Influence Of Thermal Radiation On Magnetohydrodynamic (Mhd) Boundary Layer Flow Of A Viscous Fluid Over An Exponentially Stretching Sheet

A.S. Idowu and S. Usman

ABSTRACT: Radiation on magnetohydrodynamic (MHD) boundary layer flow of a viscous fluid over an exponentially stretching sheet was considered together with it’s effects. The new technique of homotopy analysis method (nHAM) was used to obtain the convergent series expressions for velocity and temperature, where the governing system of partial differential equations has been transformed into ordinary differential equations. The interpretation to these expressions is shown physically through graphs. We observed that the effects of Prandtl and Magnetic number acts in opposite to each other on the temperature.

KEYWORDS: Boundary-layer; heat transfer; MHD; radiation; stretching sheet

I. INTRODUCTION

In many engineering processes today incompressible boundary layer flow due to an exponentially stretching sheet is useful in a good number of applications. Such applications includes the industrial manufacturing in the aerodynamic extrusion of plastic sheets, hot rolling, the boundary layer along a liquid film condensation process, cooling process of metal plate in a bath and in the polymer industries. It is seen that kinematics of stretching with both the simultaneous heating or cooling during this processes has great influence on the quality of the end products (Magyari and Keller, 1999). The work of Sakiadis, (1961) was able to look into the stretching flow problem where from there Crane, (1970) became the first to study the boundary layer flow caused by a stretching sheet which accelerates with a velocity varying linearly with the distance from a fixed point. Carragher and Crane, (1982) investigated the heat transfer area under this problem, with the conditions that the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. The steady boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution was also discussed by Magyari, et al. (1999). Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface was studied by Partha, et al. (2005). Sajid and Hayat, (2008) in recent time considered the radiation effects on the flow over an exponentially stretching sheet, where the problem was solved analytically using the homotopy analysis method. To deal with the problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field(Ganesan and Palani, 2004), MHD has important applications. Various processes in engineering areas occur at high temperature where radiation heat transfer becomes of great importance in the design of pertinent equipments(seddeek, 2002). Anuar, (2011) studied the MHD boundary layer flow on an exponentially stretching sheet taking the velocity gradient in the energy equation to be zero. The motivation to this present work is the variation of velocity gradient on the problem of MHD boundary layer flow over an exponentially stretching sheet in the presence of radiation where the velocity gradient in the energy equation not zero, which has not been studied.
II. PROBLEM FORMULATION

Consider the two-dimensional flow of an incompressible, steady viscous fluid bounded by a stretching sheet and conducted electrically which is placed in a fluid of uniform temperature \( T_\infty \), given in Fig 1. with magnetic field \( B(x) \) applied normal to the sheet and the induced magnetic field neglected, which is justified for MHD flow at small magnetic Raynold number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} = \frac{\sigma B^2 u}{\rho}
\]

(2)

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y}
\]

(3)

Fig 1: physical Model and Coordinate System

where \( u \) and \( v \) are the velocities in the \( x \)- and \( y \)-directions respectively, \( \rho \) is the fluid density, \( \nu \) the kinematic viscosity, \( \mu \) is the dynamic viscosity, \( \kappa \) the thermal conductivity, \( C_p \) the specific heat, \( T \) the fluid temperature in the boundary layer and \( q_r \) is the radiation heat flux. The boundary conditions are given by:

\[
\begin{align*}
    u &= U_w = U_e \exp\left(\frac{x}{l}\right), \quad v = 0, \\
    T &= T_w = T_\infty + T_e \exp\left(\frac{x}{2l}\right) \text{ at } y = 0, \\
    u &\to 0, T \to T_\infty \text{ as } y \to \infty
\end{align*}
\]

(4)

where \( U_w \) is the reference velocity, \( T_e \) and \( T_\infty \) are respectively the temperature at and far from the plate and \( L \) the reference length. Understanding fluid radiation is devoted to the derivation of reasonable simplifications (Aboeldahab and El Gendy 2002). One of these simplifications was made by Cogley et al. (1968) who assumed that the fluid does not absorb its own radiation, but it only absorbs radiation emitted by the boundaries. Hence, the problem can be simplified by using the Rosseland approximation (Rosseland 1936; Siegel and Howell 1992; Sparrow and Cess 1978) which simplifies the radiation heat flux as: 
\[
q_y = -\frac{4\sigma^* \partial T^4}{3\kappa^* \partial y}
\]

where \(\sigma^*\) and \(\kappa^*\) are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. This approximation is valid at points optically far from the boundary surface and it is good only for intensive absorption which is far for an optically thick boundary layer (Bataller 2008; Siegel and Howell 1992; Sparrow and Cess 1978). Assumed that the temperature differences within the flow such that the term \(T^4\) may be expressed as a linear function of temperature. Expanding \(T^4\) in a Taylor series about \(T_\infty\) and neglecting higher order terms gives:

\[
T^4 \approx 4T_\infty^2T - 3T_\infty^4
\]

Using Equations (5) and (6), Equation (3) reduces to:

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{16T_\infty^3 \sigma^*}{\rho C_p 3k^*} \frac{\partial^2 T}{\partial y^2}
\]

To get the similarity solutions, we assumed that the magnetic field \(B(x)\) is of the form:

\[
B = B_0 \exp(\frac{x}{2L})
\]

where \(B_0\) is the constant magnetic field.

Equation (1) is satisfied by introducing a stream function \(\psi\) such that:

\[
u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}
\]

Equation (2) and Equation (3) are transformed into the corresponding ordinary differential equations by the following transformation (Sajid and Hayat 2008):

\[
\begin{align*}
U_0 \exp(\frac{x}{2L}) f'(\eta), & \quad v = -(\frac{U_0}{2L})^{\frac{1}{2}} \exp(\frac{x}{2L}) (f(\eta) + \eta f'(\eta)) \\
T = T_\infty + T_e \exp(\frac{x}{2L}) \Theta(\eta), & \quad \eta = (\frac{U_0}{2L})^{\frac{1}{2}} \exp(\frac{x}{2L}) y
\end{align*}
\]

where \(\eta\) is the similarity variable, \(f(\eta)\) is the dimensionless stream function, \(\Theta(\eta)\) is the dimensionless temperature and prime denotes differentiation with respect to \(\eta\). The transformed ordinary differential equations are:

\[
f'''' + ff''' - 2f'^2 - Mf' = 0
\]
\[ 1 + \frac{4K}{3} \theta''(\eta) + P_i [\theta'(\eta) f(\eta) - \theta(\eta) f'(\eta) + E f''''(\eta)] = 0 \]  

(12)

Where \( M = \frac{2\sigma B^2 L}{\rho U_0} \), \( K = \frac{4\sigma T^3}{kk^3} \), \( P_i = \frac{\mu C_p}{k} \) and \( E = \frac{U^2}{T C_p} \) are respectively the Magnetic, Radiation, Prandtl and Eckert parameter respectively.

The transformed boundary conditions are:

\[
\begin{align*}
  f(0) &= 0, \quad f'(0) = 1, \quad \theta(0) = 1 \\
  f'(&\eta) \rightarrow 0, \quad \theta(&\eta) \rightarrow 0 \quad \text{as} \quad &\eta \rightarrow \infty
\end{align*}
\]  

(13)

nHAM SOLUTION

In order to solve Equations (11) - (13) using nHAM, assume that \( f''(0) = \alpha \) and \( \theta'(0) = \beta \). we construct system of differential equations, as follows:

(14)

\[
\begin{align*}
  f'(\eta) &= v \\
  v'(\eta) &= w \\
  w'(\eta) &= 2f'^{''} - ff'' + Mf' \\
\end{align*}
\]

with intial approximations

\[
\begin{align*}
  f_0(\eta) &= 0, \quad v_0(\eta) = 1, \quad w_0(\eta) = \alpha
\end{align*}
\]  

(15)

the auxiliary linear operators are

\[
\begin{align*}
  Lf(\eta) &= \frac{\partial f}{\partial \eta}, \quad Lv(\eta) &= \frac{\partial v}{\partial \eta}, \quad Lw(\eta) &= \frac{\partial w}{\partial \eta}, \quad \text{where} \quad L \quad \text{is an auxiliary linear operator.}
\end{align*}
\]

and

(16)

\[
\theta'(\eta) = C
\]

\[
C'(\eta) = -\frac{P_i}{4K} \left[ f\theta' - f'\theta + Ef'''' \right]
\]

using initial approximations

\[
\begin{align*}
  \theta_0(\eta) &= 1, \quad C_0(\eta) = \beta
\end{align*}
\]  

(17)
and the auxiliary linear operators are

\[
L\theta(\eta) = \frac{\partial \theta}{\partial \eta}, \quad LC(\eta) = \frac{\partial C}{\partial \eta}
\]  

(18)

we then have

\[
f_i(\eta) = h_1 \int_0^\eta [-v_i(\eta)] d\eta
\]

(19)

\[
v_i(\eta) = h_1 \int_0^\eta [-w_i(\eta)] d\eta
\]

\[
w_i(\eta) = h_1 \int_0^\eta [-Mv_0 - 2v_i^2 + f_0 w_i] d\eta
\]

and

\[
C_i(\eta) = \frac{p h_2}{4K} \int_0^\eta [f_i C_0 - f_i^{'} \theta_0 + Ef_i^{''2}] 1 + \frac{1}{3}
\]

For \( m \geq 2 \),

\[
f_m(\eta) = (1 + h_1) f_{m-1}(\eta) + h_1 \int_0^\eta [-v_{m-1}(\eta)] d\eta
\]

\[
v_m(\eta) = (1 + h_1) v_{m-1}(\eta) + h_1 \int_0^\eta [-w_{m-1}(\eta)] d\eta
\]

\[
w_m(\eta) = (1 + h_1) w_{m-1}(\eta) + h_1 \int_0^\eta [-Mf_i^{'} + \sum_{i=0}^{m-1} (-2 f_{i-1}^{''} f_i^{'} + f_{i-1}^{'} f_i^{''})] d\eta
\]

and

\[
C_m(\eta) = \frac{p h_2}{4K} \int_0^\eta [f_m C_0 - f_m^{'} \theta_0 + Ef_m^{''2}] 1 + \frac{1}{3}
\]

\[
\theta_m(\eta) = (1 + h_2) \theta_{m-1}(\eta) + h_2 \int_0^\eta [-C_{m-1}(\eta)] d\eta
\]

(20)

(21)

(22)
\[ C_m(\eta) = (1 + h_2)C_{m-1}(\eta) + \frac{P_r h_2}{4K} \int_{\eta_0}^{\eta_{m-1}} \left[ \sum_{i=0}^{m-1} f_{m-1-i} \theta'_i - f_{m-1-i}'' \theta_i + Ef_{m-1-i}'' f_{m-i}'' \right] d\eta \]

The systems of Eqs. (19) - (22) have been solved using symbolic computation software MAPLE. It is found that

\[ f_i(\eta) = -h_i \eta \]

\[ v_i(\eta) = -h_i \alpha \eta \]

\[ w_i(\eta) = h_i (-2\eta - M\eta) \]

\[ f_2(\eta) = -(1 + h_1)h_i \eta + \frac{1}{2} h_i^2 \alpha \eta^2 \]

\[ v_2(\eta) = -(1 + h_1)h_i \alpha \eta - \frac{1}{2} h_i^2 (-2 - M) \eta^2 \]

\[ w_2(\eta) = (1 + h_1)h_i (-2\eta - M\eta) + \frac{1}{2} (Mh_i \alpha + 3h_i \alpha) \eta^2 \]

and

\[ \theta_i(\eta) = -h_2 \beta \eta \]

\[ C_i(\eta) = -\frac{P_r h_2 \eta}{1 + \frac{4K}{3}} \]

\[ \theta_2(\eta) = -(1 + h_2)h_2 \eta \beta + \frac{1}{2} \frac{h_2^2 P_r \eta^2}{1 + \frac{4K}{3}} \]
\[ C_2(\eta) = \frac{-(1+h_2)P_1h_2\eta}{1+\frac{4K}{3}} + \frac{0.50P_2h_2(0.70 + h_2\beta)\eta^2}{1+\frac{4K}{3}} \]

\[
\}

\[ f_m(\eta, \alpha; h_1)(m = 3,4,5,...) \quad \text{and} \quad \theta_m(\eta, \beta; h_2)(m = 3,4,5,...) \] can be calculated similarly. Then the series solution expressions by nHAM can be written in the form

\[
F_0(\eta, \alpha, h_1) = \sum_{m=0}^{\infty} f_m(\eta, \alpha, h_1)
\]

(27)

\[
\theta_N(\eta, \beta, h_2) = \sum_{m=0}^{N} \theta_m(\eta, \beta, h_2)
\]

(28)

we note that analytic expressions (27) and (28) contains two auxiliary parameters \( h_1 \) and \( h_2 \) as suggested by Liao (1992) and (2003) one can choose the values of \( h_1 \) and \( h_2 \) properly for \( h \)-curves which ensure the convergence of the series solutions. Using the boundary condition \( f' \rightarrow 0 \) as \( \eta \rightarrow \infty \) and \( \theta \rightarrow 0 \) as \( \eta \rightarrow \infty \) we get \( \alpha = 0 \) for \( M = 0 \) and also \( \beta = 0 \) for \( K = 0, M = 0, P_r = 0, E = 0 \).

fig 2: The \( h_1 \) curve
In figs. 2 and 3 the $\eta$-curves are shown for the range of admissible values of $h_1$ and $h_2$. Figs. 2 and 3 clearly indicate that the ranges for the admissible values of $h_1$ and $h_2$ are $[-5.6, 4.2]$ and $[-5.1, 3.1]$. Our calculations show that the series solution (27) and (28) converge in the whole region of $\eta$, when $h_1 = -0.7, h_2 = -1$.

III. RESULTS AND DISCUSSION

The system of ordinary differential equations (11) - (13) which have been solved numerically using nHAM as described by Hassan and El-tawil (2012). This method has been used to solve several boundary layer problems, we show the graphical results of velocity and temperature. Attention has been focused to the variations of $P_r, M, E$ and $K$. For this purpose Figs. 4 – 8 have been displayed. Fig. 4 shows the effect of magnetic number on velocity $f' (\eta)$. Figs. 5 – 8 elucidate the influence of Radiation number $K$, Magnetic number $M$, Prandtl number $P_r$ and Eckert number $E$ on the temperature $\theta (\eta)$. From the present study, the main findings can be summarized as follows:

- Increase in magnetic parameter $M$ have accelerating effect on velocity of the flow field.
- Increase in radiation parameter $K$, magnetic parameter $M$ and Eckert number $E$ retard the magnitude of temperature of the flow field and the thickness of thermal boundary layer.
- Increase in the prandtl number $P_r$ shows that there is a rise in the magnitude of temperature of the flow field and the thickness of thermal boundary layer.

fig 4: The Effect of Magnetic Parameter, $M$ on Velocity, $f'$ for $\alpha = 0, h = -0.7$
fig 5: The Effect of Radiation Parameter, $K$ on Temperature, $\theta$ for $Pr = 1, Ec = 0.2, M = 1$

fig 6: The Effect of Magnetic parameter, $M$ on Temperature, $\theta$ for $Pr = 1, Ec = 0.2, K = 1$
IV. CONCLUSIONS

Radiation on steady MHD boundary layer flow over an exponentially stretching sheet was investigated and its effects observed. The similarity transformations are used to reduce the partial differential equations into ordinary differential equations. Analytical solutions for the velocity and temperature distributions are obtained using a nHAM. It was found that the heat rate increases with Prandtl number $Pr$, but decreases with both magnetic parameter $M$ and radiation parameter $K$. Thus Magnetic and Radiation parameter brought about cooling effect on the sheet, because the higher the increase in the parameter reduces the heat rate.

REFERENCES


