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The boundary layer flow of nanofluids over an isothermal stretching sheet influenced by magnetic field.

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ABSTRACT : *An analysis is carried out to study the effect of the magnetic field on the boundary layer flow of the nanofluids over an isothermal stretching surface. Two types of nanofluids namely Ag–water and Cu–water are considered. Similarity transformation is used to convert the governing nonlinear equations into coupled higher order nonlinear ordinary differential equations. Fourth-order Runge–Kutta method with shooting technique is employed for the numerical solution of the obtained equations. Numerical results are obtained for distribution of velocity, temperature and concentration, for both cases .The numerical results indicate that an increase in the nanoparticle volume fraction will decrease the velocity boundary layer thickness. Meanwhile, the presence of nanoparticles results in an increase in the magnitude of the skin friction along the surface. Such effects are found to be more evident in the Ag–water solution than in the Cu–water solution. The obtained numerical results have been presented graphically and discussed in details.*

KEYWORDS – *MHD Flow, Magnetic field , Nanofluids, Runge-Kutta Method .*

I. INTRODUCTION

A large number of theoretical investigations dealing with magnetohydrodynamic (MHD) flows of nanofluids have been performed during the last decades due to their swiftly increasing applications in many fields of technology and engineering, such as MHD power generation, . Recently, the application of magnetohydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Heat and mass transfer processes by means of external force effects is one of the most important problems in modern applied physics. The study of the magnetic field effect on heat and mass transfer is of substantial significance in various industries. Sattar and Alam [1] studied the thermal diffusion as well as transportation effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate. Choi [2] who coined the term nanofluid proposed that the thermal conductivity of the base fluid can be increased by adding low concentration of nanoparticles of materials having higher thermal conductivity than the base fluid.Putra et,al [3]studied about the temperature dependence of thermal conductivity enhancement for nanofluids. Raptis and Perdikis [4] studied the viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field. Singh et al. [5] studied the effects of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream velocity. Khan and Pop [6] examined the boundary-layer flow of a nanofluid past a stretching sheet. Momentum and energy equations for a nanofluid over a linearly impermeable stretching surface in the absence of slip and magnetic field were studied by Noghrehabadi et al [7] they also studied the theoretical investigations of SiO_2-water nanofluid heat transfer enhancement over an isothermal stretching sheet. Makinde and Aziz [8] analyzed the boundary layer flow of nano fluids over a stretching sheet with convective heat transfer boundary condition. Rana and Bhargava [9] numerically studied the flow and heat transfer of a nanofluid over a nonlinearly stretching sheet. Yacob et al.[10] analyzed enhancement of two types of nanofluids, namely, Cu-water and Ag-water nanofluids over an stretching sheet with convective boundary condition. Khare and Rai [11] examined the MHD flow of non-Newtonian fluid through a rectangular channel. Khare and Srivastava [12] studied the effect of Hall current on MHD flow of a dusty viscoelastic liquid through porous medium past an Infinite Plane. Several researches investigated the MHD boundary layer flow [13,14,15] in various situations.

In this paper, an analysis is carried out numerically to study the effect of the magnetic field on the boundary layer flow of the nanofluids over an isothermal stretching surface. Two types of nanofluids namely Ag–water and Cu–water are considered. . The effects of the governing parameters on the velocity and temperature have been discussed and presented in tables and graphs.

II. FORMULATION OF MATHEMATICAL MODEL FLOW

Consider an incompressible steady two-dimensional boundary layer flow past an isothermal stretching sheet in a water-based nanofluid which can contains different volume fraction of Cu (copper) and Ag (silver) nanoparticles and uniform magnetic field is applied on it. It is assumed that the induced flow of nanofluid is laminar, and the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the fluid and nanoparticles are given in Table 1. It is assumed that the sheet surface has constant temperature of T_w, and the temperature of ambient fluid is T_∞. The fluid outside the boundary layer is quiescent, and the stretching sheet velocity is U(x) = cx where c is a constant. Under the usual boundary layer approximations, the continuity, momentum and energy equations for the nanofluid, in the Cartesian coordinates can be represent as.

The continuity equation;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial^2 y} \tag{2}$$

The energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The initial and boundary conditions are;

$$at \quad y=0; \quad v=0\,, \quad u=U_w(x)\,, \quad T=T_w$$

$$at \quad y=\infty; \quad u=0, \quad v=0, \quad T=T_\infty \tag{4}$$

where, u and v are the velocity components in the x and y directions respectively. T is temperature of the nanofluid, T_∞ temperature of the ambient nanofluid, B_0 the uniform magnetic field strength, σ electrical conductivity of base fluid, ρ_{nf} effective density of the nanofluid, α_{nf} thermal diffusivity of the nanofluid, μ_{nf} dynamic viscosity of the nanofluid are respectively given as:

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{5}$$

$$\alpha_{nf}=\frac{K_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}} \tag{7}$$

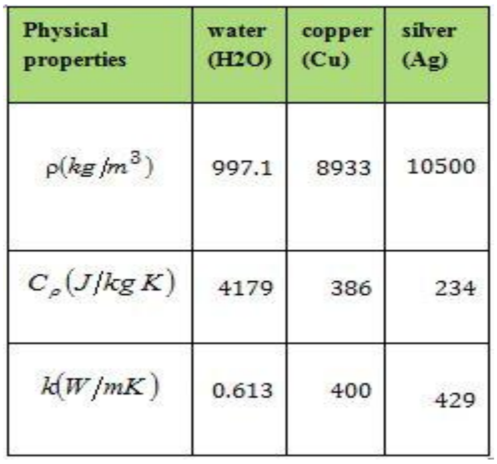
The effective density of the nanofluid is given by

$$(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s \tag{8}$$

$$\frac{K_{nf}}{K_f}=\frac{(K_s+2K_f)-2\emptyset(K_f-K_s)}{(K_s+2K_f)+\emptyset(K_f-K_s)} \tag{9}$$

Here $\vartheta_f, \mu_f, \rho_f, K_f$ are the Kinematic viscosity, dynamic viscosity, density and thermal conductivity of the base fluid respectively $\rho_s, K_s, (\rho C_P)_s$ are the density, thermal conductivity and heat capacitance of the nanoparticles respectively; φ is the solid volume fraction of nanoparticles and K_{nf} is thermal conductivity of the nanofluid. Subscripts f and s are used for base fluid and nanoparticles respectively. In this case, water as base fluid with nonoparticles of copper and silver are studied.

Table1: Thermophysical Properties For Pure Water And Various Types Of Nanoparticles.



Physical properties	**water (H2O)**	**copper (Cu)**	**silver (Ag)**
$\rho(kg/m^3)$	997.1	8933	10500
$C_p(J/kg\,K)$	4179	386	234
$k(W/mK)$	0.613	400	429

III. SOLUTION OF THE PROBLEM

To simplify the mathematical analysis of our study we introduce the following similarity transformations

$$\psi(x,y) = f(\eta)x(v_f c)^{1/2} \qquad u = cxf'(\eta)$$

$$v = -f(\eta)(v_f c)^{1/2} \qquad \eta = y(c/v_f)^{1/2}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

Where $\psi(x,y)$ is the stream function with $u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$

$\theta(\eta)$ = dimensionless temperature.

$f(\eta)$= dimensionless velocity

where primes denote differentiation with respect to the similarity variable η. By applying the introduced similarity transforms (10) on the governing equations (1-3), the equations are reduces as follows,

$$\frac{\mu_{nf}}{\mu_f} f''' + \left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right](f\,f'' - f'^2) - M\,f' = 0 \tag{11}$$

$$\frac{k_{nf}}{k_f}\theta'' + \Pr\left((1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)f\theta' = 0 \qquad (12)$$

Subject to the following boundary conditions:

$$\eta = 0, \quad y = 0, \quad f = 0, \quad f' = 1, \quad \theta = 1$$

$$\eta \to \infty, \; y \to \infty, \; f' = 0, \; \theta = 0 \qquad (13)$$

$\Pr = \dfrac{\mu_f}{k_f}$ Pr = Prandtl number.

The physical quantities of interest in this problem are the local skin friction coefficient C_f and the Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \qquad Nu = \frac{q_w}{k_f(T_w - T_\infty)} \qquad (14)$$

Where ,

τ_w is the surface shear stress and q_w is the surface heat flux ,which are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (15)$$

Using the similarity variable (10), we obtain

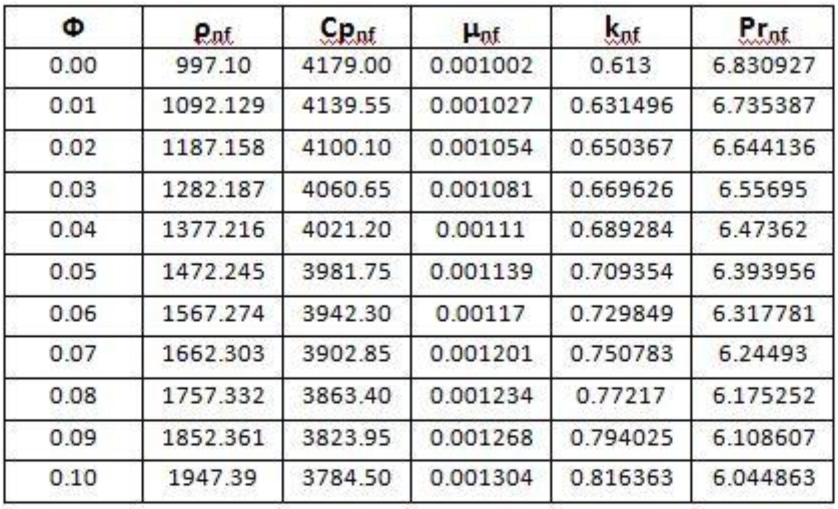
$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\phi)^{2..5}} f''(0) \qquad \mathrm{Re}_x^{-1/2} Nu = -\frac{k_{nf}}{k_f}\theta'(0) \qquad (16)$$

$\mathrm{Re}_x = u_w x / v_f$ is the local Reynolds number.

IV. RESULTS AND DISCUSSIONS

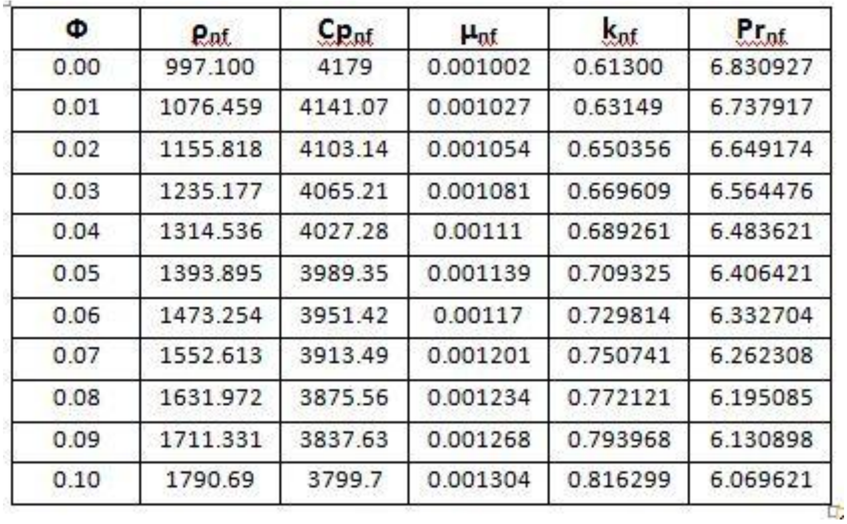
In the present paper an analysis is carried out numerically to study the effect of the magnetic field on the boundary layer flow of the nanofluids over an isothermal stretching surface. Two types of nanofluids namely Ag–water and Cu–water are considered. It is found that the behavior of the fluid flow changes with the change of nanoparticle type.As the governing boundary layer equations (11) and (12) are non linear, it is not possible to get the closed form solutions. As a result, the equations with the boundary conditions (13) are solved numerically using Runge - Kutta forth order method with a systematic guessing of $f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution. The numerical computations are carried out for velocity, temperature at different values of magnetic parameter M and are presented in figures 1- 6.

Table2: Thermophysical Properties of Ag- water Nanofluid.



Φ	ρ_{nf}	Cp_{nf}	μ_{nf}	k_{nf}	Pr_{nf}
0.00	997.10	4179.00	0.001002	0.613	6.830927
0.01	1092.129	4139.55	0.001027	0.631496	6.735387
0.02	1187.158	4100.10	0.001054	0.650367	6.644136
0.03	1282.187	4060.65	0.001081	0.669626	6.55695
0.04	1377.216	4021.20	0.00111	0.689284	6.47362
0.05	1472.245	3981.75	0.001139	0.709354	6.393956
0.06	1567.274	3942.30	0.00117	0.729849	6.317781
0.07	1662.303	3902.85	0.001201	0.750783	6.24493
0.08	1757.332	3863.40	0.001234	0.77217	6.175252
0.09	1852.361	3823.95	0.001268	0.794025	6.108607
0.10	1947.39	3784.50	0.001304	0.816363	6.044863

Table3: Thermophysical Properties of Cu- water Nanofluid**.**



Φ	ρ_{nf}	Cp_{nf}	μ_{nf}	k_{nf}	Pr_{nf}
0.00	997.100	4179	0.001002	0.61300	6.830927
0.01	1076.459	4141.07	0.001027	0.63149	6.737917
0.02	1155.818	4103.14	0.001054	0.650356	6.649174
0.03	1235.177	4065.21	0.001081	0.669609	6.564476
0.04	1314.536	4027.28	0.00111	0.689261	6.483621
0.05	1393.895	3989.35	0.001139	0.709325	6.406421
0.06	1473.254	3951.42	0.00117	0.729814	6.332704
0.07	1552.613	3913.49	0.001201	0.750741	6.262308
0.08	1631.972	3875.56	0.001234	0.772121	6.195085
0.09	1711.331	3837.63	0.001268	0.793968	6.130898
0.10	1790.69	3799.7	0.001304	0.816299	6.069621

Fig1. It shows the dimensionless velocity $f'(\eta)$ for various values of magnetic parameter M. As the value of magnetic parameter M increases,the retarding force increases because of interaction of electric and magnetic fields and consequently the velocity decreases. Fig2. Exhibits the effect of shear stress distribution for various values of M. It is observed that the magnitude of the wall of shear stress given by $(1/(1-\Phi)^{2.5})f''(0)$ decreases when the value of magnetic parameter M increases. Fig 3: Dipicts the dimensionless velocity profiles for selected values of volume fraction Φ. it shows that increase of nanoparticle volume fraction have not significant effect on the velocity profile. Fig 4: Shows effect of Prandtl number Pr on volume fraction for Cu-water and Ag-water nanofluids. It is obvious that as prandtl number Pr decreases volume fraction increases in both the cases and the decrease of Pr is slightly more for Ag- water nanofluid than Cu- water nanofluid. Fig 5: Reveals the effect of volume fraction on density of the Cu-water and Ag-water nanofluids.It is observed that when volume fraction increases, density of all nanofluids increases and the increase is more for Ag –water nanofluid than other nanofluid. Fig. 6. It exhibits the temperature profile for selected values of volume fraction Φ. It is very much evident from the fig that temperature of nanofluids increases as the value of volume fraction increases.

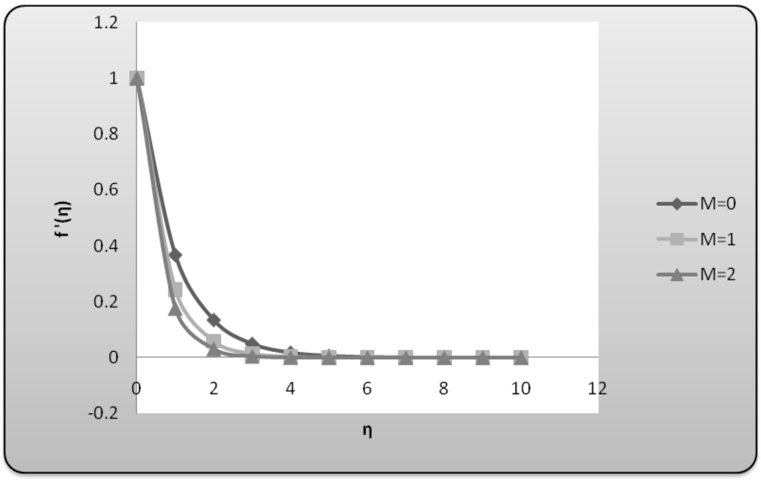


Figure1.Effect of magnetic parameter M on dimensionless velocity profiles f '(η)

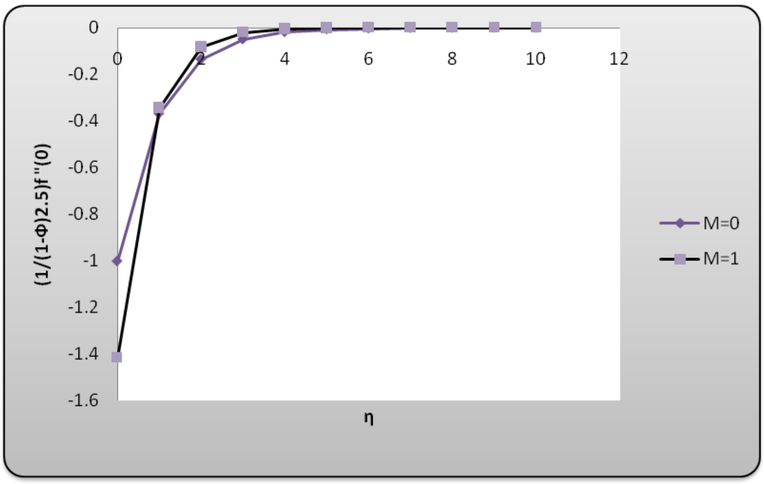


Figure2: Shear stress distribution for various values of M

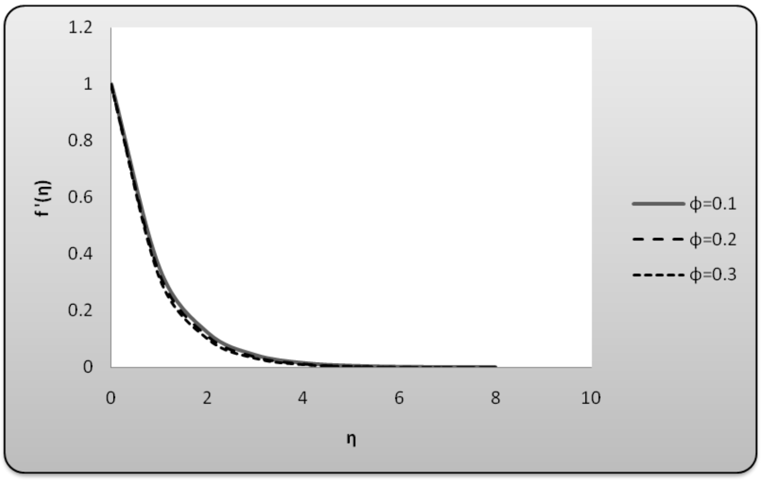


Figure3: Dimensionless velocity profiles for selected values of volume fraction Φ

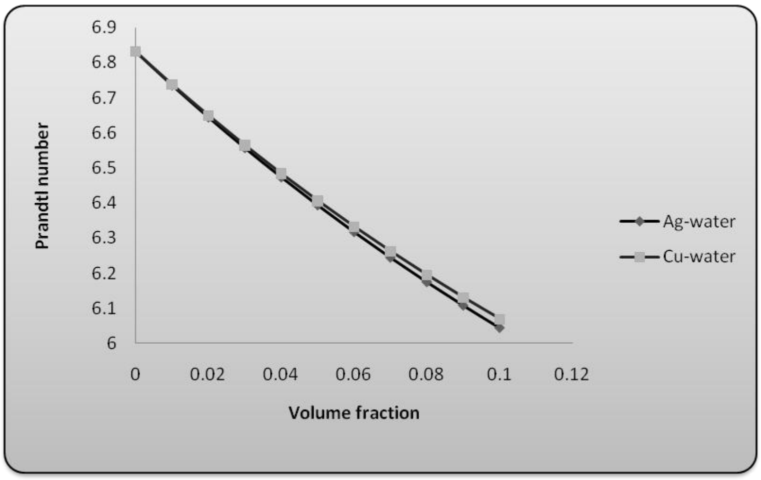


Figure 4: Effect of Prandtl number Pr on volume fraction for Cu-water and Ag-water nanofluids

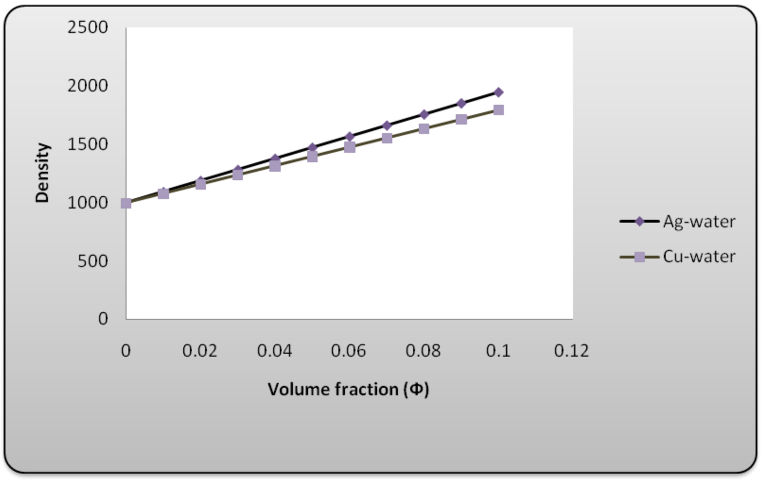


Figure 5: Effect of volume fraction on density of the Cu-water and Ag-water nanofluids

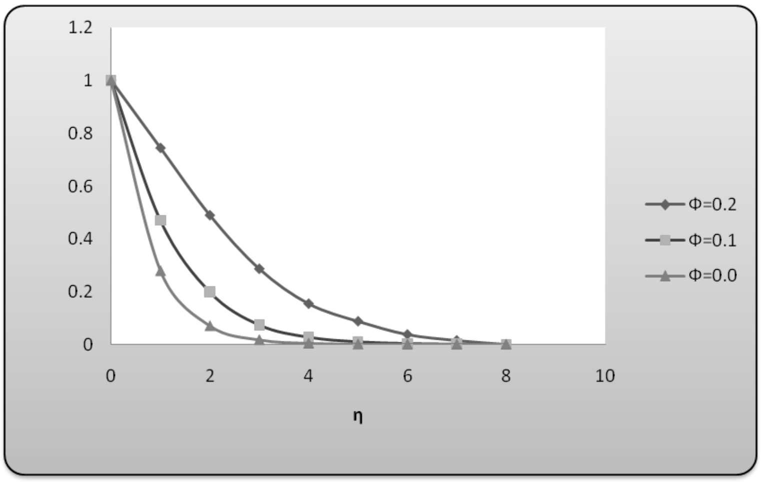


Figure 6: Temperature profile for selected values of volume fraction Φ

V. CONCLUSION

In this paper the effect of the magnetic field on the boundary layer flow of the nanofluids over an isothermal stretching surface was presented. Velocity and temperature distribution in the flow and thermal boundary layers studied. Numerical results prove that nanofluids under the influence of magnetic field lead to drop of dimensionless velocity and magnitude of the wall of shear stress at the surface. However, it was also observed that when volume fraction increases, density and temperature of all nanofluids increases. Such effects were found to be more evident in the Ag–water solution than in the Cu–water solution

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