American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-4, Issue-8, pp-93-97
www.ajer.org
Research Paper Open Access

An Evaluation of the Plane Wave properties of Light using Maxwell's Model

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ABSTRACT: *The only two forces of nature that human beings can directly experience through their 5 senses are gravity and light. The other sensations such as a smell, heat, sound etc detect macroscopic properties of mater and not its constituents. Electrostatic and electromagnetic charges are deeply likened with light. It is the accelerated movement of these charges that gives light. The movement is at the speed of light. This part of electromagnetic radiation (light) is the only one that is visible to human being. Here, the language of engineering electromagnetism; the second grand unification of science that unifies the 3 most powerful forces of light, magnetisms and electricity (Maxwell's four equations) and underlying vector calculus equations were invoked to decouple the component parts of a typical electromagnetic radiation to bring out this phenomenon called light, as a basic component part of an electromagnetic radiation.*

Keywords **-** *current, displacement, field, magnitude, phasor, vector*

I. INTRODUCTION

Light is a transverse wave having an oscillating electric and magnetic field. Both fields are perpendicular to each other and to the direction of their propagation. The ratio of the electric field intensity ($\bar{E}$) and magnetic field intensity ($\hat{H}$) is equal to a constant thus;

$$\rightarrow \frac{E}{B} = \text{constant (c)}$$

Therefore an electromagnetic waves is defined as plane if at any given instant of time, the electric intensity ($\bar{E}$) and the magnetic intensity ($\overline{H}$), remain the same (in magnitude and phase), over a plane normal to the direction of wave propagation and are completely propagating in the Z direction and the X-Y plane is called the plane of the wave or the wave front.

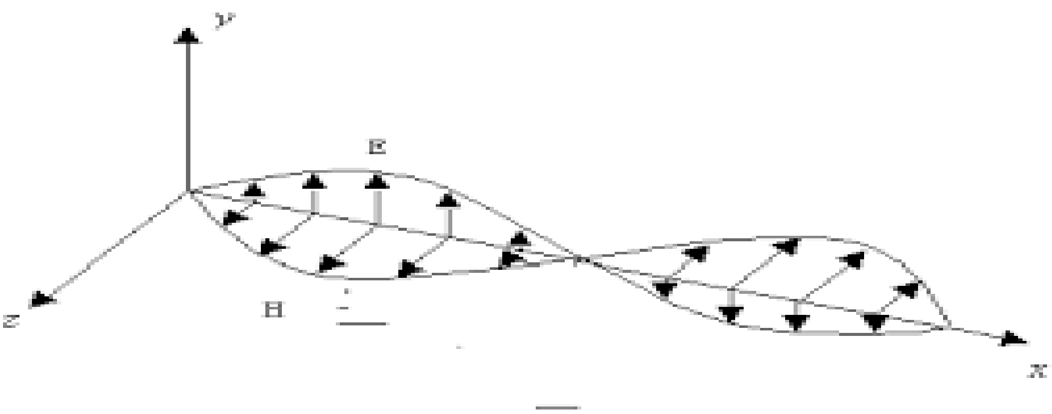

Figure 1: Plane Wavefront.

From the above definition;

$$\frac{\partial B}{\partial X} = \frac{dB}{dy} = 0$$

$$\frac{\partial E}{\partial X} = \frac{dE}{dy} = 0$$

$$:.\ \frac{dEX}{dBX} = \frac{dEY}{dBy} = \text{Constant} = \text{field strength} \cong 377 \text{ (intrinsic impedance)}$$

According to Maxwell equationfor plane wave:

$E=E_{\circ}Cos\Phi X^{;H} = H_{\circ}Cos\Phi\bar{y}$ where X and Y are unit vector in a given direction.

II. METHODOLOGY

Maxwell's original equation where broken down into four equations. These four equations are a refined form of the 22 previous equations. It unifies the 3 most powerful forces of nature: Light, Electricity and Magnetisms. It consists of the electromagnetic language that binds these 3 universal forces [4]. Also Maxwell's equations play a central role in the analysis in the transmission media [2].

The four sets of equations satisfied everywhere by the electric field vectors ($\bar{E}$ and $\bar{D}$), and magnetic field vectors ($\bar{H}$ and $\bar{B}$) are;

$$\text{div } \bar{D} = p$$

$$\text{div}ß = 0$$

$$\text{curl } \bar{E} = -\frac{d\bar{B}}{dt} = \mu_r\mu_{\circ}\frac{-dH}{dt}$$

$$\text{curl } \bar{H} = \bar{J} + \frac{d\bar{D}}{dt} = \delta\varepsilon + \epsilon_r\epsilon_{\circ}\frac{d\bar{E}}{dt}$$

Where $\bar{D}$ and $\frac{\delta\bar{D}}{dt}$ are electric displacement and displacement current density respectively and $\bar{J}$ is the conduction current density.

$\bar{D} = \epsilon_r\epsilon_{\circ}\bar{E}$ and $H = \frac{\bar{B}}{UrUo}$ with ρ = free charge density

$\epsilon_{\circ}$ = permittivity of free space = 8.854 X 10^{-2}fm $^{-1}$

ϵ_r=dielectric constant or relative permittivity of the material

$\epsilon_r = 1$ in a vacuum, $\epsilon_r > 1$ in homogenous, isotopic medium

$\mu_{\circ}$ = permeability of free space

$\mu\epsilon_{\circ} = \frac{1}{C2}$ where c speed of light.

However, in the words of Maxwellians [4]; in the beginning God said;

$$\bar{\nabla} \text{ X } \bar{H} = \bar{J} + \frac{\delta\bar{D}}{dt}$$

$$\bar{\nabla} \text{ X } \bar{E} = -\frac{\delta\bar{B}}{dt}$$

$$\bar{\nabla}.\bar{B} = 0$$

$\bar{\nabla}.\bar{D}$ =p and there was light.

This Maxwell's equation is made up of electromagnetism and light. The equations can be decoupled and by applying constitutive relations and other vector calculus conditions, thus as a component of electromagnetic wave can be unveiled.

III. PHYSICAL INTERPRETATIONS

$\bar{\nabla}$ X $\bar{H} = \bar{J} + \frac{\delta\bar{D}}{dt}$

$\bar{\nabla}$ X $\bar{E} = -\frac{\delta\bar{B}}{dt}$

$\bar{\nabla}.\bar{B} = 0$

$\bar{\nabla}.\bar{D} = p$

Where,

$\bar{J}$ = Conduction current density

$\frac{\delta\bar{D}}{dt}$ = time varying electric flux density

$\bar{B}$ = magnetic flux density

$\frac{\delta\bar{B}}{dt}$ = time varying magnetic flux density

ρ = electric charge density

The four equations describe the experimental al facts that [4].

Equation 1: Magnetic field is not only produced by a conductor current but also by a displacement current or time varying electric flux density (Ampere's law)

Equation 2: Electric field can be produced by a time varying magnetic field (Faraday and Lenz's law)

Equation 3: There is no isolated magnetic change in nature i.e. the existence of a monopole.

Equation 4: Establishes the occurrence of isolated electric charges in nature (Gauss law).

IV. MAXWELL'S EQUATION IN FREE SPACE(CHARGE FREE ZONE)

A free space or charge free zone is space with nothing in it at all. It doesn't exist in the known universe, but interstellar space is a good approximation [6]

Features of free include: Uniformity everywhere, no electric charge in and around it. It carries no electric current and nothing that electromagnetic radiations are propagated in free space by electromagnetic theorem, we use Maxwell's equation as well as a constitutive tool to understand and decipher plane waves into their constituent parts.

Using Maxwell's equations (M.E) (1) and decoupling it by multiplying it by a vector $\bar{\nabla}$ we have that [4]:

$\bar{\nabla}$ X [$\bar{\nabla}$ X $\bar{H}$] = $\bar{\nabla}$ X $\bar{J}$ + $\bar{\nabla}$ x $\frac{\delta\bar{D}}{dt}$ ....(1)

Invoking constitute relations to decouple the L.H.S. of the equation (1), we have, assuming

$\bar{D} = \epsilon\bar{E}$

:. R.H.S = $\bar{\nabla}$ X $\bar{J}$ + $\bar{\nabla}$ X[$\frac{d\epsilon\bar{E}}{dt}$]= $\bar{\nabla}$ X $\bar{J}$ +€$\frac{d}{dt}$ [$\bar{\nabla}$ X $\bar{E}$]…………………………………..(1a)

To decouple L.H.S of the equation, we use the vector identity theorem, which state the curl of the curl of a vector is equal to the gradient of divergence minus the laplacian. Thus:

L.H.S = $\hat{V}$ [$\hat{V}$.$\hat{H}$] – $\hat{V}^2\hat{H}$

Where $\hat{V}$ [$\hat{V}$.$\hat{H}$] = gradient of the divergence of the vector = 0

$\bar{\nabla}^2\bar{H}$ = Lapalcian

Bringing the L.H.S and R.H.S of the equations together, we have that

0-$\bar{\nabla}^2\bar{H}$=$\bar{\nabla}$X$\bar{J}$+€$\frac{d}{dt}$[$\bar{\nabla}$X$\bar{E}$]… ……………..(2)

Form the L .H.S, $\bar{\nabla}$X$\bar{E}$ = Maxwell's equation (M.E) (2), applying the constitutive relations to further decouple, we have;

[$\bar{\nabla}$ X $\bar{E}$] = $\frac{dB}{dt}$

B=μ$\bar{H}$:.[$\bar{\nabla}$X$\bar{E}$]=$-\frac{\delta\mu\bar{H}}{\delta t}$……………..……(3)

Substituting equation (3) back in equation (2) we have,

$$0 - \bar{\nabla}^2\bar{H} =- \frac{\mu\epsilon d2\bar{H}}{dt2} = - \bar{\nabla} X \bar{J}$$

The above equation (4) is called the general wave equation for the H field [4]. We can further deduce the wave equation on the $\bar{E}$ field by decoupling Maxwell's equation 2 by taking the curl of the equation:

$\bar{\nabla}$X [M.E (2)]

$\bar{\nabla}$ X $\bar{\nabla}$ X $\bar{E}$ = $\bar{\nabla}$ X -$\frac{\delta\bar{B}}{dt}$

$\bar{\nabla}$ X $\bar{\nabla}$ X $\bar{E}$ = - $\frac{d}{dt}$ [$\bar{\nabla}$ X $\bar{B}$]………... …….(5)

By applying vector identity to the L.H.S and the constitutive relations on the R.H.S, we have that;

$\bar{\nabla}$ ($\bar{\nabla}$ X $\bar{E}$) - $\bar{\nabla}^2$ X $\bar{E}$ =μ $\frac{d}{dt}$ ($\bar{\nabla}$ X $\bar{H}$) = μ $\frac{d}{dt}$ (J $\bar{J}$+ € $\frac{d\bar{E}}{dt}$)

The L.H.S. can also be decoupled further with M.E (4)

$\bar{\nabla}$.($\bar{\nabla}$ X $\bar{E}$) where $\bar{\nabla}$.$\bar{D}$ = ρ, substituting we have that $\bar{\nabla}$ ($\frac{p}{€}$)

Bringing the two sides of the equations, together we have; ……………………………………...…(6)

$$\bar{\nabla}^2 \bar{E} - \mu\epsilon \frac{d2\bar{E}}{dt2} = \mu \frac{d\tau}{dt} + \bar{\nabla} \left(\frac{p}{E}\right)$$

equation for the $\bar{E}$ field. But for a source free region

$\bar{J} = 0$ and $p = 0$

Substituting equation (4) and (6) we can deduce the general vector wave equation for a source free region in the $\bar{H}$ and $\bar{E}$ field respectively:

$$\bar{\nabla}^2 H - \mu \epsilon \frac{d2H}{dt2} = 0$$

$$\bar{\nabla}^2 E - \mu \epsilon \frac{d2E}{dt2} = 0$$

time sinusoidally , the time derivative can be replaced by factors of jw to

$\frac{\delta 2}{dt2} = (jw)^2 = -w^2$ substituting in the wave equations, we have [4]:

$$\bar{\nabla}^2 \ \bar{H} + w^2 \mu\epsilon\bar{H} = 0$$

$$\bar{\nabla}^2 \ \bar{E} + w^2 \mu\epsilon\bar{E} = 0$$

Comparing the general equations with the source velocity equation

$\bar{\nabla}\ ^2 w - \frac{1}{\surd 2}\frac{\delta co2}{\delta t2} = S^o$ (free space)

where $\surd$ =source velocity

We have that

$$:.\ \sqrt{sp} = \frac{1}{\surd 2} = \mu\epsilon = \frac{1}{\surd\mu\epsilon}$$

Assume free space, $\epsilon = \epsilon_\circ = \frac{1}{36\pi}$ X $10^{-9} = 8.854X10^{-6}C^2/nm^2$,

$\mu = \mu_\circ = 4\pi\ X10^{-7}Tm/A$

$$= \sqrt{sp} = \frac{1}{\surd 4\pi X10-7X\frac{1}{36\pi}X\ 10-9} = 3X10^8 m/s = \text{speed of light.}$$

This shows that light is an integral part of both the electric field $\bar{E}$ and the magnetic field $\bar{H}$.

V. CONCLUSION

The above derived equation shows that light can behave like a projecting magnetic or electric field. That is, we have light embedded in both the electric and magnetic fields. Electromagnetic waves are produced that propagates through a vacuum at the speed of light.

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