0090</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
<td>148</td>
<td>0.6145</td>
<td>0.2339</td>
<td>0.01091</td>
<td>0.0042</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>148</td>
<td>0.4072</td>
<td>0.1144</td>
<td>0.0095</td>
<td>0.0027</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>148</td>
<td>0.2960</td>
<td>0.0676</td>
<td>0.0088</td>
<td>0.0020</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>148</td>
<td>0.2350</td>
<td>0.0447</td>
<td>0.0063</td>
<td>0.0015</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>148</td>
<td>0.1949</td>
<td>0.0318</td>
<td>0.0081</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

**Source:** Authors Computation
Table 4: The queuing system characteristics of Skye bank

<table>
<thead>
<tr>
<th>Number</th>
<th>λ</th>
<th>µ</th>
<th>(L_s)</th>
<th>(L_q)</th>
<th>(W_s)</th>
<th>(W_q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>324</td>
<td>251</td>
<td>1.3625</td>
<td>0.000000031</td>
<td>0.0040</td>
<td>0.00000000091</td>
</tr>
<tr>
<td>2</td>
<td>171</td>
<td>251</td>
<td>2.1377</td>
<td>1.4564</td>
<td>0.0125</td>
<td>0.0085</td>
</tr>
<tr>
<td>3</td>
<td>114</td>
<td>251</td>
<td>0.8322</td>
<td>0.3780</td>
<td>0.0073</td>
<td>0.0033</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>251</td>
<td>0.5165</td>
<td>0.1759</td>
<td>0.0060</td>
<td>0.0021</td>
</tr>
<tr>
<td>5</td>
<td>68</td>
<td>251</td>
<td>0.3746</td>
<td>0.1021</td>
<td>0.0055</td>
<td>0.0015</td>
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<tr>
<td>6</td>
<td>57</td>
<td>251</td>
<td>0.2937</td>
<td>0.0667</td>
<td>0.0052</td>
<td>0.0012</td>
</tr>
<tr>
<td>7</td>
<td>48</td>
<td>251</td>
<td>0.2416</td>
<td>0.0470</td>
<td>0.0049</td>
<td>0.00096</td>
</tr>
<tr>
<td>8</td>
<td>42</td>
<td>251</td>
<td>0.2053</td>
<td>0.0350</td>
<td>0.0048</td>
<td>0.0008</td>
</tr>
<tr>
<td>9</td>
<td>38</td>
<td>251</td>
<td>0.1784</td>
<td>0.0270</td>
<td>0.0047</td>
<td>0.0045</td>
</tr>
<tr>
<td>10</td>
<td>34</td>
<td>251</td>
<td>0.1578</td>
<td>0.0215</td>
<td>0.0046</td>
<td>0.00060</td>
</tr>
<tr>
<td>11</td>
<td>31</td>
<td>251</td>
<td>0.1414</td>
<td>0.0175</td>
<td>0.0045</td>
<td>0.00056</td>
</tr>
</tbody>
</table>

Source: Authors Computation

III. DISCUSSION OF FINDINGS

The comparative analysis of the two banks under review differs significantly with respect to the queuing theory. The result of the respective banks shows that waiting lines is highly reduced if the number of servers is drastically increased so as to satisfy customers at an optimum advantage. Based on the number of servers of the two banks, when an arriving customer will have to wait until he or she is attended to, Wema bank has the highest waiting probability service value (0.019%) when compared with Skye bank value (0.000022%). This value indicates that for optimum efficiency in the bank, there is need to increase the service station. This gave Skye bank a practical advantage over Wema bank that no queue exists in their banking system. That is, the probability that a customer will have to wait is very infinitesimal when compared with Wema bank. This can also be seen in the parameters of the queuing theory under consideration that the expected number or waiting time of customers on the queue or in the banking system reduces irrespective of the waiting lines. The higher the number of servers, the lesser the waiting lines, the lower the number of servers, the more the waiting lines. This is the practical cause of the two banks in this research study.

IV. CONCLUSION

The queuing number, the number of servers, and the optimal probability service as investigated by means of queuing theory are the three measures that improve the efficiency of commercial banks. The analysis in this study as carried out by the two banks is effective and practical. It was also investigated that the optimal queuing model is feasible.

REFERENCES


