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A Method for Damping Estimation Based On Least Square Fit

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Abstract: *A new approach based on least square fit method is proposed to estimate damping. Noise resistance of the proposed method and half-power bandwidth method are analyzed and compared by plenty of simulations with different signal-to-noise ratios (SNR). The proposed method is more accurate and stable than half-power bandwidth method in all SNRs, especially when the noise level is high. If* $SNR \le 30dB$ *, the proposed method should be used for damping estimation instead of half-power bandwidth method. A damping estimation experiment is carried out with both methods, and the results indicate and verify that there is smaller variability for the proposed method.*

Keywords: *Least square fit, damping estimation, half-power bandwidth*

I. INTRODUCTION

Since damping is a valuable parameter for resonant response of structures or systems, damping is of great significance for structural dynamics. It is difficult to obtain damping through a theoretical method so that the universal way for damping estimates is experimental investigation[1,2]. Half-power bandwidth method[3,4], which calculates resonant damping with frequency bandwidth for vibration energy decreases 3dB and the resonant frequency, is a widely used approach for damping estimation. Half-power bandwidth method is introduced as main method for damping estimates in the American test standard ASTM E756-05. Bertha[5] obtained damping of a system that does not possess real modes with half-power bandwidth method. Guo[6] improved the half-power bandwidth method and proposed a new method based on integral opinion. Badsar[7] determined the material damping ratio in shallow soil layers with the half-power bandwidth method.

As the half-power bandwidth method uses only three data points, its accuracy will be affected by the signal-to-noise ratio (SNR). When some of the three data points are seriously affected by noise, a large error will appear and cannot be neglected. The aim of this work is to put forward a new method which has high accuracy and strong noise resistance by using the amplitude of frequency response.

II. HALF-POWER BANDWIDTH METHOD

A widely used method, namely half-power bandwidth method, is introduced as follows. The damping loss factor can be obtained by the quotient of half-power bandwidth and the resonant frequency, as shown in equation (1). The diagrammatic sketch of the method is shown in Fig. 1.

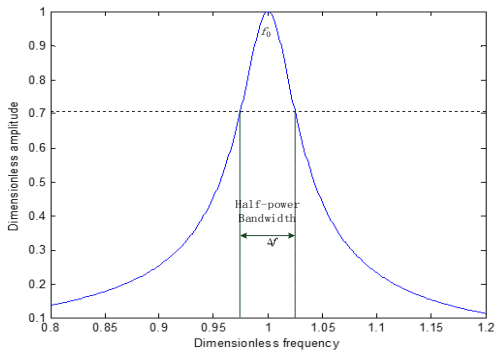

Fig.1. Half-power bandwidth method

$$\eta = \frac{\Delta f}{f_0} \qquad (1)$$

The half-power bandwidth method is a classical and widely used method, but there is a fatal short coming that if the signal-to-noise ratio (SNR) is not high enough. It is difficult to recognize the half-power bandwidth, seen in Fig. 2.

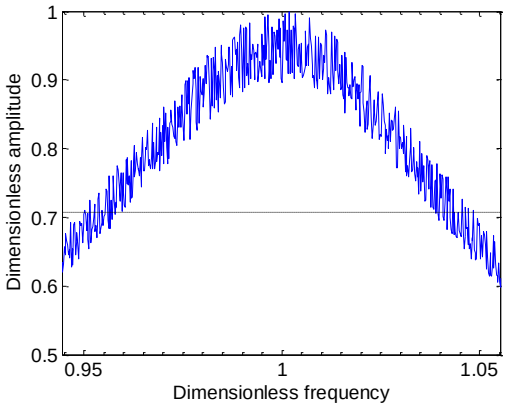

Fig. 2. Half-power bandwidth method with low SNR

It can be seen in Fig. 2 that the half-power bandwidth is hard to decide because there are more than two frequencies related to half-power response.

III. DAMPING ESTIMATION BASED ON LEAST SQUARE FIT

The parameter of damping is often estimated with data near the resonant frequency. The widely used half-power bandwidth method for damping investigation only uses three data points of frequency response so that it will cause a large error when the experimental data is affected by noise. It is expected to obtain a more accurate damping if more data points of frequency function are used. Therefore, a new approach based on least square fit method is proposed in which a numbers of data points near resonant frequency are used.

The amplitude of frequency function is

$$|H| = \frac{F/K}{\sqrt{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \eta^2}} \qquad (2)$$

where F is excitation force, K the stiffness, ω_0 natural circle frequency, and η the loss factor.

It is obvious that the maximum amplitude can be obtained

$$|H|_{\max} = \frac{F}{K\eta} \qquad (3)$$

The dimensionless amplitude is defined as

$$|\bar{H}| = \frac{|H|}{|H|_{\max}} = \frac{\eta}{\sqrt{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \eta^2}} \qquad (4)$$

Rewrite equation (4) as

$$\frac{1}{|\bar{H}|^2} = 1 + \frac{1}{\eta^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 \qquad (5)$$

Make that

$$x = \left(1-\frac{\omega^2}{\omega_0^2}\right) \qquad (6)$$

and

$$y = f(x,\eta) = 1 + \frac{1}{\eta^2}x^2 \qquad (7)$$

A number of data points $(x_i, y_i)(i = 1, 2, \cdots, n)$ can be obtained according to the experimental frequency response. The residual error is defined as

$$err(\eta) = \sum_{i=1}^{n} \left[y_i - f(x_i, \eta) \right]^2 \qquad (8)$$

The damping identification can be transferred into getting a proper value of η to make the residual error the most minimum. Gauss-Newton iterative method is used to find the optimal parameter of damping.

Expand $f(x_i, \eta)$ to Taylor series at point of initial value η_0 as

$$f(x_i, \eta) \approx f(x_i, \eta_0) + f'(x_i, \eta_0) \cdot (\eta - \eta_0) \qquad (9)$$

where $f'(x_i, \eta_0) = \left. \frac{df(x_i, \eta)}{d\eta} \right|_{\eta = \eta_0}$.

Rewrite equation (8) as

$$err(\eta) = \sum_{i=1}^{n} \left[y_i - f(x_i, \eta_0) - f'(x_i, \eta_0) \cdot (\eta - \eta_0) \right]^2 = \sum_{i=1}^{n} \left[\tilde{y}_i - g(x_i - \eta_0)\eta \right]^2 \qquad (10)$$

where $\tilde{y}_i = y_i - f(x_i, \eta_0) + g(x_i, \eta_0) \cdot \eta_0$, $g(x_i, \eta_0) = f'(x_i, \eta_0)$. Equation (10) is a typical least square optimization problem. The estimated η_1 can be obtained for the first generation by solving equation (11).

$$A^{\mathrm{T}} A \cdot \eta = A^T b \qquad (11)$$

where $A = \left(g(x_1, \eta_0), g(x_2, \eta_0), \cdots, g(x_n, \eta_0) \right)^{\mathrm{T}}$, $b = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_n)$.

After η_1 is obtained, η_0 will be replaced by η_1 and repeat the above step to get the estimated value η_2 for the second generation. The iteration will be finished if the difference between the neighbor generations of η is small enough for satisfactory demand. The last generation of η is the estimated damping. The experimental and fitting response are shown in Fig. 3 and damping can be obtained in the fitting process.

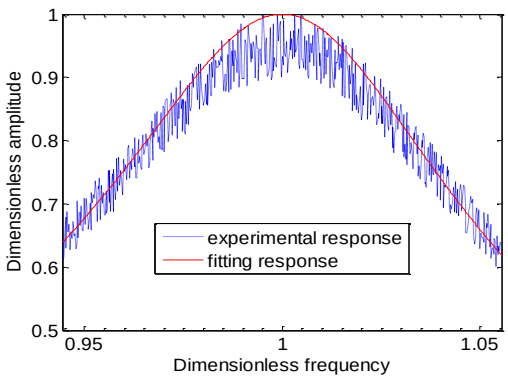

Fig. 3. Experimental and fitting response

IV. RESISTANCE TO NOISE

This simulation is conducted to illustrate the noise sensitivity of the proposed methods. A single-degree-of-freedom system is adopted to verify the validity of least square fit method. A comparison is made between least square fit method and half-power bandwidth method. Gaussian random noise is applied to the simulated frequency response, and the signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 20 \log_{10} \left(|H|_{\mathrm{signal}} / |H|_{\mathrm{noise}} \right) \qquad (12)$$

where $|H|_{\mathrm{signal}}$ is amplitude of effective signal and $|H|_{\mathrm{noise}}$ is amplitude of noise.

The simulated signal parameters are $\eta = 0.001$, $f_0 = 500\ \mathrm{Hz}$, $\omega_0 = 2\pi f_0$, and $\Delta f = 0.1\ \mathrm{Hz}$. It is simulated on 1000 samples with the SNR in the range from 10 dB to 50 dB. Errors of estimated damping with the two methods are shown in Fig. 4.

In Fig. 5, each method is tested for 41 noise levels, which is from 10dB to 50dB with 1dB interval, with 1000 samples in each noise level. The average error is calculated by equation (13).

$$\text{average error} = \frac{1}{N} \sum_{i=1}^{N} \frac{|\eta_i - \eta_{\mathrm{exact}}|}{\eta_{\mathrm{exact}}} \qquad (13)$$

where η_i is the identified damping, η_{exact} is the exact loss factor set in the simulation, N is the number of samples.

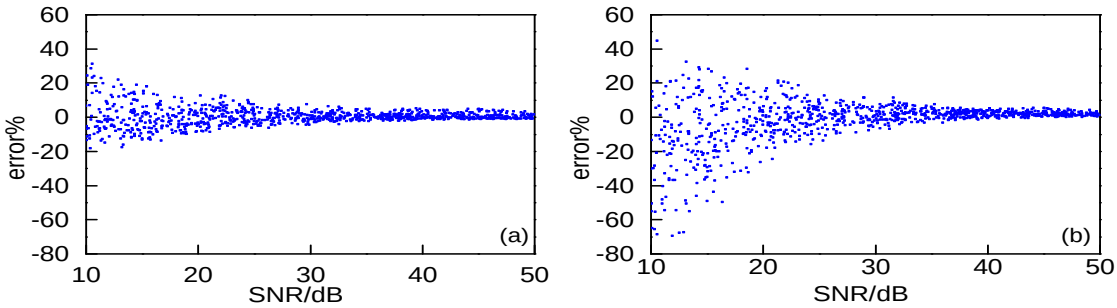

Fig. 4. Error of the estimated damping versus SNR: (a) proposed method, (b) half-power bandwidth method.

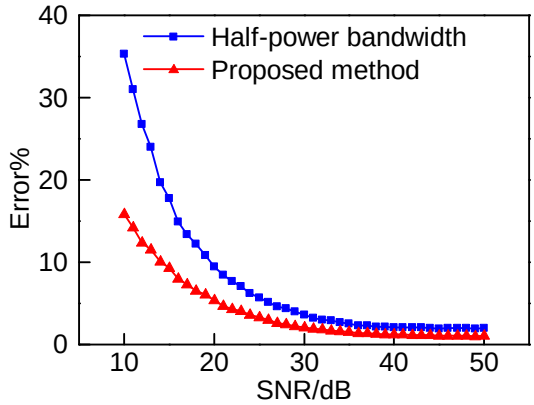

Fig. 5. Average error for the two methods

It can be seen in Fig. 4 that identified damping by the two methods are more and more accurate with the increased SNR. The accuracy of proposed method is more than half-power bandwidth method at all SNRs. It may also be noticed that there is considerable variability of half-power bandwidth method especially in the lower SNR.

As seen in Fig. 5, both methods are of satisfied accuracy if $SNR > 30dB$. But if $SNR \le 30dB$, error of half-power bandwidth is much larger than proposed method. It is clear that the proposed method should be used to estimate damping instead of half-power bandwidth if $SNR \le 30dB$

V. EXPERIMENT

The experiment is carried out on an alloy beam with the dimensions of $0.23m \times 0.01m \times 0.002m$, as shown in Fig. 6. Fig. 7 is the dimensionless response of the third mode. Fig. 5 is the estimated damping obtained from six repeat experiment results by the two methods.

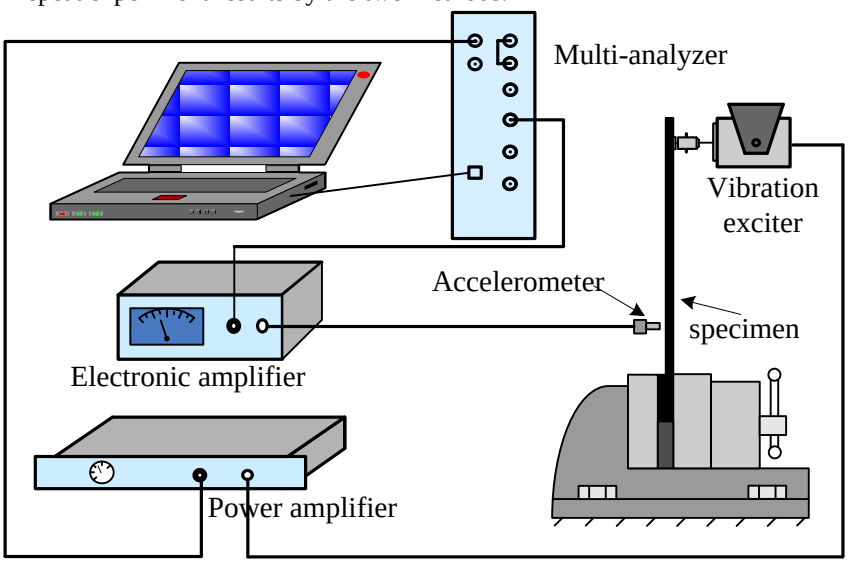

Fig. 6. Experimental setup

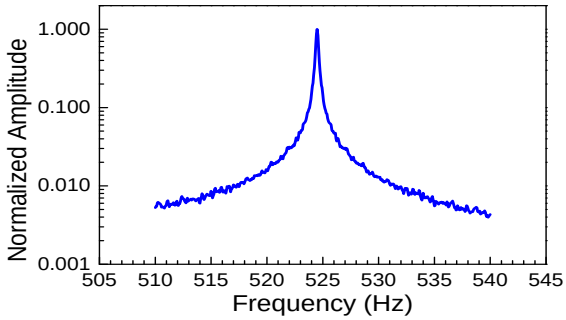

Fig. 7. Dimensionless response

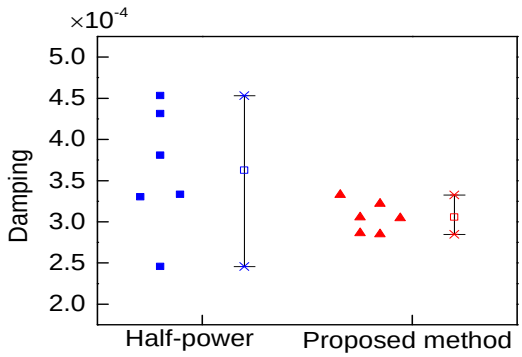

Fig. 8. Damping estimated by the two methods

As seen in Fig. 8, it is clear that there is smaller variability for the proposed method than half-power bandwidth method. Thus, damping estimated by the proposed method is more accurate than half-power bandwidth method.

VI. CONCLUSION

A noval method based on least square fit is proposed to estimate damping in this paper. There is smaller variability for the proposed method than half-power bandwidth method. The proposed method is more accurate than half-power bandwidth method, especially in the lower SNR circumstances. The proposed method should be used instead of half-power bandwidth method for damping estimation if $SNR \leq 30dB$.

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