American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-4, Issue-8, pp-38-40
www.ajer.org
Research Paper Open Access

A Random Evolution of Stochastic difference Equations related to M/M/1 Queueing System

M.Reni Sagayaraj[1], P.Manoharan[2], S. Anand Gnana Selvam[3]
R.Reynald Susainathan[4]
[1,2,3]*Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur - 635601, Vellore District.Tamil Nadu, S.India.*
[4]*Reckitt Benckiser, India Limited (Gurgaon), New Delhi. India.*

ABSTRACT : *We propose a stochastic model for evolution with single server queue. Birth and death of species occur with constant probabilities. It is shown that the Poisson like arrival process (births) with slowly diffusing like time dependent average inter-event time may be represented as a super statistical one and exhibits* $1/_{\mu}$ *arrival (birth). We obtain customers induces less to the system and they alter the dynamics of the net profit. We associate various costs and analysis the Markov evolution of the net profit functions.*
Keywords - *Random evolution, stochastic Difference Equation, Birth and death process. net profit functions, point process*

I. INTRODUCTION

A random evolution is a natural but apparently new generalization of this notion. In this note we hope to show that this concept leads to simple and powerful applications of probabilistic tools to initial-value problems of both parabolic and hyperbolic type. The term `random evolution' first appeared in the year 1968 paper of Griego and Harris [9] where the further acknowledgments paper The two-state Markov chain, often called the telegraph process, is the continuous-time counterpart of the classical Bernoulli process of Discrete-parameter probability theory. A Simpler proof of the telegraph equation was obtained by Kaplan [4].Recent approaches to large deviation of random evolution are contained in the works of Bezuidehout [2] and Eizenberg-Freidin [3].

A random evolution $M(t,\omega)$ is the product. It is shown that the Poisson process with diffusing average time may be represented by the equation.

$$M(t) = T_{\tau 0}\,(t_1) T v_{(t_1)}(t_2 - t_1) \ldots \ldots . T v_{(t_N(t))}\big(t - T_{N(t)}\big) \qquad (1)$$

The study of Markov diffusion process has played an important role in the development of stochastic processes both from theoretical as well as the applied points of view, one of the earliest applications was to the study of the fluctuations of arrival variables due to various aspects were studied.

Assume that stochastic differential equations is

$$\frac{dx}{dt} = a(x;t)$$

Describe a one dimensional dynamical system. Assume that (1) fulfils conditions such that a unique solution exists, thus $x(t) = x(t;x_0;t_0)$ is a solution satisfying the initial condition $(t_0) = x_0$. Given the initial condition, we know how the system behaves at all times t, even if we cannot find a solution analytically. We can always solve it numerically up to any desired precision.
A way of modelling these elements is by including stochastic influences of birth. A natural extension of a deterministic differential equations model is a system of stochastic differential equations, where relevant parameters are modelled as suitable stochastic processes, or stochastic processes are added to the driving system

equations. This approach assumes that the dynamics are partly driven by arrival of the variable.
A natural extension of a deterministic ordinary differential equations model is given by a stochastic differential equations model, where relevant parameters are randomized or modelled as random processes of some suitable form, or simply by adding a arrival term to the driving equations of the system. This approach assumes that some degree of arrival is present in the dynamics of the process. Here we will use the Wiener process. It leads to a mixed system with both a deterministic and a stochastic part in the following way:

$$dX_t = \mu(X_t;t)dt + \sigma(X_t;t)dW_t \quad (3)$$

When $X_t = X(t)$ is stochastic processes, not a determined function like in[1].This isindicated by the capital letter. Here $W_t = W(t)$is a wiener process and since it is now here differentiable, we need to define what the differential means. It turns out that it is very useful to write $dw_t = \xi_t dt$ where a white arrival process is, define as being normally distributed for any fixed `t' and uncorrelated.
We have proposed stochastic model of $\frac{1}{\mu^\beta}$ arrival with 0.5, β < 2, based on the simple point process models [10] and on nonlinear stochastic differential equations [5], [6]. Here we show that the Poisson like point process with slowly diffusing time-dependent average interevent. The distribution of the Poisson like interevent time may be expressed as an exponential distribution of the Non extensive statistical Mechanics. We describe the problem as Random evolution with single server Queue. The Markov evolution of a net profit function determined.

II. MODEL DESCRIPTION OF THE PROBLEM

We consider a counter where the customers arrive according to a Poisson Process wit rate λ .There is a single server at the counter and he serves the customers according to the order of their arrival. The service time for a customer has exponential distribution with mean $\frac{1}{\mu}$ .An arriving customer joins the queue with probability 1 if the system size is 0.If the system size is 1 when the customers arrives, he join the queue with probability p and he balks with probability q[11].
The basic concept of queueing theory

$$p + q = 1$$

And $P_0(t) = p$, for all n ≥ 2. We assume that $P_0(t) = b$ using probability arguments, we have

$$P_0(t+\Delta) = P_0(1-\lambda\Delta) + P_1(t)\mu\Delta + o(\Delta) \quad (4)$$
$$P_1(t+\Delta) = P_1(1-\mu\Delta) + P_0(t)\lambda\Delta + o(\Delta) \quad (5)$$

From the above equation we have

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t) \quad (6)$$
$$P_1'(t) = -\mu P_0(t) + \lambda P_1(t) \quad (7)$$

We observe the if the initial probabilities are assumed to be the steady state values, then queueing process is stationary at any time.
If we are assume that the process starts with 1 customer at time t=0, then a=0 and b=1.In this situation, we obtain the steady state probabilities are,

$$P_0(t) = \frac{\mu}{\lambda+\mu}\{1 - e^{-(\lambda+\mu)t}\} \quad (8)$$
$$P_1(t) = \frac{\mu}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}\{1 - e^{-(\lambda+\mu)t}\} \quad (9)$$

III. A RANDOM EVOLUTION WITH SINGLE SERVER QUEUE

We consider the queueing model M/M/1 starting with one customer at time t=0.As busy periods always fetch utility of the system by the customers and in turn provide profit to the system. We attach a positive cost $C_1, C_2 > 0$ per unit time, to busy periods .In the same way a negative cost -$C_1, C_2 > 0$ per unit time is attached to idle periods. Considering the epochs of beginning and ending of the busy periods ,we note that these epochs constitute an alternating point process on the time axes characteristic by the two densities like $\mu e^{-\mu t}$ and $\lambda e^{-\lambda t}$. Let C(t) be the cost per unit time at time t. Then it can be easily given that

$$C(t) = \frac{1}{2}(C_1 - C_2) + \frac{1}{2}(C_1 + C_2)(-1)^N(t) \quad (10)$$

Let P(t) be the net gain of the system up to time t. Then it can be easily seen that

$$P(t) = \int_0^t C(u)du$$

Solve the above we get two random quantities p(t)and C(t) constitute a random motion.

IV. THE MARKOV EVOLUTION OF A NET PROFIT FUNCTION

Let r(t) be the value of the service per unit time per customer in the system at time t. We define

$$r(t) = \begin{cases} -r_1 \ if \ n(t) = 0 \\ r_2 \ \ if \ n(t) = 1 \\ r_3 \ \ if \ n(t) > 1 \end{cases}$$

Where n(t) represents the number of customers in the system at time t. Here we assumed that
$r_i > 0 \, , i - 0, 2,3 \ldots$

It is to be noted that r1 is the negative cost due to the idle time of the server and r_2 and r_3 corresponds to the state dependent positive gain due to the customer who join the system. Clearly r_2 and r_3 contribute positive revenue to the net profit function. Then the net profit function L(t) is given the stochastic integral.

$$L(t) \int_0^t r(u)du \qquad (11)$$

In [8] by using r(u) as the instantaneous velocity of the server , L(t) gives the distance traveled by the server in time t. Then the time evolution of the net gain can be studied by identifying the server as a particle under a random motion [7] on the real line with three velocities in a cyclic manner. We now study the time evolution of L(t) .for this ,we assume that the server enter in to the idle state at time t=0.Then we note that n(0)=0 and $r(0) = r_1$.

It is easy to note that the discrete component of L(t) given by

$$P_r\{L(t) = -r_{(t)}\} \, e^{-\lambda t} \qquad (12)$$

From the assumption made on the input and output processes, it may be verified that the stochastic process.

V. CONCLUSION

The special non-linear stochastic difference equation, generating the distribution density with $\frac{1}{\mu}$ mean arrival. We derive two random quantities P(t) and C(t) constitute of a random variable with random motion. We analyze cost of the Markov evolution of the net profit. We describe the explicit expression of the transient probabilities Pn(t) are found in a direct way along with steady state solution. It can be applied for telecommunication network.

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