

Unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction

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ABSTRACT: An analysis is presented to study the forced convection in unsteady magneto-hydrodynamic boundary layer flow of a nanofluid over a permeable shrinking sheet in the presence of thermal radiation and chemical reaction. A variable magnetic field is applied normal to the sheet. The nanofluid model includes Brownian motion and thermophoresis effects are also considered. The boundary layer equations governed by the partial differential equations are transformed into a set of ordinary differential equations the help of local similarity transformations. The coupled and nonlinear differential equations are solved by the implicit finite difference method along with the Thomas algorithm. We have explained the effect of various controlling flow parameters namely unsteadiness parameter A , magnetic parameter M , thermal radiation parameter R , Prandtl number Pr , Brownian motion parameter N_b , thermophoresis parameter N_t and Lewis number Le on the dimensionless velocity, temperature and nanoparticle volume fraction profiles are analyzed.

KEY WORDS: Nanofluid, Magnetic field, Thermal radiation, Chemical reaction, Shrinking sheet, implicit finite difference method.

I. INTRODUCTION:

The flow over a shrinking surface is an important problem in many engineering processes with applications in industries such as the hot rolling, wire drawing and glass wire production. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The present trend in the field of magnetic strength analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with magnetic effect is of considerable importance in chemical and hydrometallurgical industries. Bhattacharyya and Gupta [1], Gupta and Gupta [2] and Cheng and Lin [3] studied the heat and mass transfer on nonlinear MHD boundary layer flow in various situations.

Magneto-hydrodynamic (MHD) mixed convection heat transfer flow in porous and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, high temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids and power generation systems. A few representative fields of interest in which combined heat and mass transfer with chemical reaction play important role, are design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, food processing and cooling towers. Cooling towers are the cheapest way to cool large quantities of water. Chemical reaction can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. This depends on whether they occur at an interface or as a single phase volume reaction. For example, formation of smog is a first order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke-stacks. This NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope of what is termed as photochemical smog.

The boundary layer flow over a shrinking surface is encountered in several technological processes. Such situations occur in polymer processing, manufacturing of glass sheets, paper production, in textile industries and many others. Crane [4] initiated a study on the boundary layer flow of a viscous fluid towards a linear stretching sheet. An exact similarity solution for the dimensionless differential system was obtained. Carragher and Carane [5] discussed heat transfer on a continuous stretching sheet. Afterwards, many investigations were made to examine flow over a stretching/shrinking sheet under different aspects of MHD, suction/injection, heat and mass transfer etc. [6–13]. In these attempts, the boundary layer flow, due to stretching/shrinking has been analyzed. Magyari and Keller [14] provided both analytical and numerical solutions for boundary layer flow over an exponentially stretching surface with an exponential temperature distribution. The combined effects of viscous dissipation and mixed convection on the flow of a viscous fluid over an exponentially stretching sheet were analyzed by Partha et al. [15], Elbashbeshy [16] numerically studied flow and heat transfer over an exponentially stretching surface with wall mass suction. Madhu.M and Naikoti Kishan[17] studied the Two-dimensional MHD mixed convection boundary layer flow of heat and mass transfer stagnation-point flow of a non-Newtonian power-law nanofluid towards a stretching surface in the presence of thermal radiation and heat source/sink.

On the other hand, the flow over a shrinking sheet is a new field of research at present and few literatures is available on this area of research now. Wang [18] first studied a specific shrinking sheet problem. Recently, Miklavcic and Wang [19] obtained the existence and uniqueness of the solution for steady viscous hydrodynamic flow over a shrinking sheet with mass suction. Hayat et al. [20] derived both exact and series solution (using HAM) describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of MHD viscous flow due to a shrinking sheet was solved by Sajid and Hayat [21] using HAM.

It is interesting to note that the Brownian motion of nanoparticles at molecular and nanoscale levels are a key nanoscale mechanism governing their thermal behaviors. In nanofluid systems, due to the size of the nanoparticles, the Brownian motion takes place, which can affect the heat transfer properties. As the particle size scale approaches to the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in the heat transfer. In view of these applications, Nield and Kuznetsov ([22, 23]) analyzed the free convective boundary layer flows in a porous medium saturated by nanofluid by taking Brownian motion and thermophoresis effects into consideration. In the first article, the authors have assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology and hence they have concluded that this prevents particles from agglomeration and deposition on the porous matrix. Chamkha *et al.* [24] carried out a boundary layer analysis for the natural convection past an isothermal sphere in a Darcy porous medium saturated with a nanofluid. Nield and Kuznetsov [25] investigated the cross-diffusion in nanofluids, with the aim of making a detailed comparison with regular cross diffusion effects and the cross-diffusion effects peculiar to nanofluids, and at the same time investigating the interaction between these effects when the base fluid of the nanofluid is itself a binary fluid such as salty water. Recently, a boundary layer analysis for the natural convection past a horizontal plate in a porous medium saturated with a nanofluid is analyzed by Gorla and Chamkha [26], N. Kishan et.al [27], studied the unsteady MHD flow of heat and mass transfer of Cu-water and TiO₂-water nanofluids over stretching sheet with a non-uniform heat/source/sink considering viscous dissipation and chemical reaction.

The effect of an applied magnetic field on nanofluids has substantial applications in chemistry, physics and engineering. These include cooling of continuous filaments, in the process of drawing, annealing and thinning of copper wire. Drawing such strips through an electrically conducting fluid subject to a magnetic field can control the rate of cooling and stretching, thereby furthering the desired characteristics of the final product. Such an application of a linearly stretching sheet of incompressible viscous flow of MHD was discussed by Pavlov[28] . In other work, Jafar et al.[29] studied the effects of magnetohydrodynamic (MHD) flow and heat transfer due to a stretching/shrinking sheet with an external magnetic field, viscous dissipation and Joule effects. Recently, Samir kumar at.el.[30] studied the forced convection in unsteady boundary layer flow of a nanofluid over a permeable shrinking sheet in the presence of thermal radiation.

It is now propose to study the effects of chemical reaction and the influence of magnetic field on the flow and heat transfer due to the unsteady two dimensional laminar flow of a incompressible viscous nanofluid caused by a permeable shrinking sheet with thermal radiation effects. The results are presented focus on how the chemical reaction parameter, thermal radiation, magnetic field, Brownian motion, thermophoresis effects of the heat transfer and characteristic of the flow.

II. FLOW ANALYSIS:

Consider unsteady two-dimensional laminar boundary-layer flow of incompressible electrically conducting viscous nanofluid past a permeable shrinking sheet. The flow is subjected to a transverse magnetic field of strength B which is assumed to be applied in the positive y -direction, normal to the surface. It is

assumed that the velocity of the shrinking sheet is $u_w(x, t)$ and the velocity of the mass transfer is $v_w(x, t)$, where x is the coordinate measured along the shrinking sheet and t is the time. It is also assumed that the constant surface temperature and concentration of the sheet are T_w and C_w , while the uniform temperature and concentration far from the sheet are T_∞ and C_∞ , respectively. Under these assumptions, the unsteady boundary-layer equations governing the flow, heat and mass transfer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho_f} u - \frac{\mu}{k\rho} u \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_f c_p} \frac{\partial q_r}{\partial y} + \tau [D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} (\frac{\partial T}{\partial y})^2] \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_c(C - C_\infty) \quad (4)$$

where u and v are the velocity components in the x and y -directions respectively, m is the kinematic viscosity, r is the electrical conductivity (assumed constant), ρ_f is the density of the base fluid, α_m is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and C_p is the specific heat at constant pressure. Here τ is the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid, T is the fluid temperature and C is the nanoparticle volume fraction.

The term $\frac{\sigma B^2}{\rho_f} u$ in the R.H.S. of Eq. (2) is the Lorentz force which arises due to the interaction of the fluid velocity and the applied magnetic field and $\frac{\mu}{k\rho} u$ is permeability parameter. In writing Eq. (2), we have neglected the induced magnetic field since the magnetic Reynolds number for the flow is assumed to be very small. This assumption is justified for flow of electrically conductive fluids such as liquid metals e.g. mercury, liquid sodium, etc. Eq. (3) depicts that heat can be transported in a nanofluid by convection, by conduction and also by virtue of nanoparticle diffusion and radiation.

The term $u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}$ is the heat convection; the term $\alpha_m \frac{\partial^2 T}{\partial y^2}$ is the heat conduction; the term $\tau D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y}$ is the thermal energy transport due to Brownian diffusion; the term $\tau \frac{D_T}{T_\infty} (\frac{\partial T}{\partial y})^2$ is the energy transport due to thermophoretic effect and $\frac{1}{\rho_f c_p} \frac{\partial q_r}{\partial y}$ is the nanoparticle heat diffusion by radiation. Eq. (4) shows that the nanoparticles can move homogeneously within the fluid (by the term $\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y}$), but they also possess a slip velocity relative to the fluid due to Brownian diffusion $D_B \frac{\partial^2 C}{\partial y^2}$; the thermophoresis $\frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}$ and $k_c(C - C_\infty)$ is chemical reaction parameter.

Here the boundary conditions are

$$u = u_w(x, t) = -\frac{cx}{(1-\lambda t)}, \quad v = v_w(x, t), \quad T = T_w, \quad C = C_w, \quad \text{at } y = 0,$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty, \quad (5)$$

The wall mass transfer velocity then becomes

$$v_w(x, t) = -\sqrt{\frac{cv}{(1-\lambda t)}} s, \quad (6)$$

where s is the constant wall mass transfer parameter with $s > 0$ for suction and $s < 0$ for injection, respectively. Using Rosseland's (see Brewster [31]) approximation for radiation we can write

$$q_r = -\frac{4\sigma_1}{3K_1} \frac{\partial T^4}{\partial y}, \quad (7)$$

where σ_1 is the Stefan-Boltzmann constant and K_1 is the mean absorption coefficient. Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher order terms we get $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$. Hence from Eq. (7), using the above result,

$$\text{We have } \frac{\partial q_r}{\partial y} = - \frac{16\sigma_1 T_\infty^3}{3K_1} \frac{\partial^2 T}{\partial y^2} \quad (8)$$

to attain the similarity solutions of the Eqs. (1) – (4) with the boundary conditions (5), we take the transverse unsteady magnetic field strength applied to the sheet is of the form $B = B_0/\sqrt{1-\lambda t}$, where B_0 is constant. This form of $B_0(t)$ has also been considered by Vajravelu et al. [32] while analyzing the MHD flow and heat transfer over an unsteady stretching sheet. The stream function and dimensionless variable can be taken as

$$\psi = \sqrt{\frac{cv}{(1-\lambda t)}} xf(\eta), \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \varphi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \quad \eta = y \sqrt{\frac{c}{v(1-\lambda t)}} \quad (9)$$

Where the stream function ψ is defined in the usual way $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Substituting (9) into Eqs. (1)–(4), we obtain the following ordinary differential equations

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f''\right) - Mf' - \delta f = 0 \quad (10)$$

$$\frac{1}{Pr_{eff}}\theta'' + f\theta' - A\frac{\eta}{2}\theta' + Nb\theta'\varphi' + Nt\theta'^2 = 0 \quad (11)$$

$$\varphi'' + Le\left(f - A\frac{\eta}{2}\right)\varphi' + \frac{Nt}{Nb}\theta'' + Y\varphi = 0 \quad (12)$$

Where

$$M = \frac{\sigma B_0^2}{\rho_f c}, \quad \delta = \frac{\mu}{k_\rho} u, \quad A = \frac{\lambda}{c}, \quad Pr_{eff} = \frac{Pr}{1+4R/3}, \quad R = \frac{4\sigma_1 T_\infty^3}{K_1 \alpha_m \rho_f c_\rho}, \quad Pr = \frac{v}{\alpha_m}, \\ v = \frac{\mu}{\rho_f}, \quad Nb = \frac{\tau(C_w - C_\infty) D_B}{v}, \quad Nt = \frac{\tau(T_w - T_\infty) T}{v T_\infty}, \quad Le = \frac{v}{D_B}, \quad Y = k_c(C - C_\infty) \quad (13)$$

Here M is the dimensionless magnetic parameter, A is the unsteadiness parameter, δ is permeable parameter, Pr_{eff} is the effective Prandtl number, R is the thermal radiation parameter, Pr is the Prandtl number, v is the kinematic viscosity of the fluid, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Le is the Lewis number and Y is chemical reaction parameter. It is worth noting that the temperature actually does not depend on Prandtl number (Pr) and the thermal radiation parameter (R) independently, but depends only on a combination of them termed as effective Prandtl number Pr_{eff} which is directly proportional to the Prandtl number and inversely proportional to the thermal radiation parameter.

The corresponding boundary conditions are

$$f(0) = s, f'(0) = -1, \theta(0) = 1, \varphi(0) = 1.$$

$$f'(\infty) = 0, \theta(\infty) = 0, \varphi(\infty) = 0. \quad (14)$$

The physical quantities of interest are the skin friction coefficient C_f , the local Nusselt number Nu_x and the local Sherwood number Sh_x which are defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \quad (15)$$

Where τ_w is the shear stress at the stretching surface, q_w and q_m are the wall heat and mass fluxes, respectively. Hence using Eq. (10) we get

$$Re_x^{1/2} C_f = f''(0), \quad Nu_x Re_x^{-1/2} = -\theta'(0), \quad Sh_x Re_x^{-1/2} = -\varphi'(0) \quad (16)$$

Where $Re_x = u_w(x, t)x/v$ is the local Reynolds number based on the stretching velocity $u_w(x, t)$.

III. RESULTS AND DISCUSSION:

As the governing boundary layer equations 10-12 non linear, it is not possible to get the closed form solutions. Consequently, the equations with the boundary conditions (14) are solved numerically by the means of implicit finite difference scheme along with the Gauss-Sidel method. In order to investigate the flow quantities like velocity, temperature and concentration profiles and so forth a parametric study as taken to

illustrate the effects of the various physical parameters namely, magnetic parameter M , Unsteadiness parameter A , Prandtl number Pr , thermal radiation parameter R , Brownian motion parameter N_b , Thermophoresis parameter N_t , Lewis number Le , permeability parameter δ and Chemical reaction parameter γ .

The numerical computations are carried out for velocity, temperature and concentration profiles and are presented in figures 1- 9. It is found that the system of equations have dual solutions. The solutions are called as first and second solutions. The variations of dimensionless velocity, temperature and concentration profiles with the effect of magnetic field parameter M is depicted in figure 1(a) – 1(c) respectively. It is seen in figure 1(a) that the effect of magnetic field M increases, the dimensionless velocity profiles increases for first solution while it decreases in the second solution and figure 1(b) the reverse phenomenon observed in the both cases of first and second solution. From figure 1(c), it can be seen that increase the magnetic field M effect is to decrease the concentration profiles in the first solution. The trends observed for the second solution as same as for the first solution but with a difference that the effect of M is more significantly in second solution.

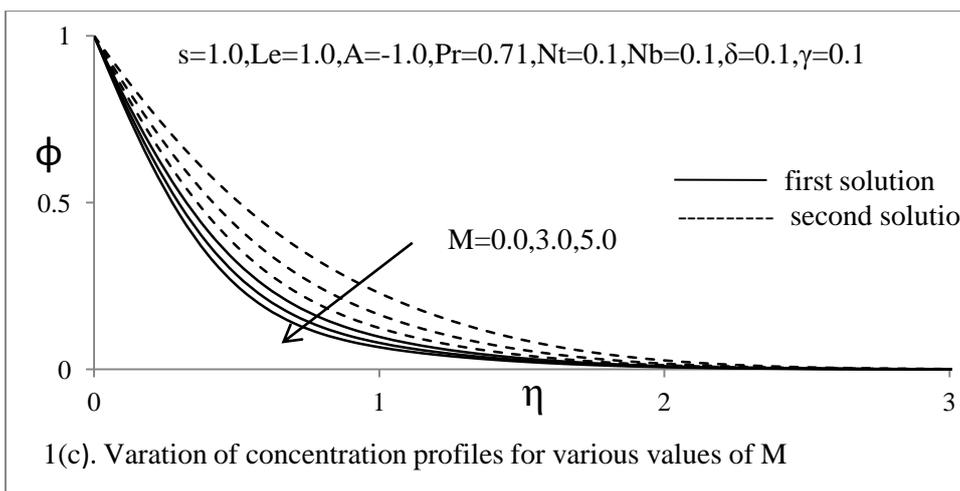
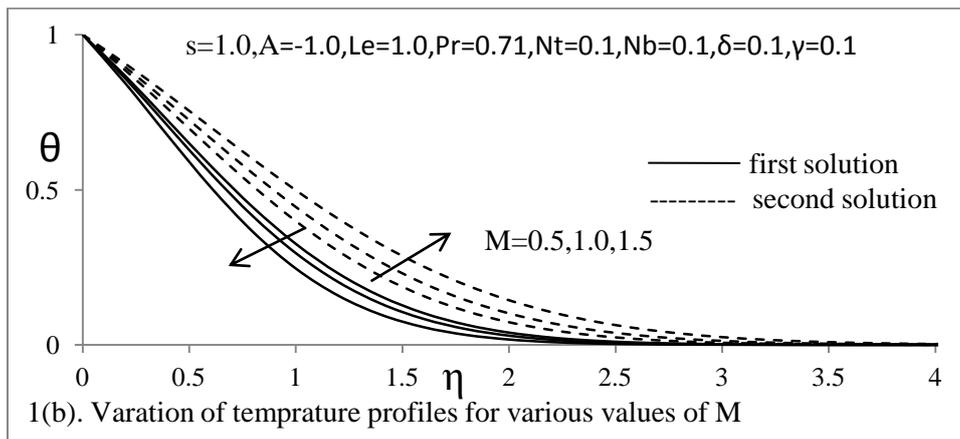
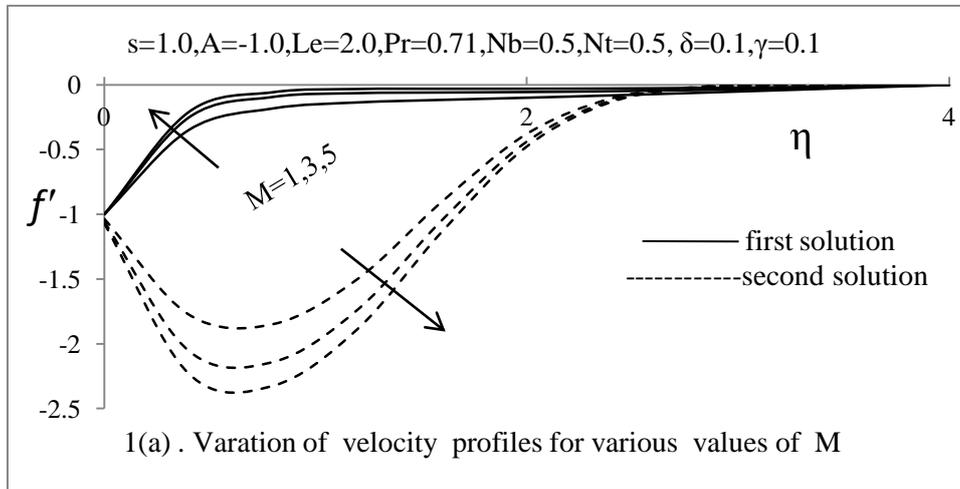
Figure 2(a) - 2(c) shows the variation of velocity, temperature and concentration profiles but different values of suction parameter s respectively. It can be seen in figure 2(a), for the first solution, if the increasing of suction parameter s the fluid velocity increases while the fluid velocity decreases with the increase of suction parameter s in the second solution. Figure 2(b) depicts that with the increase of suction parameter s , the temperature profiles decreases for both the solutions. The figure 2(c) depicts that the increasing in suction parameter s leads to enhance the concentration profiles both the solutions. Figure 3(a)-3(c) are drawn to analyze the influence of unsteadiness parameter $A (<0)$ on 3(a) velocity profiles, 3(b) temperature profiles and 3(c) concentration profiles respectively. It can be seen that the velocity of a fluid increases with increase of unsteadiness parameter A in the vicinity of the boundary in both the solutions, the reverse trend is observed when it is away from the boundary. From figure 3(b) and 3(c) reveals that the temperature at a point decreases as the magnitude of the unsteadiness parameter A increases. This is due to the fact that the heat transfer rate increases with the increase of unsteadiness parameter A which in turn reduces the temperature of fluid. This same can be observing both the solutions. From the figure 3(c), it can be seen that the concentration profiles decreases as increasing as the magnitude of unsteadiness parameter A increases in both first and second solutions.

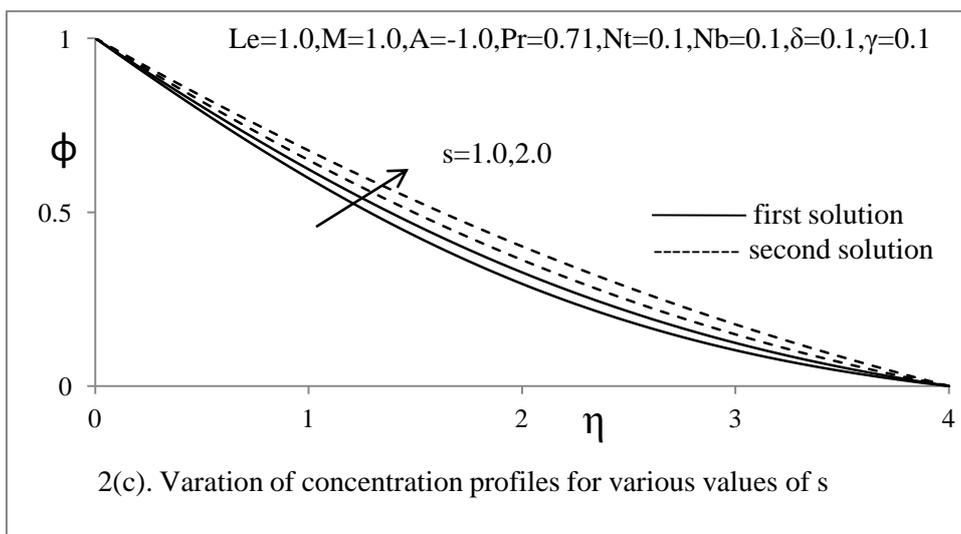
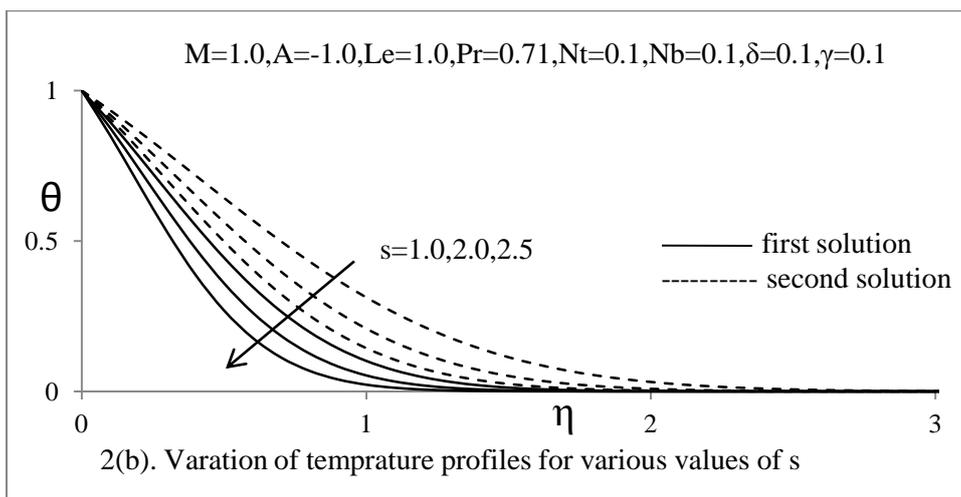
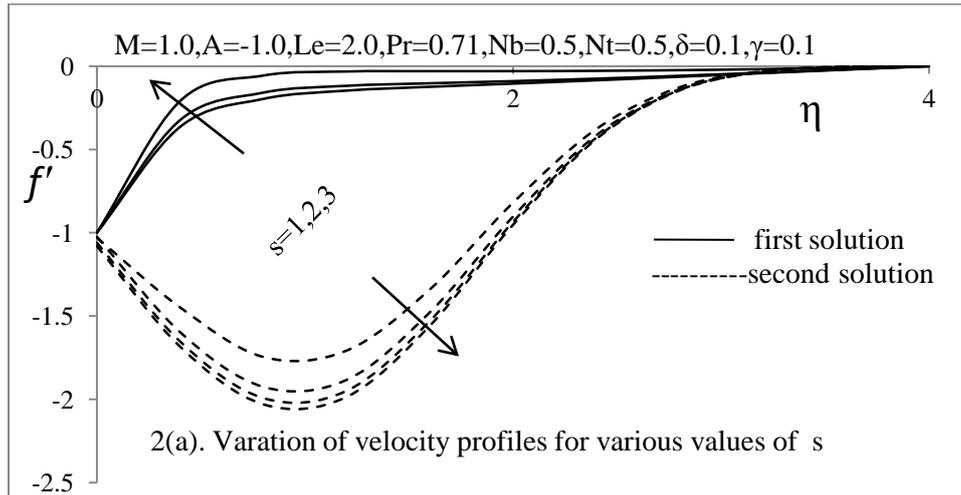
The effects of chemical reaction parameter γ on the dimensionless temperature and concentration profiles are illustrated in figure 4(a) and 4(b) respectively. It can be seen that the temperature of a fluid increases with the increase for restrictive chemical reaction parameter $\gamma (>0)$ and it increases for generating chemical reaction parameter $\gamma (<0)$ in both solutions. From figure 4(b) it is seen that the concentration profiles increase for generating chemical reaction parameter $\gamma (>0)$, while it decreases restrictive chemical reaction parameter $\gamma (<0)$ in both solutions. The reaction rate parameter is a decelerating agent when $\gamma (>0)$. The concentration boundary layer decreases in case of restrictive chemical reaction. This is due to the conversion of the species takes place as a result of chemical reaction and thereby reduces the concentration boundary layer; actually chemical reaction causes to increase the rate of interference mass transfer.

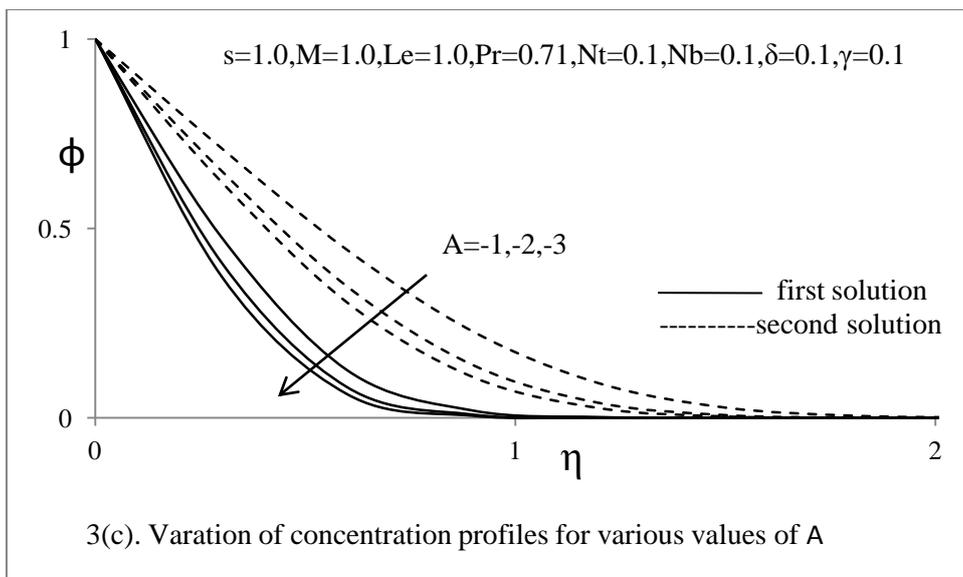
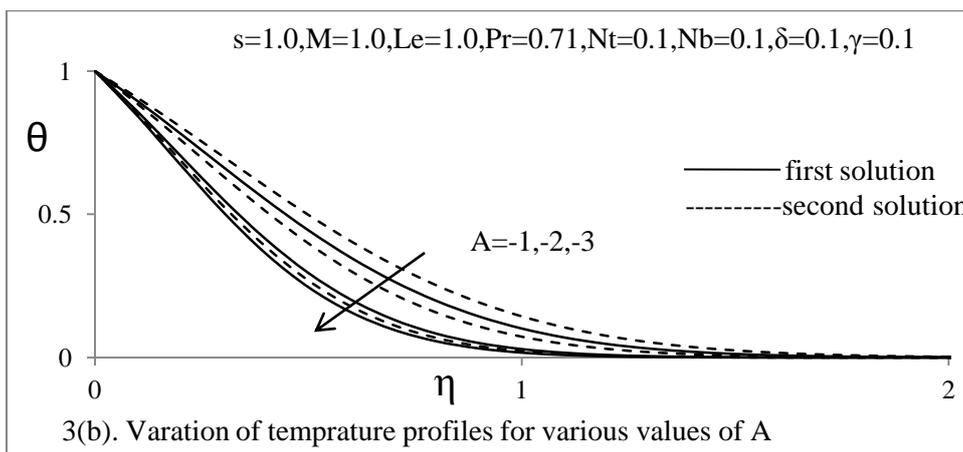
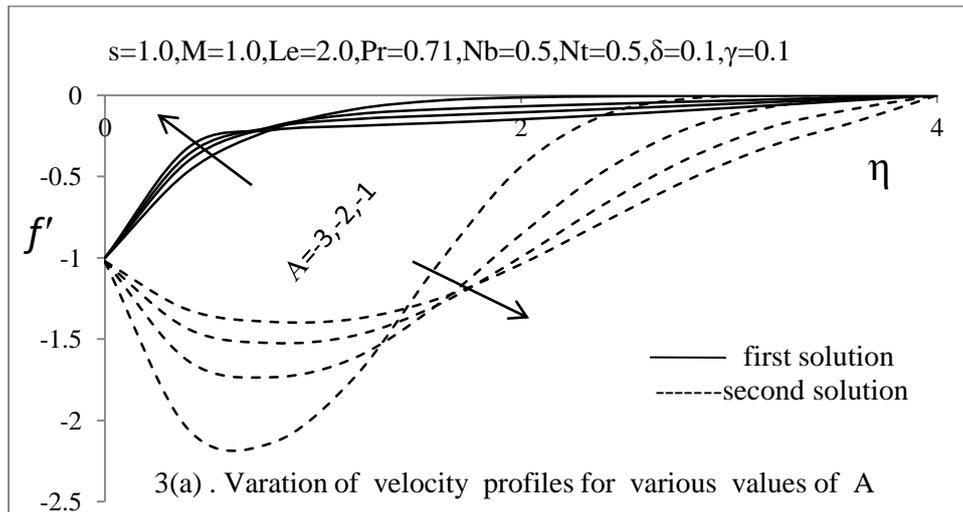
Figure 5(a) and 5(b) is plotted for the different values of Prandtl number Pr to the temperature and concentration profiles. It can be noticed from these figures. The temperature and concentration profiles decrease with the increase of Prandtl number Pr for both first and second solution. An increase in Prandtl number Pr reduces the thermal boundary layer as well as concentration boundary layer. Prandtl number Pr signifies the ratio of momentum diffusivity. As Prandtl number Pr decreases the thickness of thermal boundary layer becomes thickness of the velocity boundary layer. So the thickness of thermal boundary layer increases as Prandtl number Pr decreases and hence temperature profiles decrease with the increase of Prandtl number Pr . In heat transfer problems the Prandtl number Pr controls the relative thickness of momentum and thermal boundary layer hence Prandtl number Pr can be used to increase the rate of cooling in conducting flows.

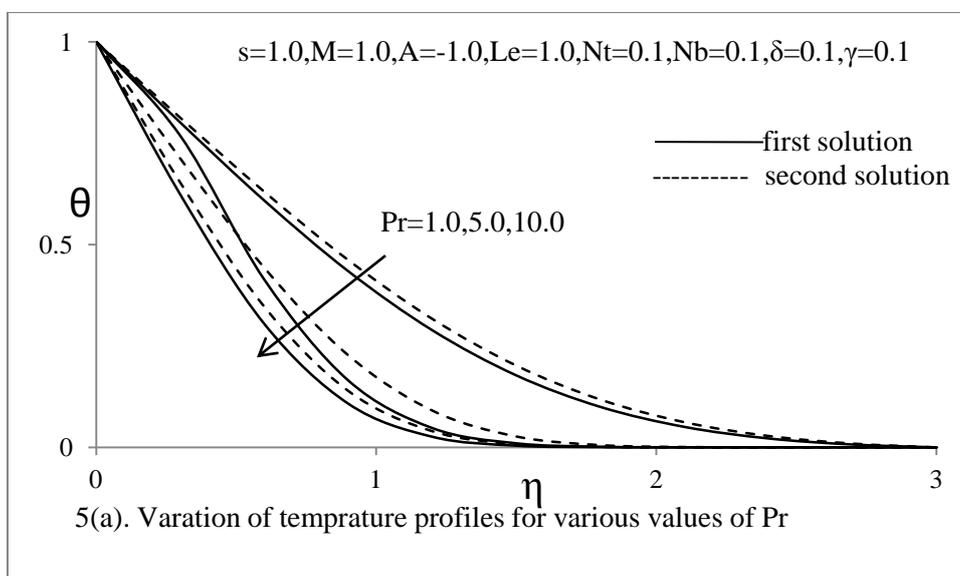
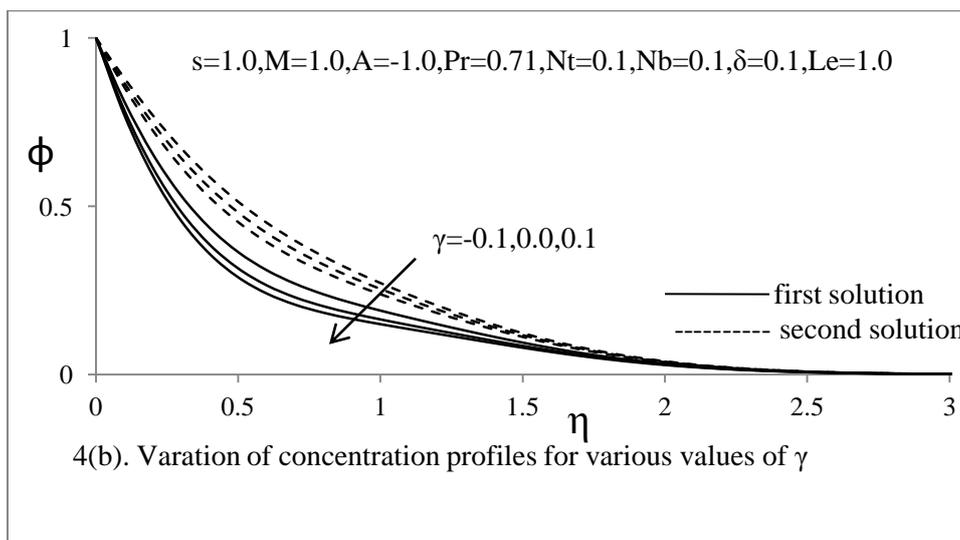
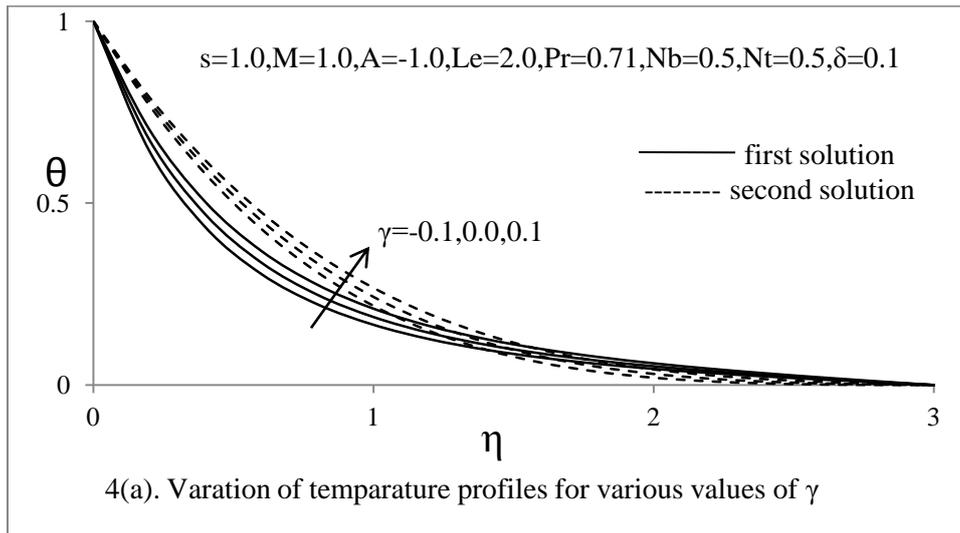
The effect of thermophoresis parameter N_t on temperature and concentration profiles are illustrated in figure 6(a) and 6(b). As the thermophoresis parameter N_t increases the temperature profiles increase and hence the thickness of thermal boundary layer with the increase of the thermophoresis parameter N_t . From figure 6(b) reveals that the concentration profiles increase with the increase of thermophoresis parameter N_t in both the first and second solutions. The thermophoresis parameter N_t phenomenon describes the fact that small micron size particles suspended in non-isothermal fluid will acquire a velocity in the direction of decreasing temperature. As increases of thermophoresis parameter N_t results in an increase of the temperature difference between sheet and the ambient fluid, consequently the thermal boundary layer thickness increases. Figure 6(b) reveals that the thermophoresis parameter N_t effects enhance the concentration profiles ϕ in both solutions.

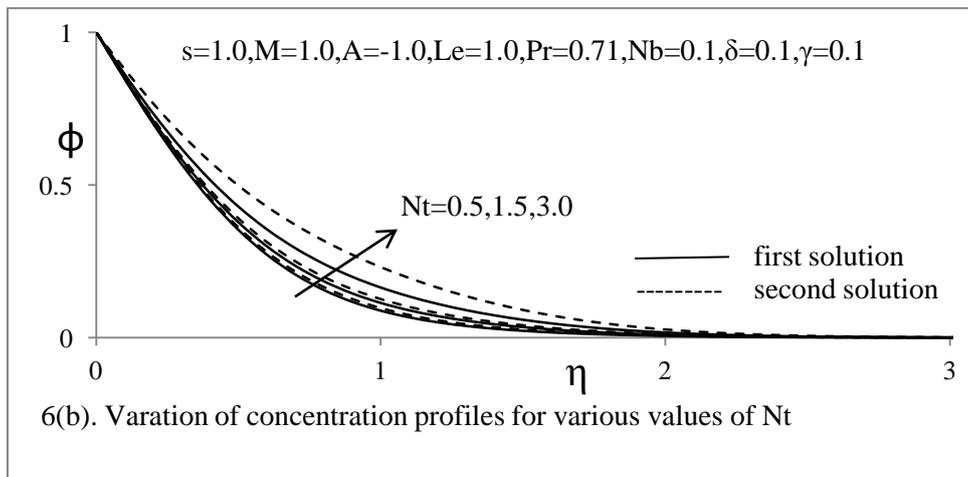
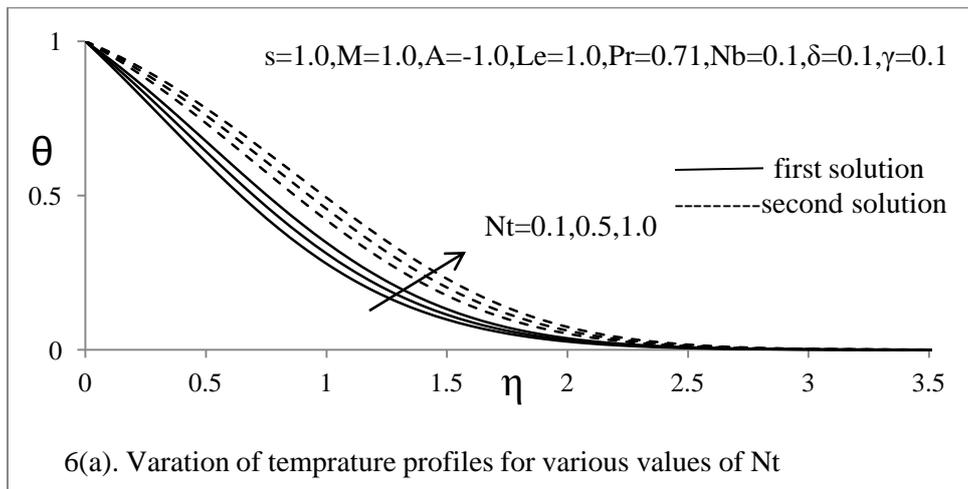
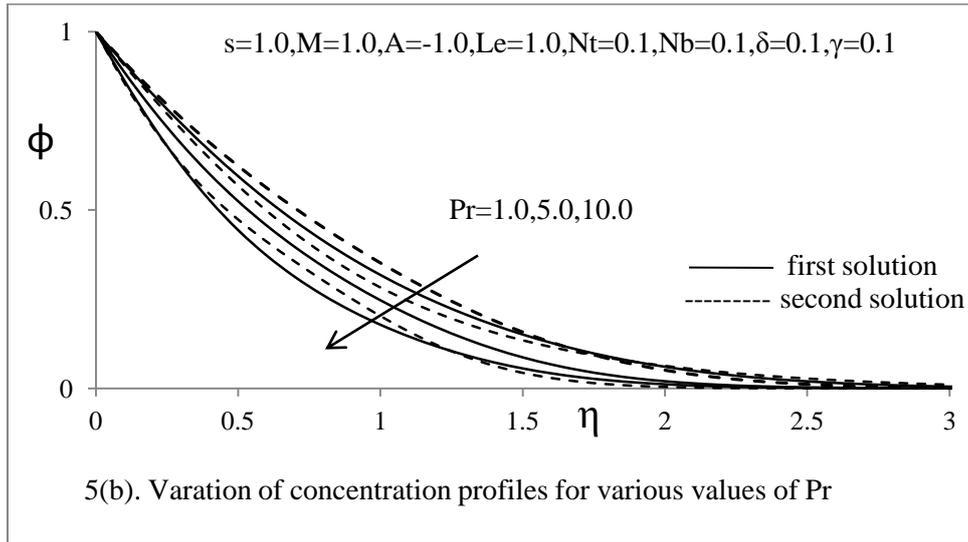
The figure 7(a) depicts for the effect of Brownian motion parameter Nb for the first and second solution respectively. The temperature profiles of nanofluid in the boundary layer region increase with the increase of Brownian motion parameter Nb. As Brownian motion parameter Nb the thickness of boundary layer increases. Figure 7(b) reveals that the impact of Brownian motion parameter Nb on concentration profiles decreases and hence the thickness of the concentration boundary layer decreases in both the solutions. From figure.8 observed that the permeability parameter δ increases the dimensionless velocity profiles decreases for first solution while it increases in the second solution away from the boundary layer. Is the effect being on concentration profiles are shown in figure 9. The concentration profiles of the fluid decreases with increase of Lewis number Le, where the velocity and temperature profiles are not significances with the increase of Lewis number Le hence it is not shown.

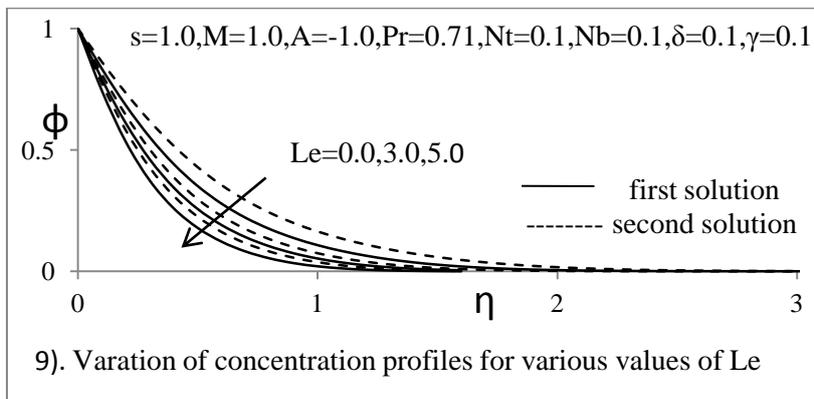
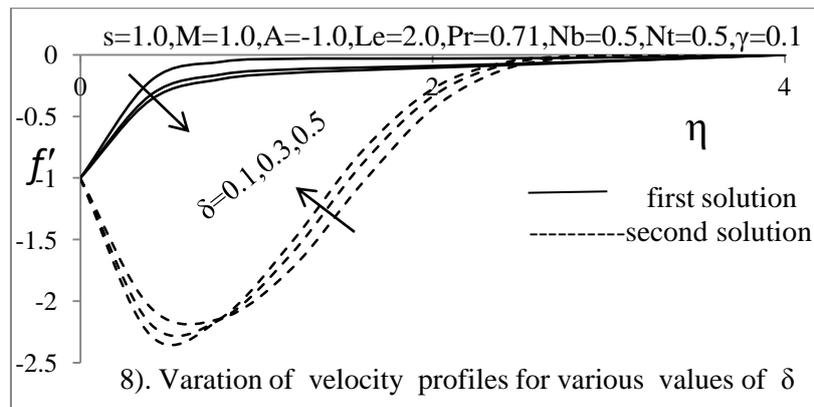
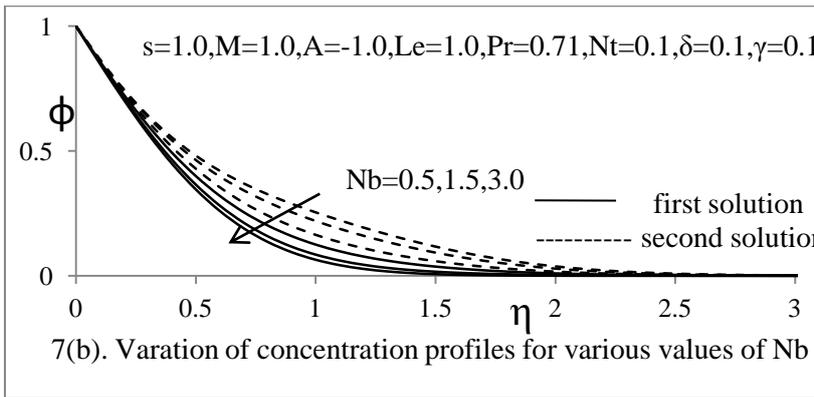
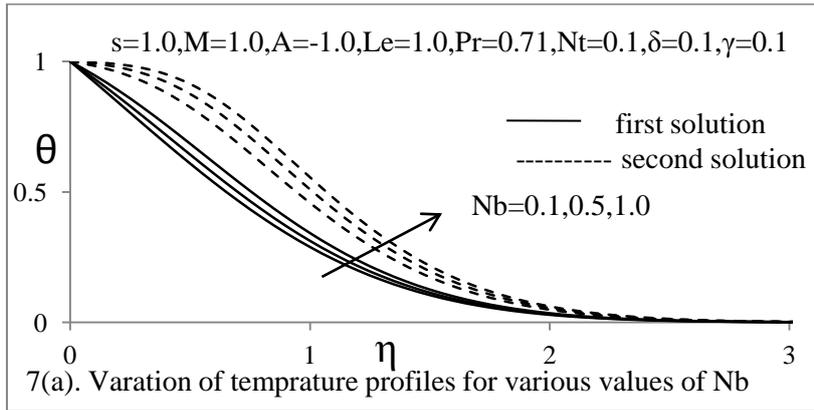












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