

## Mechanical Strength Modeling and Optimization of Lateritic Solid Block with 4% Mound Soil Inclusion

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**ABSTRACT:** The work is an investigation for the model development and optimization of the compressive strength of solid sandcrete block with mound soil inclusion. The study applies the Scheffe's optimization approach to obtain a mathematical model of the form  $f(x_{i1}, x_{i2}, x_{i3}, x_{i4})$ , where  $x_i$  are proportions of the concrete components, viz: cement, mound soil, laterite, and water. Scheffe's experimental design techniques are followed to mould various solid block samples measuring 450mm x 225mm x 150mm and tested for 28 days strength. The task involved experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. Finally a software is prepared to handle the design computation process to take the desired property of the mix, and generate the optimal mix ratios.

**Keywords:** Sandcrete, Pseudo-component, Simplex-lattice, optimization, Experimental matrix

### I. INTRODUCTION

The construction of structures is a regular operation which heavily involves sandcrete blocks for load bearing or non-load bearing walls. The cost/stability of this material has been a major issue in the world of construction where cost is a major index. This means that the locality and the usability of the available materials directly impact on the achievable development of any area as well as the attainable level of technology in the area. As it is, concrete is the main material of construction, and the ease or cost of its production accounts for the level of success in the area of environmental upgrading involving the construction of new roads, buildings, dams, water structures and the renovation of such structures. To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Where the scarcity of one component prevails exceedingly, the cost of the concrete production increases geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the scarce component when it is possible to do so without losing the quality of the concrete.

#### 1.1 Optimization Concept

The target of planning is the maximization of the desired outcome of the venture. In order to maximize gains or outputs it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is referred to as optimization [1]. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables. The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. The constraint at any time is the amount of money available to the entrepreneur at the time of investment. Hence or otherwise, an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements [2]. The function is called the objective function and the specified requirements are known as the constraints of the problem.

Everybody can make concrete but not everybody can make structural concrete. Structural concrete are made with specified materials for specified strength. Concrete is heterogeneous as it comprises sub-materials. Concrete is made up of fine aggregates, coarse aggregates, cement, water, and sometimes admixtures. [3] report that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities. For instance, Portland cement has been partially replaced with ground granulated blast furnace slag (GGBS), a by-product of the steel industry that has valuable cementitious properties [4].

### 1.2 Concrete Mix Optimization

The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base. Several methods have been applied. Examples are by [10], [17]. [11] proposed an approach which adopts the equilibrium mineral assemblage concept of geochemical thermodynamics as a basis for establishing mix proportions. [8] reports that optimization of mix designs require detailed knowledge of concrete properties. Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage. Apart from the larger deformations, the acceleration of dehydration and strength gain will cause cracking at early ages.

### 1.3 Modeling

Modeling involves setting up mathematical formulations of physical or other systems. Many factors of different effects occur in nature in the world simultaneously dependently or independently. When they interplay they could inter-affect one another differently at equal, direct, combined or partially combined rates variationally, to generate varied natural constants in the form of coefficients and/or exponents. The challenging problem is to understand and assess these distinctive constants by which the interplaying factors underscore some unique natural phenomenon towards which their natures tend, in a single, double or multi phase system.

For such assessment a model could be constructed for a proper observation of response from the interaction of the factors through controlled experimentation followed by schematic design where such simplex lattice approach of the type of [9] optimization theory could be employed. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of mathematics [10].

## II. LITERATURE REVIEW

In the past ardent researchers have done works in the behavior of concrete under the influence of its components. With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio [11]. Of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete [2]. Optimization of mix designs require detailed knowledge of concrete properties. The task of concrete mix optimization implies selecting the most suitable concrete aggregates from a data base. Mathematical models have been used to optimize some mechanical properties of concrete made from Rice Husk Ash (RHA), - a pozzolanic waste [9], and [12]. Macrofaunal activities in soil are known to affect the nutrient and organic matter dynamics and structure of the soil. Such changes in soil properties have profound influences on the productivity of the ecosystem. Termites are the dominant macrofaunal group found in the tropical soils. Termites process considerable quantities of soil materials in their mound building activities. Such activities might have potential effects on carbon sequestration, nutrient cycling, and soil texture [13].

The result of a study on some characteristics of laterite- cement mix containing termite mound soil (50% by weight of laterite) as replacement of laterite are presented by [14]. The study showed that laterite-mound soil mix stabilized with 6% cement could serve as a base course for roads for agricultural trafficking in rural areas where mound soils are abundant. The inclusion of mound soil in mortar matrix resulted in a compressive strength value of up to 40.08N/mm<sup>2</sup>, and the addition of 5% of mound soil to a concrete mix of 1:2:4:0.56 (cement: sand: coarse aggregate: water) resulted in an increase of up to 20.35% in compressive strength [15]. Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other [16]. When studying the properties of a q-component mixture, which are dependent on the component ratio only the factor space is a regular (q-1)-simplex [17]. Simplex lattice designs are saturated, that is, the proportions used for each factor have  $m + 1$  equally spaced levels from 0 to 1 ( $x_i = 0, 1/m, 2/m, \dots, 1$ ), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used [17].

Sandcrete blocks are masonry units used in all types of masonry constructions such as interior and exterior load bearing walls, fire walls party walls, curtain walls, panel walls, partition, backings for other masonry, facing materials, fire proofing over structured steel members, piers, pilasters columns, retaining walls, chimneys, fireplaces, concrete floors, patio paving units, curbs and fences [18]. The block is defined by ASIM as hollow block when the cavity area exceeds 25% of the gross cross-sectional area, otherwise it belongs to the solid category [18]. [19] stated that methods of compaction during moulding has a marked effect on the strength of sandcrete blocks. Hence, it was found that blocks from factories achieving compaction by using wooden rammers had higher strength than those compacted by mechanical vibration, except when the vibration is carried out with additional surcharge.

**III. BACKGROUND THEORY**

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying mix component variables to fix equal mixture properties. The optimization that follows selects the optimal ratio from the component ratios list that is automatically generated. This theory is the adaptation to this work of formulation of response function for compressive strength of sandcrete block.

**3.1 Simplex Lattice**

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture [16], and they are quidistant from each other. Mathematically, a simplex lattice is a space of constituent variables of  $X_1, X_2, X_3, \dots$ , and  $X_i$  which obey these laws:

$$\left. \begin{array}{l} X \neq \text{negative} \\ 0 \leq x_i \leq 1 \\ \sum_{i=1} x_i = 1 \end{array} \right\} \dots\dots\dots (1)$$

That is, a lattice is an abstract space.

To achieve the desired strength of concrete, one of the essential factors lies on the adequate proportioning of ingredients needed to make the concrete. [9] developed a model whereby if the compressive strength desired is specified, possible combinations of needed ingredients to achieve the compressive strength can easily be predicted by the aid of computer, and if proportions are specified the compressive strength can easily be predicted.

**3.2 Simplex Lattice Method**

In designing experiment to attack mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial [17]. When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a spatial representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained.

As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has  $C_{q+n}^n$  coefficients. If a mixture has a total of q components and  $x_i$  be the proportion of the  $i^{th}$  component in the mixture such that,

$$x_i \geq 0 \ (i=1,2, \dots,q), \dots\dots\dots (2)$$

then the sum of the component proportion is a whole unity i.e.

$$X_1 + x_2 + x_3 + x_4 = 1 \ \text{or} \ \sum x_i - 1 = 0 \ \dots \dots \dots (3)$$

where  $i = 1, 2, \dots, q \dots$  Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if  $q = 2$ , we have 2 points of connectivity. This gives a straight line simplex lattice. If  $q=3$ , we have a triangular simplex lattice and for  $q = 4$ , it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

- i. The factor space has uniformly distributed points,



**3.2.2 Construction of Experimental/Design Matrix**

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (Table 1) zero.

Table 1 Design matrix for (4,2) Lattice

N	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	Y <sub>exp</sub>
1	1	0	0	0	Y <sub>1</sub>
2	0	1	0	0	Y <sub>2</sub>
3	0	0	1	0	Y <sub>3</sub>
4	0	0	0	1	Y <sub>4</sub>
5	1/2	1/2	0	0	Y <sub>12</sub>
6	1/2	0	1/2	0	Y <sub>13</sub>
7	1/2	0	0	1/2	Y <sub>14</sub>
8	0	1/2	1/2	0	Y <sub>23</sub>
9	0	1/2	0	1/2	Y <sub>24</sub>
10	0	0	1/2	1/2	Y <sub>34</sub>

Hence if we substitute in Eqn. (11), the coordinates of the first point (X<sub>1</sub>=1, X<sub>2</sub>=0, X<sub>3</sub>=0, and X<sub>4</sub>=0, Fig (3.1), we get that Y<sub>1</sub>= β<sub>1</sub>.

And doing so in succession for the other three points in the tetrahedron, we obtain

$$Y_2 = \beta_2, Y_3 = \beta_3, Y_4 = \beta_4 \tag{13}$$

The substitution of the coordinates of the fifth point yields

$$Y_{12} = \frac{1}{2} X_1 + \frac{1}{2} X_2 + \frac{1}{2} X_1 \cdot \frac{1}{2} X_2$$

$$= \frac{1}{2} \beta_1 + \frac{1}{2} \beta_2 + \frac{1}{4} \beta_{12}$$

But as β<sub>1</sub> = Y<sub>1</sub> then

$$Y_{12} = \frac{1}{2} \beta_1 - \frac{1}{2} \beta_2 - \frac{1}{4} \beta_{12}$$

Thus

$$\beta_{12} = 4 Y_{12} - 2Y_1 - 2Y_2 \tag{14}$$

And similarly,

$$\beta_{13} = 4 Y_{13} - 2Y_1 - 2Y_2$$

$$\beta_{23} = 4 Y_{23} - 2Y_2 - 2Y_3$$

etc.

Or generalizing,

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4 Y_{ij} - 2Y_i - 2Y_j \tag{15}$$

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the four points defining the coefficients β<sub>ij</sub> lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X<sub>i</sub> alone and the property of the mixture is denoted by Y<sub>i</sub>.

**3.2.3 Actual and Pseudo Components**

The requirements of the simplex that

$$\sum_{X=1} X_i = 1$$

makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios say X<sub>1</sub><sup>(i)</sup>, X<sub>2</sub><sup>(i)</sup>, and X<sub>3</sub><sup>(i)</sup> and X<sub>4</sub><sup>(i)</sup> for the i<sup>th</sup> experimental points are called pseudo components. Since X<sub>1</sub>, X<sub>2</sub> and X<sub>3</sub> are subject to ∑ X<sub>i</sub> = 1, the transformation of cement:mound soil:laterite:water at say 0.30 water/cement ratio cannot easily be computed because X<sub>1</sub>, X<sub>2</sub>, X<sub>3</sub> and X<sub>4</sub> are in pseudo expressions X<sub>1</sub><sup>(i)</sup>, X<sub>2</sub><sup>(i)</sup>, and X<sub>3</sub><sup>(i)</sup>. For the i<sup>th</sup> experimental point, the transformation computations are to be done.

The arbitrary vertices chosen on the triangle are A<sub>1</sub>(1:5.6:2.4:0.3), A<sub>2</sub>(1:5:3:0.32) A<sub>3</sub>(1:4.8:3.2:0.33), and A<sub>4</sub>(1:5.8:2.2:0.28), based on experience and earlier research reports.

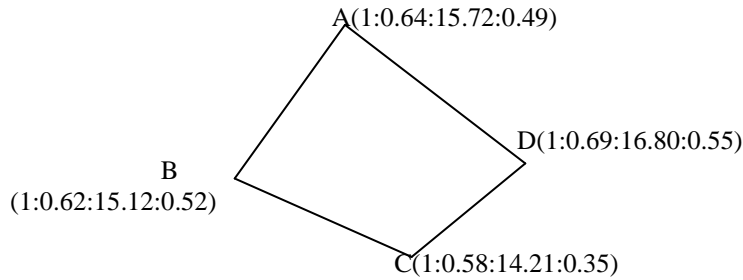


Fig. 1 Tetrahedral Simplex

**3.2.4 Transformation Matrix**

If Z denotes the actual matrix of the  $i^{th}$  experimental points, observing from Table 2 (points 1 to 3),

$$BZ = X = I \quad \text{where B is the transformed matrix.} \quad (16)$$

Therefore,  $B = I \cdot Z^{-1}$

$$\text{Or } B = Z^{-1} \quad (17)$$

For instance, for the chosen ratios  $A_1, A_2, A_3$  and  $A_4$  (fig. 1),

$$Z = \begin{pmatrix} 1 & 0.64 & 15.72 & 0.49 \\ 1 & 0.62 & 15.12 & 0.52 \\ 1 & 0.58 & 14.21 & 0.35 \\ 1 & 0.69 & 16.80 & 0.55 \end{pmatrix} \quad (18)$$

From Eqn 17,

$$B = Z^{-1}$$

$$Z^{-1} = \begin{pmatrix} -3.01 & 8.20 & 2.42 & -6.61 \\ -224.80 & 52.74 & 59.53 & 112.53 \\ 9.31 & -2.96 & 2.35 & -4.00 \\ 3.05 & 9.31 & -7.31 & -5.05 \end{pmatrix}$$

Hence,

$$\begin{aligned} B Z^{-1} &= Z \cdot Z^{-1} \\ &= \begin{pmatrix} 1 & 0.64 & 15.72 & 0.49 \\ 1 & 0.62 & 15.12 & 0.52 \\ 1 & 0.58 & 14.21 & 0.35 \\ 1 & 0.69 & 16.80 & 0.55 \end{pmatrix} \begin{pmatrix} -3.01 & 8.20 & 2.42 & -6.61 \\ -224.80 & 52.74 & 59.53 & 112.53 \\ 9.31 & -2.96 & 2.35 & -4.00 \\ 3.05 & 9.31 & -7.31 & -5.05 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Thus, for actual component Z, the pseudo component X is given by

$$X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} = B \begin{pmatrix} -3.01 & 8.20 & 2.42 & -6.61 \\ -224.80 & 52.74 & 59.53 & 112.53 \\ 9.31 & -2.96 & 2.35 & -4.00 \\ 3.05 & 9.31 & -7.31 & -5.05 \end{pmatrix} Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{pmatrix}$$

which gives the  $X_i(i=1,2,3)$  values in Table 2.

The inverse transformation from pseudo component to actual component is expressed as

$$AX = Z \tag{19}$$

where A = inverse matrix

$$A = Z^{-1}X^{-1}$$

From Eqn 16,  $X = BZ$ , therefore,

$$A = Z \cdot (BZ)^{-1}$$

$$A = Z \cdot Z^{-1}B^{-1}$$

$$A = IB^{-1}$$

$$= B^{-1} \tag{20}$$

This implies that for any pseudo component X, the actual component is given by

$$Z = \begin{pmatrix} 1 & 0.64 & 15.72 & 0.49 \\ 1 & 0.62 & 15.12 & 0.52 \\ 1 & 0.58 & 14.21 & 0.35 \\ 1 & 0.69 & 16.80 & 0.55 \end{pmatrix}$$

$$Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{pmatrix} = B^{-1} \begin{pmatrix} 0.64 & 15.72 & 0.49 \\ 0.62 & 15.12 & 0.52 \\ 0.58 & 14.21 & 0.35 \\ 0.69 & 16.80 & 0.55 \end{pmatrix} X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} \tag{21}$$

Eqn 21 is used to determine the actual components from points 5 to 10 , and the control values from points 11 to 13 (Table 2).

Table 2 Values for Experiment

N	$X_1$	$X_2$	$X_3$	$X_4$	RESPONSE	$Z_1$	$Z_2$	$Z_3$	$Z_4$
1	1	0	0	0	$Y_1$	1.00	0.64	15.72	0.49
2	0	1	0	0	$Y_2$	1.00	0.62	15.12	0.52
3	0	0	1	0	$Y_3$	1.00	0.58	14.21	0.35
4	0	0	0	1	$Y_4$	1.00	0.69	16.80	0.55
5	1/2	1/2	0	0	$Y_{12}$	1.00	0.63	15.42	0.51
6	1/2	0	1/2	0	$Y_{13}$	1.00	0.32	14.97	0.42
7	1/2	0	0	1/2	$Y_{14}$	1.00	0.67	16.26	0.52
8	0	1/2	1/2	0	$Y_{23}$	1.00	0.31	14.67	0.44
9	0	1/2	0	1/2	$Y_{24}$	1.00	0.66	15.96	0.54
10	0	0	1/2	1/2	$Y_{34}$	1.00	0.35	15.51	0.45
Control points									
11	0.25	0.25	0.25	0.25	$Y_{1234}$	1.00	0.49	15.46	0.48
12	0.5	0.25	0.25	0	$Y_{1123}$	1.00	0.48	15.19	0.46
13	0.25	0.5	0	0.25	$Y_{1224}$	1.00	0.64	15.69	0.52

### 3.2.5 Use of Values in Experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients: cement, mound soil, laterite and water for casting the samples. The values obtained are presented in Tables in section 5.

### 3.3 Adequacy of Tests

This is carried out by testing the fit of a second degree polynomial [17]. After the coefficients of the regression equation has been derived, the statistical analysis is considered necessary, that is, the equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called control points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response,  $S_Y^2$ , is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a quaternary mixture, the following points are assumed:

$X_i$  can be observed without errors [17].

The replication variance,  $S_Y^2$ , is similar at all design points, and

Response values are the average of  $n_i$  and  $n_{ij}$  replicate observations at appropriate points of the simplex

Then the variance  $S_{\hat{Y}_i}$  and  $S_{\hat{Y}_{ij}}$  will be

$$(S_{\hat{Y}}^2)_i = S_Y^2/n_i \tag{22}$$

$$S_{\hat{Y}}^2)_{ij} = S_Y^2/n_{ij} \tag{23}$$

In the reduced polynomial,

$$\hat{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \tag{24}$$

If we replace coefficients by their expressions in terms of responses,

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$$

$$\begin{aligned} \hat{Y} &= Y_1 X_1 + Y_2 X_2 + Y_3 X_3 + Y_4 X_4 + (4Y_{12} - 2Y_1 - 2Y_2) X_1 X_2 + (4Y_{13} - 2Y_1 - 2Y_3) X_1 X_3 + (4Y_{14} - 2Y_1 - 2Y_4) X_1 X_4 \\ &+ (4Y_{23} - 2Y_2 - 2Y_3) X_2 X_3 + (4Y_{24} - 2Y_2 - 2Y_4) X_2 X_4 + (4Y_{34} - 2Y_3 - 2Y_4) X_3 X_4 \\ &= Y_1(X_1 - 2X_1 X_2 - 2X_1 X_3 - 2X_1 X_4) + Y_2(X_2 - 2X_1 X_2 - 2X_2 X_3 - 2X_2 X_4) + Y_3(X_3 - 2X_1 X_3 + 2X_2 X_3 + 2X_3 X_4) \\ &+ Y_4(X_4 - 2X_1 X_4 + 2X_2 X_4 + 2X_3 X_4) + 4Y_{12} X_1 X_2 + 4Y_{13} X_1 X_3 + 4Y_{14} X_1 X_4 + 4Y_{23} X_2 X_3 + 4Y_{24} X_2 X_4 + 4Y_{34} X_3 X_4 \end{aligned} \tag{25}$$

Using the condition  $X_1 + X_2 + X_3 + X_4 = 1$ , we transform the coefficients at  $Y_i$

$$\begin{aligned} X_1 - 2X_1 X_2 - 2X_1 X_3 - 2X_1 X_4 &= X_1 - 2X_1(X_2 + X_3 + X_4) \\ &= X_1 - 2X_1(1 - X_1) = X_1(2X_1 - 1) \text{ and so on.} \end{aligned} \tag{26}$$

Thus

$$\hat{Y} = X_1(2X_1 - 1)Y_1 + X_2(2X_2 - 1)Y_2 + X_3(2X_3 - 1)Y_3 + X_4(2X_4 - 1)Y_4 + 4Y_{12} X_1 X_2 + 4Y_{13} X_1 X_3 + 4Y_{14} X_1 X_4 + 4Y_{23} X_2 X_3 + 4Y_{24} X_2 X_4 + 4Y_{34} X_3 X_4 \tag{27}$$

Introducing the designation

$$a_i = X_i(2X_i - 1) \text{ and } a_{ij} = 4X_i X_j \tag{27a}$$

and using Eqns (3.22) and (3.230) give the expression for the variance  $S_Y^2$ .

$$S_{\hat{Y}}^2 = S_Y^2 \left( \sum_{1 \leq i \leq q} a_{ii}/n_i + \sum_{1 \leq i < j \leq q} a_{ij}/n_{ij} \right) \tag{28}$$



If the number of replicate observations at all the points of the design are equal, i.e.  $n_i=n_{ij}= n$ , then all the relations for  $S_{\hat{Y}}^2$  will take the form

$$S_{\hat{Y}}^2 = S_Y^2 \xi / n \tag{29}$$

where, for the second degree polynomial,

$$\xi = \sum a_i^2 + \sum_{1 \leq i < j \leq q} a_{ij}^2 \tag{30}$$

As in Eqn (30),  $\xi$  is only dependent on the mixture composition. Given the replication variance and the number of parallel observations  $n$ , the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of  $\xi$  taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$$t = \Delta_Y / (S_{\hat{Y}}^2 + S_Y^2) = \Delta_Y n^{1/2} / (S_Y (1 + \xi))^{1/2} \tag{31}$$

where  $\Delta_Y = Y_{exp} - Y_{theory}$  (32)

and  $n$  = number of parallel observations at every point.

The t-statistic has the student distribution, and it is compared with the tabulated value of  $t_{\alpha/L}(V)$  at a level of significance  $\alpha$ , where  $L$  = the number of control points, and  $V$  = the number for the degrees of freedom for the replication variance.

The null hypothesis is that the equation is adequate is accepted if  $t_{cal} < t_{Table}$  for all the control points.

The confidence interval for the response value is

$$\hat{Y} - \Delta \leq Y \leq \hat{Y} + \Delta \tag{3.33}$$

$$\Delta = t_{\alpha/L,k} S_{\hat{Y}} \tag{34}$$

where  $k$  is the number of polynomial coefficients determined.

Using Eqn (29) in Eqn (34)

$$\Delta = t_{\alpha/L,k} S_Y (\xi/n)^{1/2} \tag{35}$$

#### IV. METHODOLOGY

To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, sandcrete or concrete are none of these, nevertheless they are popular construction materials [20].The necessary materials required in the manufacture of the sandcrete in the study are cement, mound soil, laterite and water.

##### 4.1 Materials

The Laterite materials were collected at Emene area of Enugu State and conformed to BS 882 and belongs to zone 1 of the ASHTO classification. The mound soil were collected from bushes in the same area. The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag.

**4.2 Preparation of Samples**

The sourced materials for the experiment were transferred to the laboratory. The pseudo components of the mixes were designed following the background theory from where the actual variables proportions were developed. The component materials were mixed at ambient temperature according to the specified proportions of the actual components generated in Table 2. In all, two solid blocks of 450mm x225 x150mm for each of ten experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

**4.3 Strength Test**

After 28 day of curing, the blocks were crushed to determine the sandcrete block strength, using the compressive testing machine to the requirements of BS 1881:Part 115 of 1986.

**V. RESULT AND ANALYSIS**

**5.1 Determination of Replication Error And Variance of Response**

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the ten design points. Below is the table which which contain the results of two repetitions each of the 10 design points plus three Control Points of the (4,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$\hat{Y} = \sum(Y_r)/r \tag{36}$$

where  $\hat{Y}$  is the mean of the response values and

r =1,2.

$$S_Y^2 = \sum[(Y_i - \hat{Y}_i)^2]/(n-1) \tag{37}$$

where n = 13.

**Table 3 Result of the Replication Variance of the Compressive Strength Response for 450x225x150 mm Block**

Experiment No (n)	Repetition	Response Y (N/mm <sup>2</sup> )	Response Symbol	$\sum Y_r$	$\hat{Y}_r$	$\sum(Y_r - \hat{Y}_r)^2$	$S_i^2$
1	1A	2.94	Y <sub>1</sub>	5.00	2.50	0.30	0.03
	1B	2.06					
2	2A	2.41	Y <sub>2</sub>	4.39	2.20	0.09	0.01
	2B	1.98					
3	3A	2.24	Y <sub>3</sub>	4.20	2.10	0.00	0.00
	3B	1.96					
4	4A	2.30	Y <sub>4</sub>	4.38	2.19	0.02	0.00
	4B	2.08					
5	5A	2.43	Y <sub>12</sub>	4.42	2.21	0.01	0.00
	5B	1.99					
6	6A	2.31	Y <sub>13</sub>	4.61	2.31	0.00	0.00
	6B	2.30					
7	7A	2.69	Y <sub>14</sub>	4.64	2.31	0.27	0.02
	7B	1.95					
8	8A	2.54	Y <sub>23</sub>	5.21	2.61	0.01	0.00
	8B	2.67					

9	9A 9B	1.97 2.36	$Y_{24}$	4.33	2.17	0.08	0.01
10	10A 10B	2.23 2.07	$Y_{34}$	4.30	2.15	0.01	0.00
Control Points							
11	11A 11B	2.41 2.27	$C_1$	4.68	3.24	0.01	0.00
12	11A 11B	2.17 2.21	$C_2$	4.38	2.19	0.00	0.00
13	13A 13B	2.61 2.12	$C_3$	4.73	2.37	0.12	0.01

$\Sigma 0.10$

Replication Variance

$$S_{Yc}^2 = (\Sigma S_i^2)/(n-1) = 0.10$$

Replication Error

$$S_{Yc} = (S_{Yc}^2)^{1/2} = 0.10^{1/2} = 0.31$$

**5.1.1 Determination of Regression Equation for the Compressive Strength**

From Eqn 15 and Table 3 the coefficients of the reduced second degree polynomial is determined as follows:

$$\begin{aligned} \beta_1 &= 2.50 \\ \beta_2 &= 2.20 \\ \beta_3 &= 2.10 \\ \beta_4 &= 2.19 \\ \beta_{12} &= 4(2.21) - 2(2.50) - 2(2.20) = -0.55 \\ \beta_{13} &= 4(2.31) - 2(2.50) - 2(2.10) = 0.02 \\ \beta_{14} &= 4(2.31) - 2(2.50) - 2(2.19) = -0.10 \\ \beta_{23} &= 4(2.61) - 2(2.20) - 2(2.20) = 1.83 \\ \beta_{24} &= 4(2.17) - 2(2.20) - 2(2.19) = -0.11 \\ \beta_{34} &= 4(2.15) - 2(2.10) - 2(2.19) = 0.02 \end{aligned}$$

Thus, from Eqn (11),

$$\hat{Y} = 2.50X_1 + 2.20X_2 + 2.10X_3 + 2.19X_4 - 0.55X_1X_2 + 0.02X_1X_3 - 0.10X_1X_4 + 1.83X_2X_3 - 0.11X_2X_4 + 0.02X_3X_4 \quad (38)$$

Eqn (38) is the mathematical model of the compressive strength of the solid sandcrete block based on the 28-day strength.

**5.1.2 Test of Adequacy of the Compressive strength Model**

Eqn 38, the equation model, will be tested for adequacy against the controlled experimental results.

We recall our statistical hypothesis as follows:

1. Null Hypothesis ( $H_0$ ): There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2. Alternative Hypothesis ( $H_1$ ): There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

**5.1.3 t-Test for the Compressive strength Model**

If we substitute for  $X_i$  in Eqn (38) from Table 3, the theoretical predictions of the response ( $\hat{Y}$ ) can be obtained. These values can be compared with the experimental results (Table 3). For the t-test (Table 4),  $a$ ,  $\xi$ ,  $t$  and  $\Delta_y$  are evaluated using Eqns 27, 30, 31 and 32 respectively.

Table 4 t-Test for the Test Control Points

N	CN	I	J	$a_i$	$a_{ij}$	$a_i^2$	$a_{ij}^2$	$\xi$	$\hat{Y}$	$\hat{Y}_a$	$\Delta_y$	t
1	C <sub>1</sub>	1	2	-0.125	0.250	0.016	0.063	0.469	2.34	2.32	0.02	0.09
		1	3	-0.125	0.250	0.016	0.063					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.250	0.016	0.063					
		2	4	-0.125	0.250	0.016	0.063					
		3	4	-0.125	0.250	0.016	0.063					
					0.094	0.375						
2	C <sub>2</sub>	1	2	0.000	0.500	0.000	0.250	0.563	2.19	2.37	0.18	-0.67
		1	3	0.000	0.500	0.000	0.250					
		1	4	0.000	0.000	0.000	0.000					
		2	3	0.000	0.250	0.000	0.063					
		2	4	0.000	0.000	0.000	0.000					
		3	4	0.000	0.000	0.000	0.000					
					0.000	0.563						
3	C <sub>3</sub>	1	2	-0.125	0.500	0.016	0.250	0.656	2.37	2.18	0.18.	0.65
		1	3	-0.125	0.000	0.016	0.000					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.000	0.016	0.000					
		2	4	-0.125	0.500	0.016	0.250					
		3	4	-0.125	0.000	0.016	0.000					
							0.094					

Significance level  $\alpha = 0.05$ ,  
 i.e.  $t_{\alpha/L}(V_c) = t_{0.05/3}(13)$ , where L=number of control point.

From Appendix A, the tabulated value of  $t_{0.05/3}(13)$  is found to be 3.01 which is greater than any of the calculated t-values in Table 4. Hence we can accept the Null Hypothesis.

From Eqn (35), with  $k=3$  and  $t_{\alpha/k,v} = t_{0.05/k}(13) = 3.01$ ,  
 $\Delta = 1.13$  for  $C_{1234}$ ,  $1.16$  for  $C_{1124=0.26}$ , and  $1.208$  for  $C_{1224}$ ,  
 which satisfies the confidence interval equation of  
 Eqn (33) when viewed against the response values in Table 4.

## 5.2 Computer Program

The computer program is developed for the model (Appendix 1). In the program any desired Compressive Strength can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the following tolerance:

Compressive Strength -  $0.00005 \text{ N/mm}^2$ ,

Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

### 5.2.1 Choosing a Combination

It can be observed that the strength of 2.435 N/sq mm yielded 4 combinations. To accept any particular proportions depends on the factors such as workability, cost and honeycombing of the resultant lateritic concrete.

## IV. Conclusion and Recommendation

### 6.1 Conclusion

Simplex design was applied successfully to prove that the modulus of lateritic concrete is a function of the proportion of the ingredients (cement, mound soil, laterite and water), but not the quantities of the materials [16]. The maximum compressive strength obtainable with the compressive strength model is 2.61 N/sq mm. See the computer run outs which show all the possible lateritic concrete mix options for the desired modulus property, and the choice of any of the mixes is the user's. One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using sand as aggregate. This is due to the predominantly high silt content of laterite. It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if workability and other demands of the resulting lateritic concrete have to be satisfied. This is an important area for further research work. The project work is a great advancement in the search for the applicability of laterite and mound soil in concrete mortar production in regions where sand is extremely scarce with the ubiquity of laterite. The paper provides an optimized design perspective of the use of admixtures instead of undersigned percentage addition which is currently prevalent in the concrete industry of the world.

### 6.2 Recommendations

From the foregoing study, the following could be recommended:

- i) The model can be used for the optimization of the strength of concrete made from cement, mound soil, laterite and water.
- ii) Laterite aggregates cannot adequately substitute sharp sand aggregates for heavy construction.
- iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete.
- iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

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## APPENDIX 1

'QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF SANDCRETE MIXES  
'USING THE SCHEFFE'S MODEL FOR CONCRETE COMPRESSIVE STRENGTH

```
CLS
C1$ = "(ONUAMAH.HP) RESULT OUTPUT ": C2$ = "A COMPUTER PROGRAM "
C3$ = "ON THE OPTIMIZATION OF A 4-COMPONENT SANDCRETE MIX"
PRINT C2$ + C1$ + C3$
PRINT
'VARIABLES USED ARE
'X1, X2, X3,X4, Z1, Z2, Z3,Z4, Z$,YT, YTMAX, DS

INPUT "ARE MIX RATIOS KNOWN AND THE ATTAINABLE STRENGTH NEEDED?,CHOOSE Y=
YES OR N=NO"; OPTION$
IF OPTION$ = "Y" THEN
  INPUT "X1="; X1
  INPUT "X2="; X2
  INPUT "X3="; X3
  INPUT "X4="; X4
  X = X1 + X2 + X3 + X4
  X1 = X1 / X: X2 = X2 / X: X3 = X3 / X: X4 = X4 / X
  GOTO 25
ELSE
  GOTO 20
END IF

'INITIALISE I AND YTMAX

20 I = 0: YTMAX = 0

FOR MX1 = 0 TO 1 STEP .01
  FOR MX2 = 0 TO 1 - MX1 STEP .01
    FOR MX3 = 0 TO 1 - MX1 - MX2 STEP .01
      MX4 = 1 - MX1 - MX2 - MX3
      YTM = 2.5 * MX1 + 2.2 * MX2 + 2.1 * MX3 + 2.19 * MX4 - .55 * MX1 * MX2 + .02 * MX1 * MX3 -
.1 * MX1 * MX4 + 1.83 * MX2 * MX3 - .11 * MX2 * MX4 + .02 * MX3 * MX4
      IF YTM >= YTMAX THEN YTMAX = YTM
    NEXT MX3
  NEXT MX2
NEXT MX1
INPUT "ENTER DESIRED STRENGTH, DS = "; DS

'PRINT OUTPUT HEADING
PRINT
25 PRINT TAB(1); "No"; TAB(10); "X1"; TAB(18); "X2"; TAB(26); "X3"; TAB(34); "X4"; TAB(40);
"YTHEORY"; TAB(50); "Z1"; TAB(58); "Z2"; TAB(66); "Z3"; TAB(74); "Z4"
IF OPTION$ = "Y" THEN 30
PRINT
```

```

'COMPUTE THEORETICAL STRENGTH, YT
FOR X1 = 0 TO 1 STEP .01
  FOR X2 = 0 TO 1 - X1 STEP .01
    FOR X3 = 0 TO 1 - X1 - X2 STEP .01
      X4 = 1 - X1 - X2 - X3
30    YT = 2.50 * X1 + 2.20 * X2 + 2.10 * X3 + 2.19 * X4 - .55 * X1 * X2 + .02 * X1 * X3 - .10 * X1 * X4
      + 1.83 * X2 * X3 + .11 * X2 * X4 - .02 * X3 * X4

      IF OPTION$ = "Y" THEN 40
      IF ABS(YT - DS) <= .00005 THEN
        'PRINT MIX PROPORTION RESULTS
        Z1 = X1 + X2 + X3 + X4; Z2 = .64 * X1 + .62 * X2 + .58 * X3 + .69 * X4; Z3 = 15.74 * X1 + 15.12 *
X2 + 14.21 * X3 + 16.8 * X4; Z4 = .49 * X1 + .52 * X2 + .35 * X3 + .55 * X4
40    I = I + 1
        PRINT TAB(1); I; USING "##.###"; TAB(7); X1; TAB(15); X2; TAB(23); X3; TAB(32); X4;
TAB(40); YT; TAB(48); Z1; TAB(56); Z2; TAB(64); Z3; TAB(72); Z4
        PRINT
        PRINT
        IF OPTION$ = "Y" THEN 540
        IF (X1 = 1) THEN 550
        ELSE
          IF (X1 < 1) THEN GOTO 150
        END IF

150    NEXT X3
        NEXT X2
        NEXT X1
        IF I > 0 THEN 550
        PRINT
        PRINT "SORRY, THE DESIRED STRENGTH IS OUT OF RANGE OF MODEL"
        GOTO 600
540 PRINT TAB(5); "THE ATTAINABLE STRENGTH IS "; YT; "N/mm2"
        GOTO 600
550 PRINT TAB(5); "THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS "; YTMAX; "N / Sq
mm "
600 END
A COMPUTER PROGRAM(ONUAMAH.HP)RESULT OUTPUT ON THE OPTIMIZATION OF A 4-
COMPONE
NT SANDCRETE MIX

ENTER DESIRED STRENGTH, DS =? 2.435

No  X1  X2  X3  X4  YTHEORY  Z1  Z2  Z3  Z4

1  0.020  0.570  0.380  0.030  2.435  1.000  0.607  14.837  0.456

2  0.030  0.500  0.420  0.050  2.435  1.000  0.607  14.840  0.449

3  0.120  0.400  0.460  0.020  2.435  1.000  0.605  14.809  0.439

4  0.160  0.350  0.490  0.000  2.435  1.000  0.604  14.773  0.432

THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS 2.608853 N / Sq mm

Press any key to continue

```